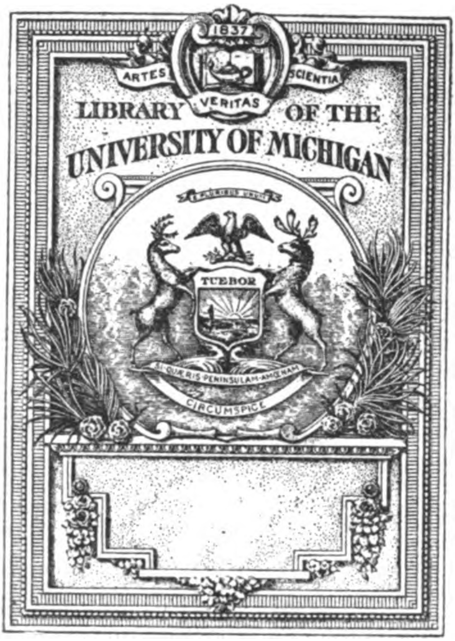
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THE GRID STRUCTURE IN ECHELON SPECTRUM LINES.

By Norton A. Kent and Lucien B. Taylor.

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THE GRID STRUCTURE IN ECHELON SPECTRUM LINES.

Norton A. Kent and Lucien B. Taylor.

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Some years ago Nutting [1] noted a peculiar, complex structure, termed by him the "fluting" or "grid," which appeared in many echelon spectrum lines, and consisted of several fine components of different and often changing intensity. Later one of us [2] independently noted this structure. Nutting crossed the 12″ Lummer plate of the Bureau of Standards with his echelon and was apparently forced to the conclusion that the structure was real — that is, that it indicated an actual discontinuity of emission in the source.

Proceeding on the assumption of reality, the writers attempted a solution of the problem using Liλ 6104 which, although known to be a spectroscopic doublet, offered peculiar advantages in that the grid was extremely brilliant, well-marked and persistent.

Apparatus.

The apparatus used consisted of:—

Two echelons: No. 1, made by Porter, 30 plates, each 14.76 mm. thick, step 1 mm., aperture 31.0 by 33.0 mm.; No. 2, made by Petitdidier, 30 plates, each 23.29 mm. thick, step 1 mm., aperture 31.0 by 35.5 mm.

The Bureau of Standards 12″ Lummer plate kindly loaned by Dr. Stratton.

A Hilger Lummer plate — length 131 mm., width 14.5 mm., depth 4.827 mm.

A Hilger constant deviation prism spectroscope combined with an echelon as in Figure 1a; also a separate Hilger spectroscope with another echelon spectroscope as in Figure 1b. The achromatic lenses of both echelon spectroscopes are of about 50 cm. focal length and 5 cm.

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1 Astrophys. Jour. 23, pp. 64 and 220. 1906.
2 Kent, Proc. Am. Acad. XLVIII, No. 5. Aug. 1912.

aperture; each echelon bed rotates on an axis at its center; the Hilger micrometer is fitted with one fixed and two movable cross-hairs as shown in Figure 2.

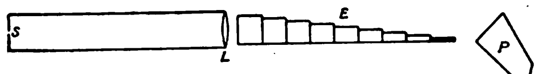


FIG. 1a.

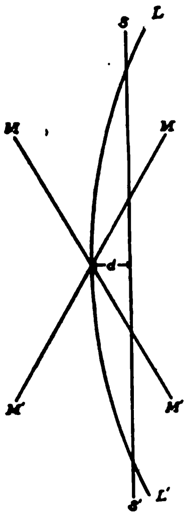
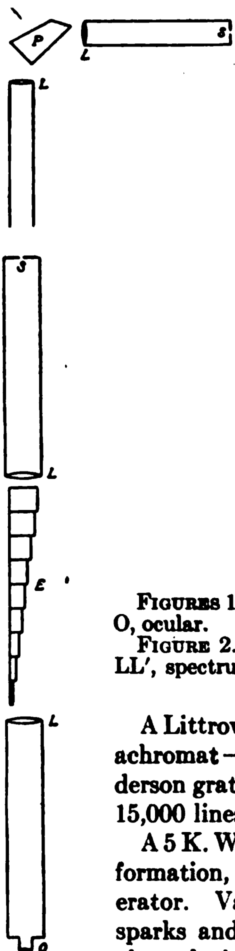


FIG. 2.



FIGURES 1a and 1b. S, slit; L, L, lenses; E, echelon; P, prism; O, ocular.

FIGURE 2. SS', fixed crosshair; MM', MM' movable system; LL', spectrum line.

A Littrow mount spectroscope consisting of a Petitdidier achromat — focal length 30 feet, aperture 6"; and an Anderson grating — aperture 3¾" vertical by 5" horizontal, 15,000 lines per inch, used in the third order.

A 5 K. W. transformer, 110 to 30,000 volts ratio of transformation, fed by a 60 cycle Holtzer-Cabot 4.5 K. W. generator. Various large induction coils capable of giving 6" sparks and operated by a rotary mercury break and an electrolytic interrupter, had proven insufficient.

FIG. 1b.

A vacuum arc of construction as indicated in Figure 3.

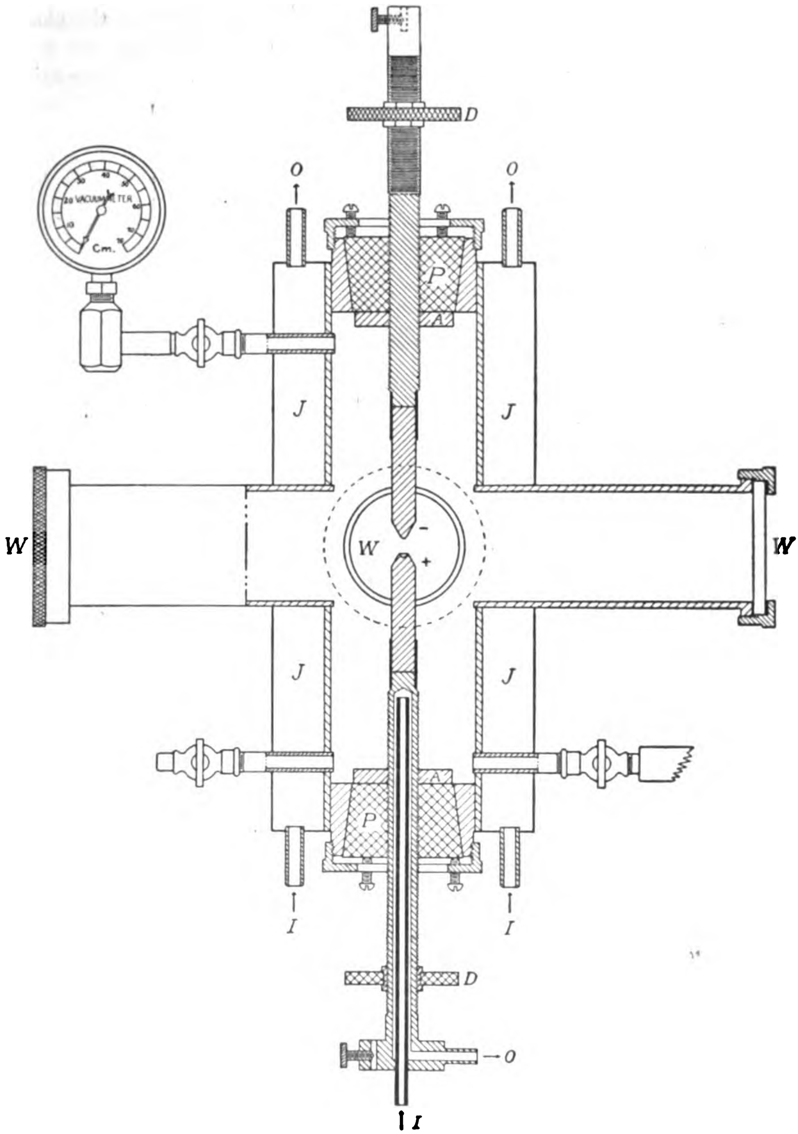


FIGURE 3. One fourth original size. D, D, fibre disks; O, O, water outlet; P, P, fibre plugs; A, A, asbestos; J, water jacket; W, W, W, windows; I, I, I, water inlet.

This was also adapted to pressures of several atmospheres as the glass windows and fibre plugs were held in place by threaded rings.

Quartz vacuum tubes — even pyrex glass having proven unsuit-

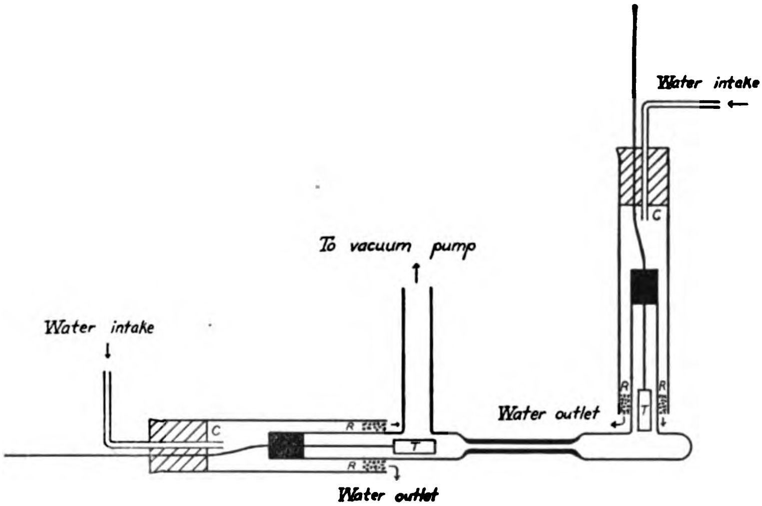


FIGURE 4. C, C, cork stoppers; R, R, rubber sponges; T, T, terminals.

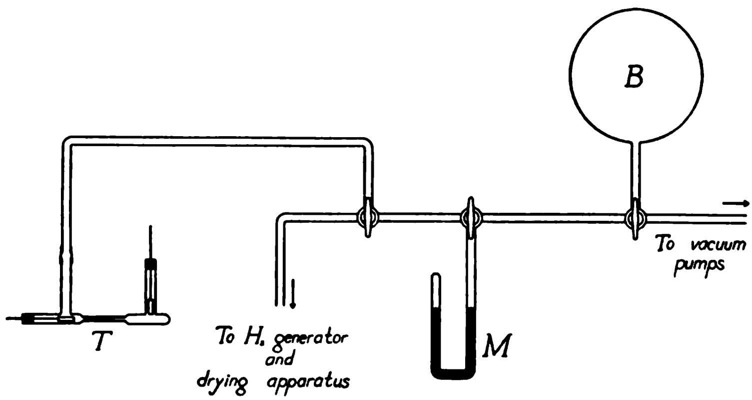


FIGURE 5. T, tube; M, manometer; B, bulb.

able — of various forms, the most successful of which, for salts such as lithium chloride, proved to be that shown in Figure 4 in which fine brass wire, often in helical form, was fitted into brass caps, 6 mm. in diameter, and sealed in with De Khotinsky cement, each joint being cooled by a water jacket. The salt is shoved into the capillary by a wire and the tube will run many hours without refilling. It may be used end on as well as side on. The capillaries varied from 2 to 0.5 mm.

Auxiliary apparatus as shown schematically in Figure 5. The bulb B, prevented too rapid changes in pressure. The system was washed out with hydrogen from a Kipp generator, dried by sulphuric acid and a calcium chloride tower. The mercury manometer, M, indicated the pressure — generally from 8 cm. to a fraction of a millimeter.

Procedure and Certain Results.

Both Lummer plates were each in succession crossed with echelon No. 1. In each case, with a carbon arc soaked with lithium chloride, both at atmospheric pressure and in a moderate vacuum, there appeared a pattern which, at this stage of the investigation, seemed to indicate that the grid was real. The following facts, (1) to (6), are, however, clearly not in accord with this conclusion, and prove conclusively that this curious structure is due to the phenomenon of "secondary maxima" observed by Stansfield[3] and resulting from successive reflections from the surfaces of the echelon plates, producing a Fabry and Perot system in the region of the primary light of the echelon. (1) to (3) deal with some of the criteria of echelon secondary maxima given by Stansfield. These criteria are, in essence, indicated below by italics.

(1) The width of Liλ 6104, given by an open carbon arc at atmospheric pressure, as seen in the Littrow grating, using a narrow slit, was found to be about 0.25 t. m. when echelon No. 2 showed the grid plainly. The suspicion, therefore, was confirmed that the line was too wide for the echelon, the difference between the adjacent orders being about 0.26 t. m. In the case of Janicki's observation[4] of Hg. λ 5461, Nutting's work on lines of many elements, and the work of one of us on the zinc lines as given by arc and spark, the indications are that with all lines for which the echelon shows the grid, their breadth is so great that the use of this instrument is not at all justifiable.

The writers then proceeded to study the structure from this new

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[3] Phil. Mag. (6) 18. 383. 1909.
[4] An. der Phys. Vol. XIX, p. 36. 1906.

standpoint, considering the primary line of width approximately 0.25 t. m., and not as formerly, one of the grid components itself.

These components are indeed, in this sense, each *narrower than the primary maximum* — 0.25 t. m.— the grid components, all of them now regarded as secondary maxima, being only about 0.05 t. m. in width in echelon No. 2.

(2) The curvature of one of the mercury yellow lines was compared with that of a grid component in λ6104. By stopping down the echelon spectroscope slit, a line of definite length was observed, and by setting the stationary cross-hair of the filar micrometer upon the ends of the image, and the movable system upon its center, the horizontal distance, d, Figure 2, from the ends of each line to its center were measured. It was found that the *curvature of the component is* about 25% *greater than that of the primary line.*

(3) With a small mirror, set at 45°, over the lower half of the echelon spectroscope slit an argon vacuum tube and the lithium arc were observed at the same time. The relative motion of the grid components in λ6104 and a nearby argon line were then studied as the echelon was rotated. The *primary* argon *line moves about one-half as fast as the grid components.*

Quantitative measurements of the relative displacements were later made with Zn λ4810. A quartz vacuum tube was fitted with coiled brass wire leads and brass terminals, exhausted, filled with hydrogen to 10 or more cm. pressure and then gradually exhausted to 1 mm. or less. The zinc lines given by the brass wire leads appeared very sharp, steady and brilliant. With λ4810, as thus produced, was compared the "gridded" line of a cored carbon arc at atmospheric pressure, in which small pieces of zinc had been placed, the small mirror arrangement allowing simultaneous observation of both sources. Upon rotation of the echelon the grid components rushed by the narrow tube line. To measure the relative speed a plane mirror was attached to a side of the echelon case. The image of an illuminated slit in a piece of cardboard was formed by a lens upon a distant scale after reflection from the mirror. The echelon was set near the $\theta = 0$ position. A reading of the position of the slit image on the scale was taken when the tube line lay upon the fixed hair of the filar micrometer. The echelon was then rotated until the slit image moved about 2 cm. The displacement of the tube line was then measured by the movable cross-hair system. A similar series was then taken with a grid line. The ratio of the displacements was 3.6 : 6.4 or about 1 : 2.

(4) The echelon was removed and the ocular focussed on the prism

image of a line. Replacing the echelon shortened the focus for a true narrow echelon image by about 0.6 mm. The focus for the grid components of the same line was 0.7 mm. shorter yet — the light forming the grid had traversed the echelon plates more than once.

(5) Although the grid components are generally very well defined (the minimum being "deep"), it is a difficult matter, with a fluctuating source such as an open arc, to obtain accurate measurements. The grid spacings appear to vary slightly at different stages. When the grid is complete the spacing is regular and, within the limits of error of measurement is equal to one fifth the distance between the orders. This was proven as follows:— A quartz tube having merely coils of fine brass wire as terminals gave extremely fine zinc lines. Echelon No. 2 was set in double order condition and Δo, the difference between the two orders, measured for λ4810 (see Table I). Then the grid was measured as given by a 3 ampere open carbon arc. Three distinct series of readings were taken. Then the tube was again used. The accuracy of an individual setting was about 0.2% in Δo and about 5% in Δg. It thus appears that in this region, at least within 2%, Δo = 5Δg. The focus of the instrument was, of course, not changed, the difference between that for primary and secondary maxima being so slight that distances between the components of the grid are not appreciably affected.

TABLE I.

Distances are measured in divisions of the micrometer head. Each Δo distance given is the mean as calculated from four settings; each Δ_g from two. Settings were made on the six centrally situated grid components.

Δo for Zn λ 4810		Δg for six grid components.						
		1–2	2–3	3–4	4–5	5–6	Mean	Mean of Means
22.60	Mean	4.3	4.8	4.6	4.8	4.7	4.6	
	22.58	4.8	4.4	4.6	4.8	4.1	4.5	4.6
22.50		4.3	5.0	5.0	4.3	4.7	4.6	
22.58 ÷ 4.6 = 4.9 or Δo = 4.9 Δg								

A similar series for Zn λ6362 gave Δo = 27.65 and Δg = 5.6, 5.9, 5.5, 5.6, 5.4: mean = 5.5. Hence Δo = 5.0 Δg.

(6) The structure given by both echelons is the same. That is, there are five secondary maxima for every primary maximum. Δg for Hydrogen $\lambda 6563$ is 0.061 t. m. for instrument No. 2, while for No. 1 it is about 0.096 t. m., which again is another fact fatally inconsistent with the existence of a definite discontinuous emission in the source.

With this evidence at hand the writers then attempted to clear up the results of the crossed dispersions. The source previously used was hardly adequate. By removing the soft core of the lower carbon it was possible to feed copiously into the arc a strong LiCl solution. Greater brilliancy and steadiness were obtained. The results were unmistakably in accord with the facts given in (1) to (6) above. Figures 6a and 6b indicate the structure observed. These will be

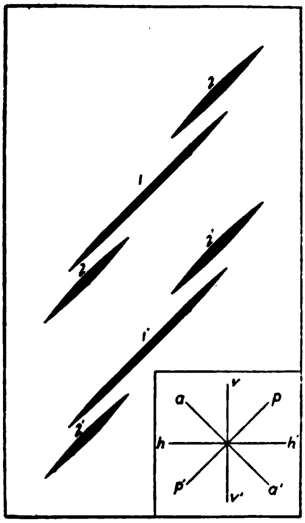

Fig. 6a.

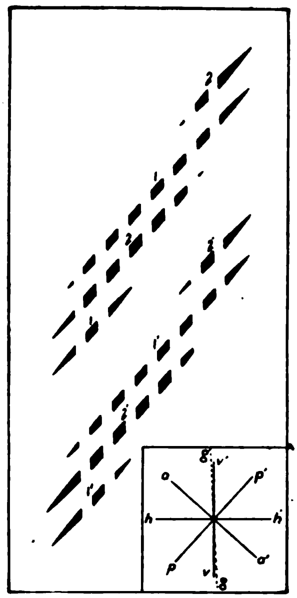

Fig. 6b.

Figure 6a. Li $\lambda 6104$ with crossed Lummer plate and echelon. Grid not indicated. λ_1 and λ_2 in single and double order condition respectively.

Figure 6b. Li $\lambda 6104$ as in Fig. 6a. Grid shown. λ_1 and λ_2 both between single and double order condition.

discussed in full below (see page 16). When one component of the spectroscopic doublet is in double and the other in single order condition three lines appear; when both are in a condition between single and double order there are four lines. It is probable that these four lines, under conditions of inferior illumination, were interpreted as four separate and true lines. It is unfortunate that at first the only line available for study was a doublet. With this latter and better source a zinc chloride solution gave λ4810 sufficiently strong. The crossed dispersions prove it to be a simple, though broad, single line when the echelon alone shows the grid.

Further Results: Characteristics of the Grid.

(a) From numerous observations upon Li λ6104 and Zn λ4810, as developed by various sources, such as vacuum tubes and arcs (on 110 and 220 volt D.C. circuits and from 1 to 20 amperes) under high, normal and low pressure, in which the cross-hairs of the filar micrometer were set successively upon the true, narrow, lines given by the tube and the grid components given by the arc, it is quite certain that the grid is built up approximately as follows:— Suppose that in a hypothetical grating of resolving power and dispersion equal to that of the echelon, a line which is at first very narrow, e.g., 0.025 t. m., gradually becomes less monochromatic, owing to changing conditions in the source, and appears as represented diagrammatically by the small letters *a* to *e*, Figure 7. Four cases must be discussed as shown in Figures 7 to 10, respectively.

Case I:— The echelon in double order condition gives successively images *A* to *E*. When the line is very narrow the echelon shows it as such, in *A*. Similarly for a line of width, Δg — the width of a grid component or an intergrid distance — it is shown as in *B*. When of width 3Δg, the echelon shows no change, *C* appearing as *B*; for, at *m*, the primary and secondary action together give a decided minimum. When the line has a width, as in *d*, the echelon shows a triplet, *D*, and when of width as in *e*, or greater, the grid is complete — five grid maxima, 1 to 5 and 6 to 10, for each maximum, such as 3 and 8, which a narrow line would give; four maxima, 4 to 7, between the double order positions, 3 and 8, of such a narrow line. For a given position of the echelon these grid components do not, in forming, move very much, if at all: they come up in situ. There exists an *apparent* motion, in and out, which is probably due to the changing width of the primary line, which may not at all times be such as to complete the entire width of a grid component.

Case II:— When the position of the echelon, its temperature and the wave length of the line observed, result in the central grid minimum

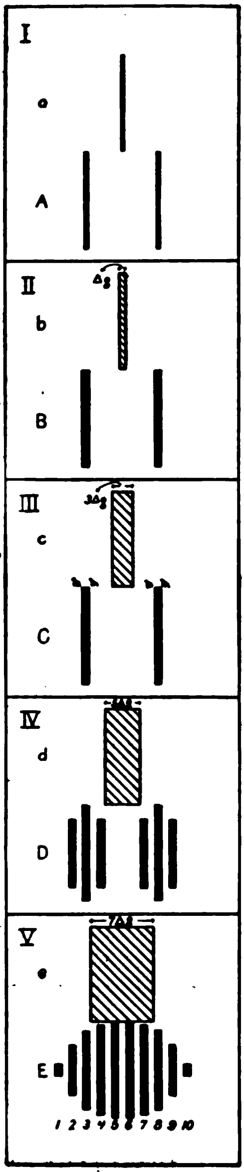

Fig. 7.

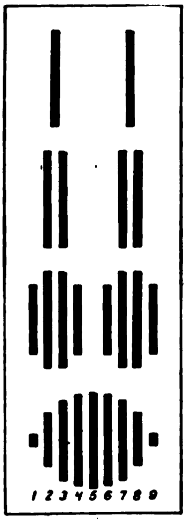

Fig. 8.

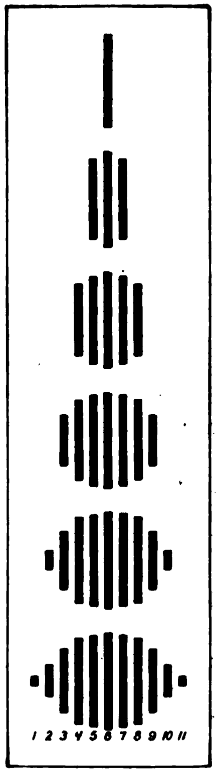

Fig. 9.

Figure 7. Case I: Echelon in double order condition and a grid maximum coincident with the primary maximum.

Figure 8. Case II: Echelon in double order condition and a grid minimum coincident with the primary maximum.

Figure 9. Case III: Echelon in single order condition and a grid maximum coincident with the primary maximum.

occurring in the position of the narrow tube line, as in Figure 8, for a double order condition of the echelon, there are nine or even eleven components when the grid is strong. Note that the grid components 2, 3, 7, 8, which at first are very brilliant when the grid is "young," grow weaker, 2 and 8 often being so faint that it is difficult to make accurate micrometer settings upon them.

Case III:— The treatment is the same for a single order condition of the echelon, as in Figure 9 which shows a triplet, quintuplet, or, with neighboring parts of adjacent orders, even as many as eleven components.

Case IV:— Here a grid minimum coincides with the primary maximum and the grid components are as shown in Figure 10.

The above statements explain why an originally narrow line, as its width increases, may appear, as it actually does, a triplet or quintuplet, as in Figures 7 and 9, or may, as it were, "reverse" and then quadruple, as in Figures 8 and 10. Actual reversal as shown by the grating probably occurs much later in the history of the line. (See page 15.)

Further, if a line be intrinsically unsymmetrical, shading off to the red for instance, the secondary action masks an early stage of broadening, and the left grid line, 2, forms as in *A*, Figure 11. Line 3, as in *A′*, then comes up as 2 strengthens.

(b) The grid begins to disappear and the line gradually becomes broad and structureless when the primary line exceeds 2Δo in width, Δo being the distance between two adjacent orders. This was determined as follows:— Using as narrow a slit as possible, a low power ocular and a mm. scale, an eye estimate was made of the breadths of various portions of an arc line shown by the grating. These were reduced to t. m. The same source was viewed simultane-

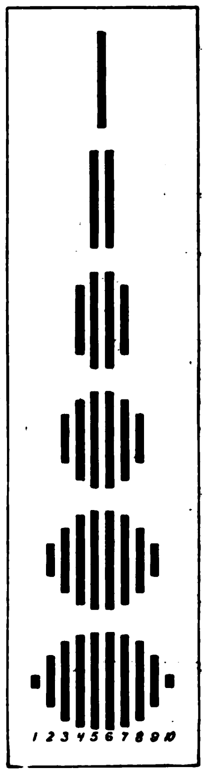

Fig. 10.

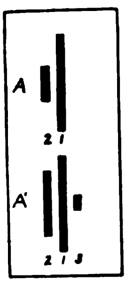

Fig. 11.

Figure 10. Case IV: Echelon in single order condition and a grid minimum coincident with the primary maximum.

ously by echelon No. 2. For Zn λ4810 three components of the grid exist when the grating shows a line 0.12 t. m. broad. Δo for λ4810 = 0.155 t. m. $\frac{3}{5} \times 0.155 = 0.09$ t. m. which compares favorably with 0.12 t. m. The complete grid exists when the line is 0.3 t. m. or 2Δo broad and the image begins to pass into a structureless line at 3Δo. Similarly for Li λ6104 a full and well-marked grid exists at a line width about 0.2 to 0.5 t. m. or Δo to 2Δo (as here Δo = 0.25 t. m.). The grid is poorly marked above about 2Δo and is gone at 3Δo.

(c) Numerous lines in the spectra of Na, Hg, Fe, Mg, Cd, Ca, Sn, Pb, and Bi, developed by an open carbon arc, show the grid whenever the line is sufficiently broad — rendered so by introducing more of the substance or increasing the current; also by increasing the capacity in the case of a spark.

(d) Li λλ6708 and 6104, Zn λλ4810, 4722 and 4680, also Hg λ5461 (mercury being fed into the lower cored carbon) show by their behavior that a line which is too broad will appear structureless in the echelon, that the center of the core of an arc may show the grid complete while light from the wings of the image gives a simple structure of but one to three components. With a sufficient amount of vapor the complete grid may be obtained even at low pressure.

(e) A study of Zn λ4810, from an arc in the vacuum or pressure tank, at pressures from 2 cm. of mercury to about three atmospheres, showed that moderate changes of pressure do not produce measurable displacements in the grid components, but merely alter somewhat their relative intensities, shifting the maximum over one or two components or even bringing up new ones. This of course means that, as long as a grid exists, the components do not change appreciably their position with changes of wavelength as small as 0.015 or 0.020 t. m.[5] Their position is affected more strongly by the position of the echelon and its temperature. Similarly, the grid components of the spectroscopic doublets Li λλ6708 and 6104 developed in vacuum tubes show intensity shifts with changes of pressure over the range of one atmosphere.

(f) The "end on" position of a vacuum tube will generally show a more complete grid than that "side on."

(g) If a line broaden unsymmetrically with increase of current the maximum of intensity will shift. Those components which are just being formed show an apparent motion outward as the number of

[5] According to Humphrey's and Mohler's results for Zn, the pressure shift reduced to λ 4000 is 0.057 t. m. for twelve atmospheres.

components increases, the first step resembling a narrow reversal as in Figures 8 and 10 or a central fixed line with two moving wings as in Figures 7 and 9. But the writers feel that this apparent motion is due to the fact that each grid component is not formed *in toto* at once: the part which lies nearest the center of the system is formed first. Certain it is that this apparent motion ceases abruptly when the component has reached a position which is one grid distance from its neighbor. If the source be an arc, many rapid fluctuations in intensity occur.

(h) Although the resolving power of the grating (225,000 in the third order) is far below that of the echelon (about 750,000 for λ6100 for echelon No. 2) it is hard to reconcile the images given by the two instruments on any other assumption than that the grid is due to secondary action.

To throw further light on the problem, Li λ6104, given by a vertical carbon arc soaked with LiCl, was viewed simultaneously by echelon and grating. Table II gives a summary obtained from various arrangements.

TABLE II.

|||| indicates the grid; ■ a broad structureless line; | a narrow unreversed line, or one very slightly reversed; ‖ a broad and strongly reversed line.

<table>
<tr><th>Arrangement</th><th>Sign of upper pole</th><th>Pole soaked with solution</th><th>Echelon shows</th><th>Grating shows</th></tr>
<tr><td rowspan="2">1</td><td rowspan="2">+</td><td rowspan="2">+</td><td>At + pole ||||</td><td>|</td></tr>
<tr><td>" − " ■</td><td>‖</td></tr>
<tr><td rowspan="2">2</td><td rowspan="2">+</td><td rowspan="2">−</td><td>" + " ||||</td><td>|</td></tr>
<tr><td>" − " ■</td><td>‖</td></tr>
<tr><td rowspan="2">3</td><td rowspan="2">−</td><td rowspan="2">−</td><td>" − " ■</td><td>‖</td></tr>
<tr><td>" + " ||||</td><td>|</td></tr>
<tr><td rowspan="2">4</td><td rowspan="2">−</td><td rowspan="2">+</td><td>" − " ■</td><td>‖</td></tr>
<tr><td>" + " ||||</td><td>|</td></tr>
</table>

Therefore which pole is soaked makes no difference, nor does it matter which pole is above. The region near the + pole generally shows the grid in the echelon, that near the − pole a broad structureless line. The grating always gives a narrow unreversed line or one very slightly reversed where the echelon shows the grid, and a strongly reversed line where the echelon shows no structure. Thus the grid does not result from conditions which produce a reversed grating line.

With Li λ6708, which usually appears widely reversed in the grating, the grid is more difficult to obtain in the echelon, while with Na λ4972 — given as an unreversed line by the grating at either edge or centre of the arc image — the echelon shows the grid at both edge and centre.

(i) We are now in a position to discuss in detail Figures 6a and 6b. These were obtained with the 131 mm. Lummer plate set between the collimator and prism of Figure 1b and crossed with echelon No. 2. The source was that described on page 10: the arc current being from 10 to 25 amps. The plate dispersed vertically, the echelon horizontally. Both figures are drawings based on visual filar micrometer measurements, a single cross hair being moved successively along the axes, *vv'* (vertical), *hh'* (horizontal), *aa'* (across the structure) *pp'* (parallel to it), as shown below the two figures.

Two Lummer plate orders are shown in each figure, the primes distinguishing these. The numerals indicate the two components of the spectroscopic doublet, the breadth along axis *aa'* their approximate relative intensity. λ_1 is the weaker line, λ_2 the stronger in both figures — λ_2 being the component of longer wavelength.

In Figure 6a λ_2 is in double order condition; in 6b both λ_1 and λ_2 are between double and single order. The echelon grid structure is not indicated in Figure 6a: in 6b its *approximate* position is shown. It was difficult to observe at the ends of the lines and so is not there indicated: it is slanted at an angle of about 2.4° (see *gg'* in Figure 6b) with the vertical. The slant of the lines themselves as well as that of the grid changes with the positions of both plate and echelon: further, the grid slant is not due to the curvature of the echelon image. This may throw some light on the disappearance of the grid at a breadth of line greater than $2\Delta o$. For, as the echelon action alone is given by the projection, on the *pp'* axis, of the grids of the lines λ_1 and λ_2, it is evident that lack of coincidence owing to slant would tend to obliterate the grid altogether, this indicating that two broad lines, the centers of which lie as far as 0.1 t. m. apart (the $\Delta\lambda$ of the two components of Li λ6104), may not give coincident grid structures; or, in other words, the grid maxima do not (for any one position and temperature of the echelon) necessarily fall together. This is not inconsistent with shift of intensity for small changes of wavelength (0.015 to 0.020 t. m.) as noted on page 14. Shift of intensity and position probably both enter with change of wavelength of the center of gravity of a primary echelon image.

These two figures show that the grid is unquestionably a secondary echelon action. Otherwise the regions between lines 1 and 2 would have been filled in with a structure along axis *aa'* similar to that along *pp'*.

With an echelon alone we have obtained only the weaker component of Li λ6104 as a single narrow line. We plan to cool the tube with liquid air, thus sharpening the stronger component so that it will no longer suffer the secondary action, to which the small satellite is probably due.

(j) We have no record of having observed in either echelon any ungridded line of width greater than Δg. Either there exists (1) a very narrow line, (2) an irregular series of such, as, for instance, in the yellow mercury lines, (3) a line of width Δg, (4) a series of such (the grid more or less complete) or (5) a broad, structureless image covering between one and two orders. And it appears extremely probable that the "reversal" of the main component of Hg λ5461, noted under certain conditions by several observers and often noticed by us, may be modified by the entrance of secondary action due to the excessive breadth of this component.

(k) The retardation producing the primary maxima of a narrow line is proportional to $n-1$, while that of the light undergoing secondary action is proportional to $3n-1$. Thus the difference in retardation in case of the two actions bears the ratio to the retardation of the primary of $\frac{2n}{n-1}$, which is a function of n alone. The value of $\frac{2n}{n-1}$ varies from 5.50 for λ6563 to 5.37 for λ4341 in echelon No. 2; and from 5.48 to 5.35 respectively in No. 1. Since echelons are generally made of substantially the same kind of glass, any two having equal separation of primary orders will have equal separation of secondary maxima, because this separation is the same fractional part of the separation of the orders; but the values of Δg in t. m., varying with the dispersion, will, of course, differ in different instruments.

We cannot state just why Δo = 5Δg. The measurements given above indicate that this is so within the limits of experimental error for both the violet and red regions.

It would be interesting to assemble an echelon under water, press the plates together and allow the superfluous water to drain off. This process might vastly reduce the secondary action. If successful Canada balsam might be substituted for water thus producing a more permanent instrument. We plan to try this experiment shortly.

Conclusion.

Summarizing the above results, we may state that the evidence is entirely against the existence of a discontinuity of emission in the source. The grid is due to a secondary action of the echelon which enters when the line under investigation is not sufficiently monochromatic. This means that the previous work of one of us [6] must be considered as of small value and also that an explanation of the apparent complexity of structure obtained by Nutting [7] can be found in secondary action.

The results obtained emphasize the fact that when an echelon is used to measure small wavelength differences, great care must be taken to obtain the lines so narrow that their width is less than $\frac{1}{8}\,\Delta o$, else secondary action may enter to cut off an edge of a line and thus give a false intensity-maximum position.

We must record our appreciation of the help rendered by various student assistants, especially Messrs. Greenleaf and Risga. We are also indebted to Dr. Lucy Wilson for her skilful aid during part of this research and to our assistants, Miss Pearson for mathematical work in connection with the calculation of the constants of the echelons, and Mr. Gilman for making the sketches accompanying this article.

We wish also to thank sincerely the Rumford Committee of the American Academy for numerous grants which made possible the purchase of the main pieces of apparatus used in this investigation.

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6 Proc. Am. Acad., Vol. XLVIII, No. 5. Aug. 1912.
7 Astrophys. Jour., XXIII, pp. 64 and 220. 1906.

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(Continued from page 3 of cover.)

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THE GENERAL CONDITIONS OF VALIDITY OF THE PRINCIPLE OF LE CHATELIER.

BY ALFRED J. LOTKA.

THE GENERAL CONDITIONS OF VALIDITY OF THE PRINCIPLE OF LE CHATELIER.[1]

By Alfred J. Lotka.

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The derivation of the principle of Le Chatelier from the laws of thermodynamics is familiar.

We may approach a converse problem. What, in the broadest terms, are the conditions which a system must satisfy in order that the principle shall apply to it? The interest of this problem arises from the fact that we have reason to suspect these conditions may prove broader than the domain within which the laws of thermodynamics are conveniently applicable.[2] We may therefore expect that a satisfactory solution of the converse problem may enable us to make rigorous application of the principle to systems to which, from lack of sufficient data it may be impossible, or from other causes it may be inconvenient to apply thermodynamic methods.

Consider a system whose state is defined in terms of a variable x and a paramenter G. The system is one of that class, the history of which follows a law

$$\frac{dx}{dt} = f(x, G) \tag{1}$$

(For example, it may consist of a mixture of $(A_1 - 2x)$ mols H_2O vapor, $(A_2 + 2x)$ mols of hydrogen, and $(A_3 + x)$ mols of oxygen at 2000 deg. C. in a rigid enclosure of volume G; A_1, A_2, A_3 being constants, namely initial masses). It is understood that other parameters besides G may enter into the function f, but it is unnecessary to set them forth explicitly, since in the reflections which follow only

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[1] Papers from the Department of Biometry and Vital Statistics, School of Hygiene and Public Health, Johns Hopkins University, No. 37.

[2] See Ehrenfest, Zeitschr. für phys. Chem. 1911, vol. 77, pp. 227, 244; Wolchonsky, Jl. Russ. Phys. Chem. Soc., 1912, vol. 44, pp. 305, 310; Chwolson, Lehrbuch der Physik, 1909, vol. 3, p. 547; Bancroft, Jl. Am. Chem. Soc., 1911, p. 92; Fournier d'Albe, Contemporary Chemistry, 1911, p. 38; Löwy, Kosmos, 1911, p. 331; Le Dantec, La Stabilité de la Vie, 1910, p. 25; L. Fredericq, Arch. de Zool. Exp. et Gén., ser. 2, vol. 3, 1885, p. XXV; Spencer, First Principles, chapter 22, section 173, Burt's Edition, p. 433. For further historical and bibliographic notes see Duhem, Traité d'Énergétique, 1911, vol. 1, pp. 523, 524.

changes in x and in one parameter G at a time will be considered, the other paramenters being constant.

According to (1) a stationary state (which need not be a true equilibrium in the thermodynamic sense) is defined by

$$0 = \frac{dx}{dt} = f(x_1, G) \tag{2}$$

where x_1 denotes the equilibrium value of x.

If the parameter G is altered by a small increment δG, the corresponding increment δx_1 in the equilibrium value x_1 of x is, in view of (2), given by

$$\frac{\partial f}{\partial x}\,\delta x_1 + \frac{\partial f}{\partial G}\,\delta G = 0 \tag{3}$$

$$\frac{\delta x_1}{\delta G} = -\frac{\partial f}{\partial G}\Big/\frac{\partial f}{\partial x} \tag{4}$$

1. Stable State.

If the stationary state defined by (2) is stable, we must have in the neighborhood of that state [3]

$$\frac{\partial f}{\partial x} < 0 \tag{5}$$

We can then distinguish two cases:

a.) $\frac{\partial f}{\partial G} > 0$. This means that the parameter G is one whose increase accelerates the transformation the progress of which is measured by x. In this case it follows immediately from (4) that $\frac{\partial x_1}{\delta G} > 0$. In other words, if the system is stable in the stationary state defined by (2), then increasing a parameter which accelerates the transformation will shift the position of the stationary state in the direction of increased transformation. From this alone, however, it does not necessarily follow that the new stationary state will actually become

[3] Condition (5) states that the velocity $f = \frac{\partial f}{\partial x}\,\delta x$ is always opposite in sign to the (small) displacement δx from equilibrium. This is evidently necessary for stability of equilibrium.

established. But, starting from the stationary state, at which $\frac{dx}{dt} = f = 0$, increase in G leads to a positive value of f. That is to say, a change actually takes place with a velocity directed towards the new stationary state, i.e. increased x.

b.) $\frac{\partial f}{\partial G} < 0$; i.e. increase in the paramenter G retards the transformation. Here it follows by similar reasoning that increase in G shifts the position of the stationary state towards diminished transformation. Furthermore, in this case the increment δG initiates a retrograde change, i.e. a change toward the new stationary state.

In both cases, (1.a) and (1.b), therefore, a change δG in the parameter G is followed by a transformation δx_1 towards the new stationary state, in the direction of the influence of the parameter G upon the velocity of transformation.

2. Unstable State.

Consider now the case in which $\frac{\partial f}{\partial x} > 0$. The stationary state defined by (2) is then unstable. A train of reasoning precisely analogous to that set forth above leads, in this case, to the conclusions:

(1) A change δG in the parameter G determines a shift of the stationary state in the direction opposed to the influence of the parameter G upon the transformation.

(2) The system, disturbed from existing stationary state by a change δG, moves, not towards, but away from the new stationary position.

Application to Influence of Initial Masses. Consider a transformation

$$S_1 + S_2 + \ldots + S_r \rightleftarrows S'_1 + S'_2 + \ldots + S'_s \qquad (6)$$

Let $\xi_1, \xi_2, \ldots \xi_r$ be the masses (expressed in mols) at time t of the components $S_1, S_2 \ldots S_r$; similarly let $\xi'_1, \xi'_2 \ldots \xi'_s$ be the masses of $S'_1, S'_2 \ldots S'_s$.

Let x measure the progress of the transformation from left to right, and let $p_i\, x$ be the amount (in mols) of S_i transformed from time $t = 0$ to time $t = t$.

Let A_i be the initial value of the mass (in mols) of some component

S_i which disappears in the transformation when x increases, and let A'_j be the initial value of some component S'_j which appears in the same transformation.

We have, according to (4),

$$\frac{\delta x_1}{\delta A_i} = - \frac{\partial f}{\partial A_i} \Big/ \frac{\partial f}{\partial x} \tag{7}$$

If we are dealing with a system of constant mass, we have an equation of constraint

$$m_1\xi_1 + m_2\xi_2 + \ldots = m_1A_1 + m_2A_2 + \ldots \tag{8}$$

where $m_1, m_2, \ldots$ are the molecular weights of the substances $S_1, S_2, \ldots$ From (8) we find by differentiation

$$\frac{\partial \xi_i}{\partial A_i} = 1 \tag{9}$$

so that we may write, instead of (7),

$$\frac{\delta x_1}{\delta A_i} = - \frac{\partial f}{\partial \xi_i} \Big/ \frac{\partial f}{\partial x} \tag{10}$$

From (10) it is seen that $\frac{\delta x_1}{\delta A_i}$ and $\frac{\partial f}{\partial \xi_i}$ are always of the same sign provided $\frac{\partial f}{\partial x}$ is negative, i.e., provided that the system is stable in the equilibrium defined by $f = 0$. That is to say, if the system is stable, and if adding a quantity of a component disappearing in the transformation increases the velocity of the transformation (at the previous equilibrium), then such addition will shift the equilibrium in the direction of increased transformation. In this case, then, the principle of Le Chatelier holds good.

On the contrary, by similar reasoning, it is found that if the addition of a quantity of a particular substance disappearing in the transformation retards the transformation, the principle does not hold as regards that substance.

Again, by similar reasoning, it is found that the principle holds or does not hold, according as the addition of a substance S'_j appearing in the transformation retards or hastens the transformation. We may therefore summarize the facts as follows:

1. Given that the system, at equilibrium, is stable with regard to changes in x, and that there is a relation of the type (8), an "equation of constraint" connecting the masses ξ and their initial values A, then the condition which must be satisfied in order that the Le Chatelier Principle may hold with regard to the effect of a change in the initial mass of some *one* component, is that the addition of such component shall accelerate or retard the transformation (at equilibrium), according as such component disappears or appears in such transformation.

2. Given that the conditions for the validity of the Le Chatelier Principle stated under (1) are satisfied for each and every component, then it is easily shown that the system is necessarily stable with regard to changes in x, so that the condition of such stability with regard to changes in x is automatically satisfied and does not need to be expressly stated. For, if $\frac{\partial f}{\partial A_i} > 0$ for every component which disappears in the transformation, and if $\frac{\partial f}{\partial A'_j} < 0$ for every component which appears, then, in view of (9), the same is true of $\frac{\partial f}{\partial \xi_i}$ and $\frac{\partial f}{\partial \xi'_j}$.

But

$$d\xi_i = -p_i\,dx \qquad (11)$$

$$d\xi'_j = +p'_j dx \qquad (12)$$

where p_i, p'_j are positive numbers, and

$$\frac{df}{dx} = \Sigma \frac{\partial f}{\partial \xi_i}\frac{d\xi_i}{dx} + \Sigma \frac{\partial f}{\partial \xi'_j}\frac{\partial \xi'_j}{dx} \qquad (13)$$

$$= -\Sigma \frac{\partial f}{\partial \xi_i} p_i + \Sigma \frac{\partial f}{\partial \xi'_j} p'_j \qquad (14)$$

which is necessarily a negative quantity if

$$\frac{\partial f}{\partial \xi_i} > 0, \quad \frac{\partial f}{\partial \xi'_j} < 0 \qquad (15)$$

3. It should be noted that the argument by which our conclusions have been drawn depends on the existence of equations of constraint, relations such as (8), connecting the ξ's and the A's. In the absence

of such constraints we are in no wise assured that the principle holds.[4] This must be clearly borne in mind in seeking to apply the Le Chatelier principle, for example, to biological systems. Thus, for instance, the malaria equilibrium under the conditions contemplated by Sir Ronald Ross,[5] is independent of the initial amount of malaria in the system (provided only this is not zero). This state of affairs arises out of the fact that there is no equation of constraint of type (8), in this case, connecting the initial amount of malaria with its status at any subsequent epoch.

Case of more than one variable. A somewhat more complicated case arises if the system under consideration is susceptible of several concurrent transformations, so that its state at any instant requires for its definition not one variable x, but a number of such variables.

It will suffice if we consider here the case for two variables x, y, as, for example, the case of a pair of consecutive reversible reactions

$$A \rightleftarrows B \rightleftarrows C \tag{16}$$

In this case we have

$$\frac{dx}{dt} = f_1 (x, y, G) \tag{18}$$

$$\frac{dy}{dt} = f_2 (x, y, G) \tag{19}$$

and equilibrium is defined by

$$f_1 = f_2 = 0 \tag{20}$$

Differentiating, in a manner analogous to that followed in the case of a single variable x, we have

$$\frac{\partial f_1}{\partial x} \delta x_1 + \frac{\partial f_1}{\partial y} \delta y_1 + \frac{\partial f_1}{\partial G} \delta G = 0 \tag{21}$$

$$\frac{\partial f_2}{\partial x} \delta x_1 + \frac{\partial f_2}{\partial y} \delta y_1 + \frac{\partial f_2}{\partial G} \delta G = 0 \tag{22}$$

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[4] For there is then no necessary relation between ξ and A, so that the derivative $\frac{\partial f}{\partial A}$ is no longer equal to $\frac{\partial f}{\partial \xi}$, but is indeterminate or meaningless.

[5] "The Prevention of Malaria," Second English Edition, John Murray, London, 1911, p. 679; Lotka, Nature, Feb. 1912, p. 497.

a system of linear equations, which we solve for $\frac{\delta x_1}{\delta G}, \frac{\delta y_1}{\delta G}$ and obtain

$$\frac{\delta x_1}{\delta G} = - \frac{\begin{vmatrix} \frac{\partial f_1}{\partial G} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial G} & \frac{\partial f_2}{\partial y} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{vmatrix}} \tag{23}$$

and a similar expression for $\frac{\delta y_1}{\delta G}$.

Condition of Stability. A general solution of (18), (19) can be written [6] in the form of exponential series

$$x = P_0 + P_1 e^{\lambda_1 t} + P_2 e^{\lambda_2 t} + P_{11} e^{2\lambda_1 t} + \dots \tag{24}$$

$$y = Q_0 + Q_1 e^{\lambda_1 t} + Q_2 e^{\lambda_2 t} + Q_{11} e^{2\lambda_1 t} + \dots \tag{25}$$

where λ_1 λ_2 are the roots of

$$\Delta(\lambda) = \begin{vmatrix} \left(\frac{\partial f_1}{\partial x} - \lambda\right) & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \left(\frac{\partial f_2}{\partial y} - \lambda\right) \end{vmatrix} = 0 \tag{26}$$

The condition for stability [7] of the equilibrium is that the real parts of all the roots λ are negative. This in turn demands that the absolute term $\Delta(0)$ be positive. But this absolute term is, evidently,

$$\Delta(0) = \begin{vmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{vmatrix} \tag{27}$$

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[6] A. J. Lotka, Proc. Am. Ac., 1920, p. 139.
[7] Idem, loc. cit., p. 144; Hurwitz, Math. Ann., 1875, vol. 46, p. 521; Blondel, Jl. de Physique, 1919, pp. 117, 153.

so that we must have, for stability,

$$\begin{vmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{vmatrix} > 0 \qquad (28)$$

In consequence, given stability of equilibrium, the sign of $\frac{\partial x_1}{\partial G}$ will be the same as that of the numerator in (23), i.e., that of the expression

$$\frac{\partial f_1}{\partial G}\frac{\partial f_2}{\partial y} - \frac{\partial f_2}{\partial G}\frac{\partial f_1}{\partial y} = D \qquad (29)$$

Example. Consecutive Reactions. By the way of example we may apply these results to the case of a pair of consecutive reversible reactions.

$$S_1 + S_2 + \ldots + S_r \rightleftarrows S'_1 + S'_2 + \ldots + S'_s \rightleftarrows S''_1 + S''_2 + \ldots + S''_t \qquad (30)$$

Let x denote the progress of the first reaction from left to right (so that, for example, a quantity, $p_i x$ of the substance S_i has been transformed at time t); and let y similarly denote the progress of the second reaction, from left to right.

Let us consider the effect upon x_1, the equilibrium value of x, of an increment $\delta A''_k$ in the initial amount of substance S''_k appearing as product of the second reaction.

We have, according to (23)

$$\frac{\delta x_1}{\delta A''_k} = - \frac{\begin{vmatrix} \frac{\partial f_1}{\partial A''_k} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial A''_k} & \frac{\partial f_2}{\partial y} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{vmatrix}} \qquad (31)$$

We shall assume stability, so that the denominator is positive.

In the numerator, evidently [8]

[8] If we exclude any possible catalytic influence.

$$\frac{\partial f_1}{\partial A''_k} = 0 \tag{32}$$

so that this numerator reduces to

$$\frac{\partial f_2}{\partial A''_k} \cdot \frac{\partial f_1}{\partial y} \tag{33}$$

Stability demands

$$\frac{\partial f_1}{\partial y} > 0 \tag{34}$$

On the other hand the principle of Le Chatelier would make

$$\frac{\delta x_1}{\delta A''_k} < 0 \tag{35}$$

This, by (31), in view of (32), (33), (34), will be true or not according as

$$\frac{\partial f_2}{\partial A''_k} \begin{matrix} < \\ > \end{matrix} 0 \tag{36}$$

Hence the principle of Le Chatelier holds good or not, as applied to the effect of A''_k upon x_1, according as

$$\frac{\partial f_2}{\partial A''_k} \begin{matrix} < \\ > \end{matrix} 0 \tag{37}$$

From this point on the discussion would follow essentially similar lines as in the case of a single dependent variable; it is therefore unnecessary to carry this out in further detail.

Influence of External Factors. We have hitherto tacitly assumed that (1), or (18), (19) are the only conditions for equilibrium, or, that, if there are any other conditions to be satisfied, these are in some way automatically taken care of.

In point of fact, in general, in addition to a condition of the form

$$\frac{dx}{dt} = f(x, G) \tag{1}$$

there will be further conditions of the form

$$H = H_e \tag{38}$$

where H is a parameter entering into the function f, while H_e is a parameter defining certain "external conditions." For example, H may be the pressure exerted by a gaseous mixture against an enclosure, and H_e may be the external pressure applied to a movable piston forming part of that enclosure. Here it is not enough, for complete equilibrium, that (1) be satisfied, but (38) also must hold.

Furthermore, the conditions (1) and (38) define equilibrium, but are insufficient to determine its stability, since they give us no information regarding the behavior of the system when $H \neq H_e$, i.e. when not in equilibrium with the external parameter H_e.

In order to settle this point we must have some further data. We are here interested in systems in which such additional data are furnished in the following manner:

In the case of these systems it is found that, in relation to the parameter G a certain parameter H having certain peculiar properties, can be defined by a relation.

$$\varphi\,(\xi_1, \xi_2, \ldots .. G, H) = \text{constant} \tag{39}$$

or its equivalent

$$\Psi\,(x, A_1, A_2, \ldots G, H) = \text{constant} \tag{40}$$

The peculiar property of G referred to above is as follows

$$\frac{dG}{dt} \gtreqless 0 \text{ according as } H - H_e \gtreqless 0 \tag{41}$$

It will perhaps be well, before proceeding any farther, to illustrate this by a concrete example. Consider the system

$$2\,H_20 \rightleftarrows 2\,H_2 + 0_2 \tag{42}$$

If ξ_1 is the mass of H_20 expressed in mols, ξ'_1 the mass of H_2 and ξ'_2 the mass of 0_2 similarly expressed; if V is the volume (parameter G) and if P is the pressure (parameter H) exerted upon the enclosure, then the equation (39) here takes the form

$$PV = (\xi_1 + \xi'_1 + \xi'_2)\,R\theta \tag{43}$$

where θ is the absolute temperature and R the general gas constant. Or, if A_1, A'_1, A'_2 are the initial masses of H_20, H_2 and 0_2 respectively, (expressed in mols), and x measures the progress of the reaction, as, for example, by the number of 0_2 mols formed, then evidently

$$\xi_1 = A_1 - 2x \tag{44}$$

$$\xi'_1 = A'_1 + 2x \tag{45}$$

$$\xi'_2 = A'_2 + x \tag{46}$$

so that (40) takes the form

$$\begin{aligned} PV &= \{(A_1 - 2x) + (A'_1 + 2x) + (A'_2 + x) \; R\theta \\ &= \{(x + A_1 + A'_1 + A'_2) \; R\theta \end{aligned} \tag{47}$$

In this case it is quite evident that the parameters P, V (corresponding to H, G of the general case) have the property defined by (41), which here appears as the characteristic property of the intensity factor and the capacity factor of an energy.

But for our present purposes we are not concerned with the question whether or not the parameters G, H defined for a given system are or are not factors of an energy. We must be prepared to deal with cases where this is either uncertain or actually known not to be true. All we need to know, for our purpose, is that the parameters G, H have the property defined by (41). An example may serve to illustrate the fact that this property may be shared by physical quantities not obviously related to energy.

Among the parameters on which the rate of increase of a human population depends is the area a occupied by them, since this determines the population density, which in turn influences the death rate in well-known manner,[9] and, presumably, in some degree the birth rate also.

Now there is an obvious relation between population density and ground rent. Regulation is effected about as follows: There is a certain demand for space, a desire for expansion, which may be measured by the rent H per unit area that the individual is willing to pay. On the other hand there is a certain market price H_e which must be paid to obtain accommodation. Now if $H > H_e$, i.e. if, on an average, an individual is willing to pay more than the market price, the population will spread over a greater area by renting more ground. If, on the other hand $H < H_e$ the individual is not willing to pay the market price, he will retrench, he will move from a six room apartment to a five room apartment say, and the area occupied by the population will contract. The parameter H_e functions, in fact, much like a "surface pressure," tending to compress the population into a smaller area. The most striking exhibition of this "surface-pressure" is seen in a great metropolis such as New York, where the population, a naturally two-dimensional structure spread like a film over the earth's surface, has been thrown into great creases towering 700 feet and more, 50 layers deep, above the street level.

[9] See, for example, Newsholme, Vital Statistics, 1899, p. 154.

It will be seen that in this case the internal parameter H and the corresponding external parameter H_e so determine changes in the area a that

$$\frac{da}{dt} \gtreqless 0 \text{ according as } H - H_e \gtreqless 0 \tag{48}$$

that is to say, the parameters H, a and H_e are related to each other and determine the course of events in a manner analogous to the intensity factor, the capacity factor of an energy, and the "applied force." But it is quite unnecessary to suppose that H and a actually are such factors of an energy in the example cited (population-spread); on the contrary, the writer is opposed to this view, which he has taken occasion elsewhere to discuss.[10] For our purposes it is quite immaterial whether P and a are factors of an energy. All we need know is that they enter into the condition (41) as there set forth.

Condition for Stability toward External Factor. Consider a system for which the condition for equilibrium with the environment is given by

$$\left.\begin{aligned} &H = H_e \\ &\varphi(G, H) = \text{constant} \\ &\frac{dG}{dt} \gtreqless 0 \text{ according as } H - H_e \gtreqless 0 \end{aligned}\right\} \tag{49}$$

Let

$$\varphi(G, H) = \text{const., i.e. } H = \chi(G) \tag{50}$$

be plotted as ordinates in a rectangular system in which G is plotted as abscissae. Then it is easily shown that the condition for stability of equilibrium is that the curve $H = \chi(G)$ must slope downwards from left to right.

For, suppose it sloped upwards. Let the system be in equilibrium at a point A_1 (Fig. 1), where

$$\left.\begin{aligned} H &= H_1 = H_e \\ G &= G_1 \end{aligned}\right\} \tag{51}$$

Suppose the system is in any way displaced to the point A_2 where

$$H_2 > H_1 \tag{52}$$

$$> H_e \tag{53}$$

10 "Economic Conversion Factors of Energy," to appear in a forthcoming issue of Proc. Nat. Ac.

Then

$$\frac{dG}{dt} > 0 \qquad (54)$$

Hence the point moves along the curve in the direction $A_1 A_2$, i.e. still farther away from equilibrium.

On the other hand, the same reasoning applied to a curve sloping downward from left to right shows that the system after displacement returns to its equilibrium position.

So, for example, the curves representing the relation between pressure and volume of a gas necessarily slope downward from left to right; the same is true of the demand and supply curves of economics. If it were true, as sometimes stated, that the more a man has, the more he wants, economic equilibrium would be an unstable condition.

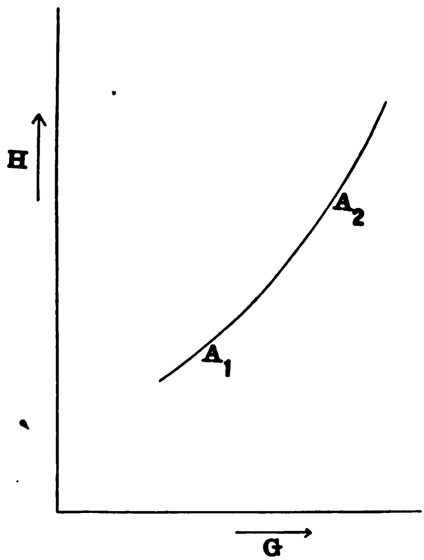

FIGURE 1.

External Stability and the Principle of Le Chatelier. Consider now a system which obeys the condition

$$\left.\begin{array}{l} \varphi(x, G, H) = 0 \\ \dfrac{dG}{dt} \gtreqless 0 \text{ according as } H - H_e \gtreqless 0 \end{array}\right\} \qquad \begin{array}{r}(55)\\(56)\end{array}$$

Let the system be stable towards H_e both when x is held constant and also when x is at the equilibrium value x_1 defined by

$$\frac{dx}{dt} = f(x_1, G) = 0 \tag{57}$$

This means that all the curves "of constant composition"

$$\left.\begin{aligned} \varphi(x, G, H) &= 0 \\ x &= \text{constant} \end{aligned}\right\} \tag{58}$$

and also the curve "of equilibrium composition"

$$\varphi(x_1, G, H) = 0 \tag{59}$$

slope from left to right downwards.

Now consider two neighboring curves of type (58) (curves of constant composition), which we will suppose solved for H and write

$$H_a = \Psi_a(G, x_a) \tag{60}$$

$$H_b = \Psi_b(G, x_b) \tag{61}$$

Suppose we start with the system in the state represented by the point Q, in internal equilibrium and also in equilibrium with an external parameter H_e (see Fig. 2).

Let x be changed at constant G, so as to increase H according to (55) until $x = x_b$, so that the representative point strikes the second curve of constant composition at R.

Since at the start of this operation

$$H = H_a = H_e \tag{62}$$

and at the end

$$\left.\begin{aligned} H &= H_b \\ &> H_e \end{aligned}\right\} \tag{63}$$

therefore the system is not in equilibrium with the external pressure H_e in the state represented by the point R, but equilibrium (for $x = x_b$) occurs at some other point T which must lie to the right of R along the curve of constant composition RT, since, whenever

$$H > H_e$$

G increases, in accordance with (56).

Furthermore, drawing a horizontal QS, T must lie below S, since the line of equilibrium composition QT must slope from right to left downwards.

It is clear therefore that lines of constant composition are steeper than lines of equilibrium composition.

It follows at once that if G be increased while the system is kept in equilibrium, so that the representative point travels along QT, then the change in x from x_a to x_b is that which at constant G increases H, or at constant H increases G.

But this is the principle of Le Chatelier. This principle therefore holds whenever the conditions (55) (56) are satisfied, and the system

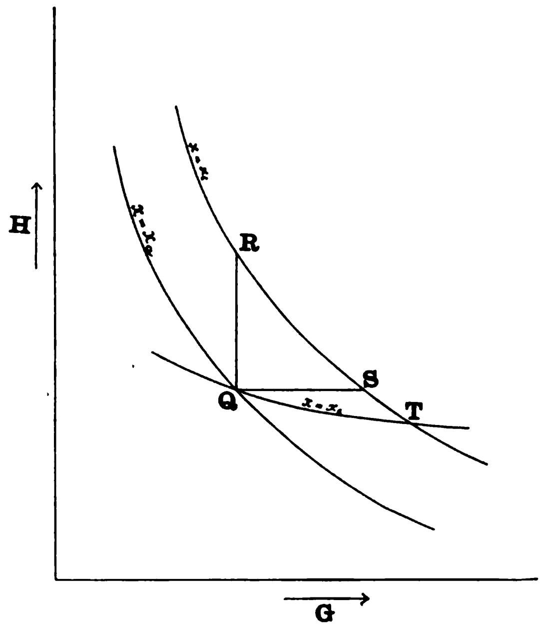

FIGURE 2.

is stable towards H_e both when x = constant and when $x = x_1$ (i.e. when x has its equilibrium value).

Similarly, it can be shown that if the conditions (55), (56) are replaced by

$$\left.\begin{array}{l} \varphi(x, G, H) = 0 \\ \dfrac{dG}{dt} \begin{smallmatrix}<\\=\\>\end{smallmatrix} 0 \text{ according as } H - H_e \begin{smallmatrix}>\\=\\<\end{smallmatrix} 0 \end{array}\right\}$$

while at the same time the system is stable towards H both when x = constant and also when $x = x_1$, then the Le Chatelier principle holds. In this case the curves $\varphi(x, G, H) = 0$ (x constant)

and $$\Psi(x_1, G, H) = 0$$

both slope from left to right upwards, and the curves of constant composition are again steeper than the equilibrium curves. An example of this type is that in which H is temperature and G is heat absorbed by the system (when $H < H_e$, G increases).

Finally, be it remarked that the results here deduced depend solely on *kinetic stability*, i.e. on the fact that the system when displaced from equilibrium has a velocity (rate of change of displacement) towards that equilibrium. The conclusions reached are therefore wholly independent of energetic (thermodynamic) consideration, since no reference whatever has been made to forces or energies or in any way whatsoever to the physical dimensions of the parameters involved.

This completes the present enquiry into the conditions of validity of the principle of Le Chatelier. It remains now only to point out the place which this communication occupies in the general plan of the series of investigations of which it forms part. This series of investigations has for its object the study of material systems evolving in accordance with a system of differential equations

$$\frac{dX_i}{dt} = F_i(X_1, X_2, \ldots X_n;\ A;\ P;\ Q) \tag{65}$$

in which the symbols X denote the masses of certain components S of the system, the symbol A has been written to denote collectively the initial values of the masses of certain components, the P's are parameters defining the state of the system (extension-in-space, topography, climatic conditions, etc.); and the Q's are parameters defining the character of the components S.

In a previous communication the kinetics of such a system were studied for the case in which the A's, P's and Q's are constant. This left open for discussion the effect of changes in these parameters. One phase of this subject has been dealt with by the writer elsewhere,[11] namely the effect of slow changes in these parameters. The present communication now extends the field of enquiry to the effect of

11 "Note on Moving Equilibria"; to appear in a forthcoming issue of the Proc. Nat. Ac.

changes of any kind in the parameters A or P, with this restriction, it is true, that we have, following Le Chatelier, interested ourselves solely in the ultimate effect upon equilibrium, leaving entirely out of consideration the path by which such equilibrium is reached. In many cases such partial information is of value, the equilibrium being the matter of chief interest, the path of the change of lesser practical importance. Herein lies the utility of such general principles as that of Le Chatelier, of Maupertuis, Lenz and others whose names have become linked with one or other form of reciprocal relations between the parameters defining a state of equilibrium. Since the application of such principles to biological systems has been essayed by various authors, without, however, any rigorous foundation upon which to build, it has appeared to the writer essential, for further progress in this field, to make a critical examination of the basis underlying these principles.

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THE EFFECT OF TENSION ON THE ELECTRICAL RESISTANCE OF CERTAIN ABNORMAL METALS.

By P. W. Bridgman.

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Introduction.

Under hydrostatic pressure the electrical resistance of most metals decreases. On the other hand the resistance of these metals increases under tension. There are, however, a few metals which are abnormal in that their resistance increases under pressure. It has been known for some years that bismuth is such a metal, and I have recently added to the number lithium, calcium, strontium, and antimony. It seemed to me of considerable interest to determine the effect of tension on the resistance of these metals. These data are presented in this paper. When I started this work, such measurements had not been published for any of these metals. During the course of my work, however, data were published in Italy for the effect of tension on the resistance of bismuth.[1] The resistance of this was found to decrease under tension, so that this metal is abnormal with respect to both tension and pressure. I have verified this result. I find that strontium is the only one of the remaining four which is also abnormal with respect

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1 E. Zavattiero, Rend. Accad. Lincei, 29 (1) 48–54, 1920.

to tension; the resistance of lithium, calcium, and antimony increases, as is normal, under tension.

When I started these measurements, only one metal was known which was abnormal with respect to tension; this was nickel. The measurements were made by Tomlinson [2] in 1876, on nickel which presumably had several per cent of impurity. Since it was my good fortune to obtain through the kindness of the Leeds and Northrup Co. some nickel of exceptionally high purity, I repeated the measurements of Tomlinson, and also extended them to find the effect of temperature and cyclic changes of tension. I have verified the sign of the effect found by Tomlinson, although there is not close numerical agreement, as was to be expected.

Since cobalt is in many respects closely related to nickel, I also determined the effect of tension on the resistance of it. The effects are entirely normal.

In addition to the five abnormal pure metals mentioned above, the alloys manganin and therlo are also abnormal with respect to their pressure coefficients. I have determined the tension coefficients of these also, and find them to be normal in sign, but to be very small.

My previous measurements of the pressure coefficient of resistance have suggested certain views as to the nature of the conduction mechanism.[3] In the following I shall discuss how far these new facts are in accord with these views.

This discussion demands a knowledge of Young's modulus. I have determined this for most of these metals.

Description of Method and Apparatus.

The apparatus and method were very simple, and for the most part were similar to those already adopted for the measurement of the effect of pressure.

The resistance was measured by a potentiometer method, the drop of potential between two potential terminals attached to the specimen being balanced against the drop due to the same current flowing through an appropriate combination of known resistances. The details of the apparatus were the same as those previously used in measuring the pressure coefficient, and have been fully described

[2] H. Tomlinson, Trans. Roy. Soc. 174, 1–172, 1883.
[3] P. W. Bridgman, Phys. Rev. 9, 269–289, 1917, and 17, 161–194, 1921.

elsewhere.[4] The wire under tension merely replaces the wire under pressure of the previous experiments.

The wires used were small, and a load of a few kilograms was sufficient in all cases. The wire was mounted vertically, and attached at the upper end to one arm of an equal arm balance, to the other arm of which known weights could be applied. For most metals it was most convenient to use the ordinary solid weights, but if there were hysteresis effects, as in the case of nickel, it was necessary to apply and remove the weight continuously; to accomplish this a water weight was used. In the case of antimony, because of its excessive fragility, a special arrangement was necessary which will be described in detail later.

The lower end of the wire was attached to a bracket, supported from above. This bracket, with the wire, dipped into an oil reservoir about 12 inches high and two inches diameter. The top and bottom of the reservoir were connected through side tubes with a turbine stirrer, by means of which a continuous stream of oil was maintained past the specimen. The wire itself was about 6 inches between potential terminals. In most cases the measurements were made at room temperature only, and the stirrer adequately maintained approximate equality of temperature. A correction could be easily determined and applied for the change in resistance due to drift of temperature of the oil bath. In a few cases, however, when more careful regulation of temperature was necessary, a large bath of water, maintained at constant temperature thermostatically, was raised around the oil bath. The walls of the latter were of thin brass, and exchange of heat between the oil and the water was sufficiently rapid to maintain constancy of temperature in the oil.

The magnitude of the tension applied was usually considerably less than the elastic limit. The behavior of the resistance gives a sensitive test of the perfect elasticity under the applied tension. The wire was usually seasoned by a number of applications of a tension higher than that of the final measurements. The behavior of the resistance beyond the elastic limit is complicated, and would make an interesting study on its own account. I felt this to be beyond the scope of the present work. I have, however, in nearly all cases determined at least the sign of the permanent change of resistance produced by exceeding the elastic limit, and in some cases have examined the phenomena a little more in detail. It appears that in all cases the permanent change of resistance beyond the elastic limit is an increase.

4 P. W. Bridgman, Proc. Amer. Acad. 52, 571–646, 1917.

For the theoretical considerations suggested by these measurements it is necessary to know the mechanical deformation produced by the tension, that is, to know Young's modulus. I have determined this for most of the metals. The metals were in most cases too soft to allow a direct determination by hanging a weight on the wire and observing the change of length, so that an indirect method was necessary. The method I used was that of flexure. A horizontal wire of known length and section was bent by a weight hung on the free end, and the amount of the flexure of the free end determined. From this Young's modulus can be calculated. The calculation assumes the perfect isotropy of the wire. It is probable that this condition is not always satisfied to as high a degree of approximation as would be desirable, but under the conditions it seemed the best that I could do. The modulus for manganin and therlo was determined directly.

The difficulties of the resistance measurements vary greatly for the different metals. The effect is in any event small, and for those metals which are soft and have a low elastic limit, the magnitude of the maximum effect is sometimes not greatly in excess of the order of magnitude of the accidental errors. An additional difficulty for the metals Li, Sr, and Ca is that of making electrical connections. These metals cannot be soldered, and mechanical spring clamps had to be used. The resistance at the contact would sometimes vary sufficiently to produce perceptible fluctuations in the main current, and also the potential terminals were sometimes subject to slight displacements under the stirring by the oil. The details of these difficulties will be described under the metals separately.

I am indebted to the skill of my assistant, Mr. J. C. Slater, for practically all the actual readings.

Here follows a description of the details for each metal.

Detailed Data for Individual Metals.

Lithium. The material was from Merck, for which I have no analysis. It was apparently entirely free from inclusions of slag, and mechanically homogeneous. Metal from the same lot was used in previous determinations of the pressure coefficient of resistance.[5] It was formed into wire of about 0.032 inches diameter by cold extrusion, in the usual way. The surface of the wire so formed is bright, and it

[5] P. W. Bridgman, Phys. Rev. 56, 59–154, 1921.

remained bright throughout the course of the experiment. The oil of the bath, a neutral heavy white petroleum which is used for medicinal purposes, was without perceptible chemical action, at least at room temperature. The current and potential connections were maintained by mechanical contact, the potential connections by small spring clamps of special design.

The measurements of lithium were the most difficult of any, because of the extreme softness of the metal. Four sets of runs were made in all. The first two, made without the thermostat, established the sign of the effect and its probable magnitude. The last two runs, with the thermostat at 30°, were somewhat more satisfactory. The elastic limit is so low that I could not determine the linearity of the effect within the elastic range. There was permanent stretch under a load of 40 gm., and there were also irregular initial effects under the first few grams of load, which may have been due to straightening of the wire. All the best measurements were made between a load of 15 gm. as zero and 35 gm. The change of resistance under this maximum increase of load of 20 gm. is an increase of only 0.02%. The extreme variation of the individual determinations for the best specimen was in the ratio from 7 to 20. Twelve determinations were made. The probable error of the mean, calculated by least squares, was 6.7%.

The tension coefficient of resistance, that is the proportional change of resistance under a tension of 1 kg/cm^2 was $+4.9 \times 10^{-5}$ for the best specimen; the other run with the thermostat gave 4.6×10^{-5}, and the only one of the preliminary specimens which was worth computing gave 4.7×10^{-5}. In the following I shall assume for the most probable coefficient $+4.8 \times 10^{-5}$.

Young's modulus was determined from the bending of three specimens, of the diameter given above and approximately 7 cm. long. The maximum load applied to these specimens was 0.066 gm. Within the limits of error the displacement was proportional to the load. The values obtained for Young's modulus were respectively 4.72, 5.26, and 4.93×10^{10} Abs. C.G.S. units. Take as the most probable mean 4.9×10^{10} C.G.S. or 5.0×10^4 in kg/cm^2.

The cubic compressibility of lithium has been found by Richards [6] to be 9.0×10^{-12}, pressure expressed in Abs. C.G.S. units. This may be combined with Young's modulus by the formula of elasticity $\sigma = \frac{1}{2}\left(1 - \frac{E\kappa}{3}\right)$ to find Poisson's ratio. The formula gives 0.42. The

[6] T. W. Richards, Jour. Amer. Chem. Soc. 37, 1643–1656, 1915.

high value is in line with our other experience that a comparatively soft metal has a Poisson's ratio near 0.5.

Calcium. This material I owe to the kindness of the Research Laboratory of the General Electric Company. It was from a different batch than that whose pressure coefficient of resistance I have previously measured.[7] So far as any chemical analysis can detect, all the calcium of the General Electric Co. contains no impurity, but in my previous discussion I remarked on the fact that there was nevertheless some difference between different batches. Some of the material can be extruded easily, while the extrusion of other is difficult. The wire which I previously used was extruded with difficulty, and was inclined to be brittle. The present wire was extruded easily, and could be readily bent into a comparatively short radius. The wire as supplied by the General Electric Co. had been extruded to a diameter of about 0.055 inches. In order to better adapt it to the magnitude of the tension which I could readily apply, I drew it down through steel dies from this size to 0.030 inches, first scraping the surface bright under oil. I did not attempt to anneal it after this drawing. Another piece, whose behavior beyond the elastic limit was specially examined, was drawn to 0.019 inches. The breaking load was at the rate of 1200 kg/cm^2; it was the same for the two sizes of wire. The break takes place with very little elongation or reduction of area.

Measurements were made of the elastic rate of change of resistance on two different samples. The range of tension was not more than one fifth of the breaking load. The effect with this metal is large enough to allow good readings. Within the limits of error the change of resistance is linear with tension up to the stresses mentioned above. Readings were made at eight or ten different loads. The maximum departure of any single observation from a straight line was 5% of the maximum change for one of the specimens, and 4% for the other. The maximum change of resistance was 0.14% of the initial value.

The tension coefficient of resistance was $+ 8.24 \times 10^{-6}$ for one specimen, and 8.50×10^{-6}, tension in kg/cm^2, for the other. Take as the most probable value the mean $+ 8.37 \times 10^{-6}$.

Young's modulus was determined from the bending of two samples, whose dimensions were of the same order as those of lithium. The maximum load applied was 0.24 gm. Within the limits of error the bending was proportional to the load. One specimen gave for Young's modulus 2.080×10^{11}, and the other 2.065×10^{11}, in Abs.

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[7] Reference 5, p. 91.

C. G. S. units. Take as the average 2.07×10^{11} C. G. S., or 2.11×10^{5}, tension in kg/cm^2. Richards [6] has found for the cubic compressibility 5.7×10^{-12} Abs. C. G. S. Combined with Young's modulus by the formula of elasticity gives for Poisson's ratio the value 0.303.

There is a very considerable range of tension above that of perfect elastic behavior and below the breaking point where there are departures from linearity and hysteresis. The immediate effect of exceeding the elastic limit is to permanently increase the resistance. The wire may be seasoned for any particular range of tension beyond the elastic limit by repeated application and removal of the load. After seasoning, the resistance, as a function of tension, described hysteresis loops exactly similar in appearance to the familiar hysteresis loops of the relation between tension and elongation. The width of the loop in the extreme case of a tension just below the breaking point may amount to one third of the maximum change. This maximum change of resistance was 1.73% for one specimen, and 2.43% for the other. The average coefficient of resistance for the extreme hysteresis loop was 20.5×10^{-6} for one specimen, and 15.2×10^{-6} for the other, against a coefficient in the elastic range of 8.4×10^{-6}.

Strontium. This material was from the same lot as the specimen whose pressure coefficient of resistance was previously determined. I am indebted for it to Dr. B. L. Glascock. The probable purity, and some of its properties have been already discussed.[8] The metal was formed into wire by extrusion in the manner already described. Two sizes were used, 0.035 and 0.019 inches in diameter.

Two samples were used; the first was not geometrically perfect and did not give as good results as the second. Two series of measurements were made on the second. The diameter of this was 0.019 inches. The wire breaks without much preliminary yield at a load of about 800 gm. At a load of 700 gm. there was a very small permanent increase of resistance. The tension coefficient of resistance was determined through a range of 400 gm., readings being made at eight different loads within this range. The effect of tension is to *decrease* the resistance. Within the limits of error the relation between tension and decrease of resistance is linear. The maximum departure from the straight line of any single observed point was 7% of the maximum change. The maximum change of resistance was 0.14% of the initial resistance.

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[8] Reference 5, p. 96.

The tension coefficient of resistance given by the two runs on the better specimen was -8.2 and -8.4×10^{-6} respectively for a tension of 1 kg/cm^2. The other specimen, which was very much more uncertain in its indications, gave a coefficient of -10.5×10^{-6}. Take as the best mean -8.3×10^{-6}.

Young's modulus was determined from the bending of two samples. Measurements were also made on the sample of the imperfect resistance measurements, but these were discarded because of the uncertainty introduced by failure of geometrical regularity. The dimensions were approximately the same as for lithium. The maximum load was 0.066 gm. and within the limits of error the relation between bending and load was linear. The first specimen gave for Young's modulus 1.24×10^{11}, and the second 1.36×10^{11} Abs. C.G.S. units. Take as the best value 1.30×10^{11}, or 1.33×10^{5} when tension is expressed in kg/cm^2. The compressibility of strontium has never been determined experimentally so far as I am aware. A probable value for it may be found by interpolating in Richard's chart [9] giving the compressibility as a periodic function of the atomic weight. The value which I have assumed for the compressibility is 6.5×10^{-12} Abs. C.G.S. The value of Poisson's ratio computed with these values for Young's modulus and compressibility is 0.359. An error of 1% in the compressibility changes Poisson's ratio by 0.5%.

The behavior of the resistance beyond the elastic limit was not investigated, except to establish that there is a permanent increase in resistance on exceeding the limit. The permanent change is therefore of the opposite sign from the elastic change.

Antimony. So-called chemically pure antimony from the J. T. Baker chemical company was used. It was extruded into wire in the way previously described.[10] This wire is not from the same source as that whose pressure coefficient of resistance was previously measured, which was from Eimer and Amend. The present material has also been used in a determination of the effect of pressure on thermal conductivity, and the data will be given elsewhere.

Great difficulty was experienced with antimony because of its extreme brittleness, and a special procedure was adopted. At the suggestion of Mr. Slater, who made the measurements, the vertical position of the other metals was replaced by a horizontal position for antimony. The wire was placed in an oil bath, without a stirrer, resting on a massive bar of copper. Tension was applied by a spring

[9] Reference 6, p. 1649.
[10] P. W. Bridgman, Phys. Rev. 9, 138–141, 1917.

balance connected to the antimony by a wire passing through a stuffing box in the side of the oil bath. This stuffing box was very loose, and was without appreciable friction. The current and potential terminals were soldered to the antimony wire. It is very difficult to make a soldered connection to this brittle material which shall be sufficiently in axial alignment to permit the application of an appreciable tension. The wires by which the tension was applied were cemented to the antimony wire by DeKhotinski cement.

Readings were made on three different samples; those on the first were the best. The diameter of the wire was 0.0146 inches. Three runs were made on it; the first two were rough in character and agreed within the limits of error with the third run. Within the limits of error the effect is linear with tension, and is positive, the resistance increasing with increasing tension. The maximum load applied to this specimen without rupture was 80 gm.; it broke at 90 gm. Readings were made at eight loads. The greatest departure from the straight line of any single observation was 8% of the maximum effect, which was a change of resistance of 0.025 %. The tension coefficient of this sample was $+4.5 \times 10^{-6}$ for 1 kg/cm^2 tension.

The second sample was extruded at a different time, and was excessively fragile. Its diameter was 0.024 inches, and it broke at a load of 20 gm. Only two readings were made; not enough to establish the linearity of the effect or to eliminate chance errors. The tension coefficient which would correspond to the mean of the readings with this sample was $+ 20.0 \times 10^{-6}$.

The third sample was 0.0295 inches diameter. It was much stronger mechanically than the second sample, and allowed loads beyond the elastic limit to be applied. The results were much less regular than for the first sample, however. Within the limits of error the effect is linear with tension up to a load of 140 gm. Ten readings were made; the worst of these departed from the smooth curve by 33% of the maximum effect. The tension coefficient shown by this sample was $+7.5 \times 10^{-6}$ for a tension of 1 kg/cm^2.

In estimating the most probable coefficient, the first sample must be given considerably more weight; I take as the most probable coefficient $+5.0 \times 10^{-6}$.

I have previously determined Young's modulus for antimony;[10] The value 7.8×10^{11} Abs. C. G. S. units was found. It was possible to definitely establish that the wire was not homogeneous, because the rigidity bears an impossible relation to Young's modulus. It is therefore not allowable to apply the formula of elasticity to compute

Poisson's ratio. The above Young's modulus, combined with Richard's value for the compressibility, 2.4×10^{-12} C. G. S.,[6] would give 0.18 for Poisson's ratio, which is improbably low. I shall in the following use 0.30, which is an average value for many metals, as more probably correct.

The third of the specimens above allowed some examination of the behavior beyond the elastic limit. Permanent increases of resistance were produced by loads in excess of 140 gm.; rupture took place at 270 gm. The phenomena were somewhat unusual in that there were no time effects. The permanent change of resistance under a given load assumed at once its final value, and there was no creep, as is usually the case. This does not seem very surprising in so brittle a material.

Bismuth. Measurements were made on three different grades of material, of three different grades of purity. The first was ordinary commercial metal, which has about 3% impurity. The second was electrolytic bismuth, for which I am indebted to the kindness of the United States Metals Refining Co. Analysis showed only 0.03% impurity of silver, and only traces of anything else. In spite of this very small impurity, however, the temperature coefficient of resistance is only about one half normal; apparently the silver exerts some very large specific effect. The third sample was electrolytic bismuth of my own preparation, which I had made several years ago for a determination of the pressure coefficient of resistance.[11] I have no chemical analysis, but spectroscopic analysis by Professor F. A. Saunders shows less impurity of silver than the other electrolytic bismuth. I was not able to repeat the preparation of this material, and had only a small quantity available. I verified the high temperature coefficient on the special piece used in this work. The question of the curious behavior caused by the small quantity of silver is more fully discussed in my paper on the effect of pressure on thermal conductivity.

Tension decreased the resistance of all the samples of bismuth. The effect is comparatively large, and with little care it was possible to obtain very good sets of readings, with deviations from the smooth curves by individual points of not more than 1%. The effects beyond the elastic limit are complicated. There is a very considerable initial range, however, within which the effect is linear, and there is no evidence for departure from the usual relations of perfect elasticity. The coefficients quoted in the following were determined within this range.

Three determinations were made on commercial bismuth at room

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11 Reference 4, p. 624.

temperature. The diameter of the wire was 0.028 inches, and the range of tension 300 gm. The coefficients found were −4.65, −4.54, and -4.78×10^{-6} respectively, the unit of tension being 1 kg/cm².

Measurements were made on one sample of the commercial electrolytic bismuth at two different temperatures. The dimensions of the wire, and the range of tension were the same as for the commercial material. At 31.1° the tension coefficient is -4.27×10^{-6}, and at 0.0° 5.20×10^{-6}. It is perhaps surprising that the coefficient should be lower at the higher temperature, but this is also the case with the pressure coefficient of resistance.

One set of measurements was made on my own pure electrolytic bismuth at 30°. The diameter of the wire was 0.0207 inches, and the range of tension 100 gm. Within this range the effect is perfectly linear, and no reading departs from the straight line by as much as 1%. The total change of resistance under this load was 0.13%. The tension coefficient of this sample was -2.92×10^{-6}. This is considerably less than the coefficients of the other samples, but will be accepted in the following as the best value for pure bismuth.

Young's modulus of my pure electrolytic bismuth was determined in the regular way by the bending experiments on two samples. The diameter was as above, and the length about 6.5 cm. The maximum load was 0.10 gm. The bending is linear with load within this range. The two samples gave for Young's modulus 2.29 and 2.45×10^{11} Abs. C. G. S. units respectively. This is much less than the value given in Kaye and Laby's tables, which is 3.19×10^{11}. It is of course possible that this wire is not homogeneous, like antimony. Richards [6] has found the compressibility to be 2.8×10^{-12}. Combined with my value for Young's modulus this gives for Poisson's ratio 0.39; combined with Kaye and Laby's value it gives 0.35.

The phenomena beyond the elastic limit are complicated and would be worth study for their own sake. In the first place, the resistance increases beyond the elastic limit, and hence the change is in the same direction as for other metals which are normal with respect to the elastic effect of tension. The time effects are very large, and may continue for many days under loads which are much below the breaking load and so small as to produce no marked change in the geometrical dimensions. Under a fixed constant load, the resistance increases at a time rate gradually becoming less, according to some law which I did not attempt to discover. Commercial electrolytic bismuth showed creep for two days under a load of 350 gm. whereas at 300 gm. the effects were still elastic. In these two days the initial rate of creep had dropped to one quarter of its initial value, which was at the rate of an increase of resistance of $\frac{2}{3}$% per hour. After the removal of a load which has exceeded the elastic limit there are also time effects, not nearly so large in magnitude as the effects on applying load, but in the same direction; the resistance continues to increase for some time after the removal of load. In this particular the effect is like an ordinary elastic after effect.

Nickel. The sign of the pressure coefficient of this metal is normal; I have already mentioned in the introduction my reasons for repeating the measurements of Tomlinson. The metal was provided by the Research Laboratory of the Leeds and Northrup Co. in the form of bars approximately 6 cm. long, 1.5 cm. wide, and 3 mm. thick. I cut from this bar a piece of suitable dimensions, and drew it down in steel dies to 0.0101 inches. It was annealed after drawing by heating to redness.

The temperature coefficient of resistance between 0° and 100° was found to be 0.00634. This is higher than the highest previous value with which I am acquainted, 0.00618 by Fleming, and is evidence of the unusual purity.

Tomlinson had found that nickel is abnormal in two particulars. In the first place the resistance decreases under tension, and in the second place the decrease is not linear with tension, but passes through a minimum, so that at a tension of the order of two thirds of the elastic limit the resistance begins to increase with increasing tension.

I verified both these particulars of behavior, and made a few additional observations. If instead of increasing the tension steadily to the elastic limit, measuring the resistance as a function of tension on the way, as Tomlinson did, the wire is subjected to a seasoning for some fixed load by a number of applications and removals of the same load, and then a cycle of resistance measurements is made, it will be found that the decreasing measurements do not follow the increasing measurements, but an open hysteresis loop is described. This loop differs in one important particular from ordinary hysteresis loops. By hysteresis we usually mean an effect that for some reason lags behind. In this case, this would mean that with decreasing tension the resistances correspond to some greater tension on the increasing run. The curious fact here is that at the initial stages of the decreasing limb of the loop the resistance may be lower than that corresponding to any value of the tension on the increasing run. The mechanism of the loop must be something quite different from the ordinary hysteresis effects.

The range of tension through which the wire has been accommodated has a slight effect on the character of the loop; the tension of the minimum increases somewhat as the range is increased.

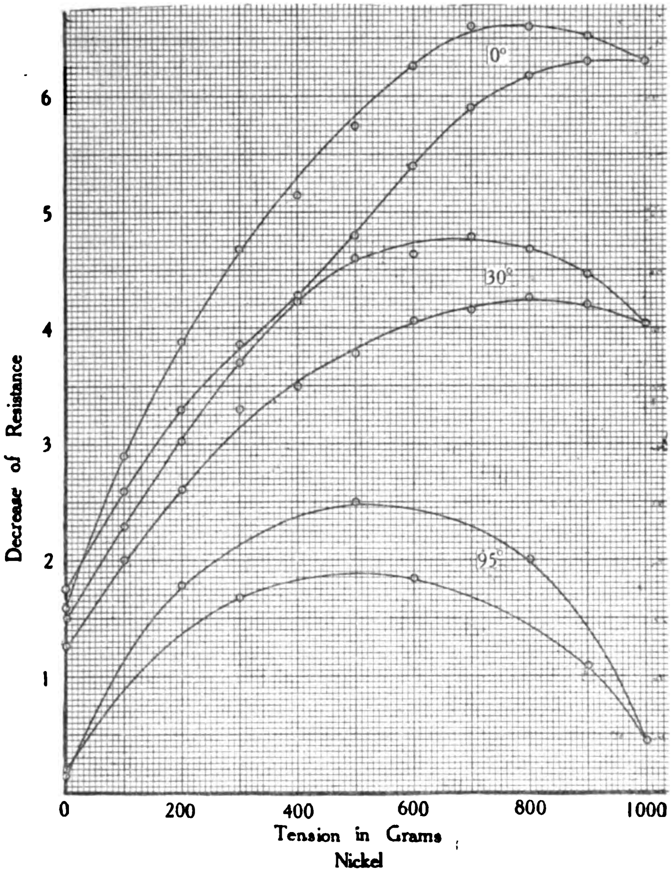

FIGURE 1. The decrease of resistance of Nickel in arbitrary units plotted against the tension in grams at three different temperatures. In each case the higher curve of the loop is that obtained with decreasing tension. One arbitrary unit of resistance corresponds to a decrease of 0.0345 per cent. The cross section of the wire was such that a load of 1000 gm. means 1900 km/cm².

There is a very large effect of temperature on the shape of the loop. I made measurements at 0°, 30°, and 95°. The minimum becomes much flatter and moves in toward smaller tensions at higher temperatures. The initial magnitude of the change, for small alterations of tension, becomes larger at the lower temperatures. Here again we find a temperature coefficient with a sign the reverse of what we would expect.

The changes of resistance as a function of tension are reproduced in Figure 1. The range of tension is the same at the three temperatures, namely 1 kg., which corresponds to about 1900 kg/cm^2. This is not far below the elastic limit.

Cobalt. An examination of the behavior of cobalt was of interest because of its close relationship to nickel, and the abnormal behavior of nickel.

There was available a piece of the same cobalt wire, 0.0030 inches diameter, as that on which the previous determinations of the pressure coefficient of resistance were made.[12] It was annealed by heating to redness. Measurements were made to a maximum load of 100 gm.; the breaking load is about 180 gm. Within the range of 100 gm. the effect is normal with respect to sign, and the relation is perfectly linear within errors of the single readings of not more than 2 or 3%. There is no trace of the abnormal behavior shown by nickel.

The tension coefficient is $+9.94 \times 10^{-7}$ at 30°, the tension being measured in kg/cm^2.

Young modulus was not determined for this wire; because of the small diameter a special procedure would have been necessary. I shall in the following computations assume that it is the same as for iron and nickel, namely 2.0×10^{12} C. G. S. I shall also assume that Poisson's ratio is 0.30.

Manganin. The interest of this alloy lies in the fact that its pressure coefficient of resistance is abnormal in being positive.[13]

Determinations of the tension coefficient were made on samples from the same spool as that from which the pressure coefficient samples were obtained and from which the pressure gauges used in all my work were cut. Measurements were made on the wire in both the hard drawn and the annealed condition.

Three runs were made on the hard drawn specimen. The wire was 0.0054 inches in diameter. The breaking load was about 900 gm. The maximum load applied before the measurements was only 300 gm., and the runs themselves reached a maximum of only 250 gm. The wire was seasoned for this range of tension by a number of applica-

12 Reference 4, p. 607.
13 P. W. Bridgman, Proc. Amer. Acad. 53, p. 370, 1918.

tions and removals of 250 gm. The effect was found to be normal in sign, the resistance increasing under tension. The effect is smaller than for some of the pure metals, and accordingly was not so regular. The maximum departure of any of the observed points from a smooth curve was 4.5% of the maximum change. The relation between tension and change of resistance is very nearly linear, but there are nevertheless consistent small departures from linearity greater than the errors of measurement; the change becomes proportionally greater at the greater tensions. The average tension coefficient over the entire range of tension is not more than one or two per cent greater than the initial coefficient.

The three runs on the hard drawn sample gave the following values for the initial coefficient respectively: $+5.78$, 5.72, and 5.63×10^{-7} for a tension of 1 kg/cm^2. The average, $+5.71 \times 10^{-7}$, is taken as the most probably correct coefficient.

A length of wire contiguous to the hard drawn specimen was annealed by heating to redness. Measurements were made on the first application of tension. 250 gm. was distinctly beyond the elastic limit. The specimen was seasoned by several applications of 250 gm., and then by a number of applications of 200 gm., until finally the changes of resistance had become steady. The initial coefficient of the annealed wire was $+5.88 \times 10^{-7}$. After accommodation to the range of 200 gm. the coefficient had increased to 6.75×10^{-7}. This latter is the average coefficient over the range of tension. After accommodation the relation between resistance and tension is not linear or single valued, but the relation involves hysteresis of the usual character, the maximum width of the hysteresis loop being about 10% of the maximum effect.

Young's modulus of this wire was determined directly from the increase of length under a given load. A piece about 3.5 m. in length was hung in an elevator shaft, and observations made on fiducial marks at the top and bottom with microscopes. The supports of the microscope were entirely independent of the supports of the wire, so they could not be affected by the load applied to the wire. The hard drawn wire was used for this determination. It was seasoned by a number of applications of 500 gm.; the maximum load for the determination of the modulus was 400 gm. There was a slight amount of hysteresis, but there were no perceptible time effects. The maximum width of the hysteresis loop was 5% of the maximum extension. The mean of points with increasing and decreasing tension lie on a straight line within the limits of error, which were not more than 0.3%.

The value found for Young's modulus was 1.35×10^{12} Abs, C. G. S. or 1.39×10^{6} when the unit of tension is 1 kg/cm^2. Neither the coefficient of cubic compressibility nor any other of the elastic constants of manganin seem to have been determined. I shall assume in the following that a probable value of Poisson's ratio is $\frac{1}{3}$.

Therlo. This alloy is made by the Driver Harris Co., and is much like manganin in its properties. Its pressure coefficient of resistance is also positive,[14] and is very close numerically to that of manganin.

Measurements were made on both hard drawn and annealed wire, as for manganin. Two samples of hard drawn wire were used. The range of tension was 250 gm. The diameter of the wire was 0.005 inches. Within this range the relation between change of resistance and tension is sensibly linear; the departure from linearity shown by the manganin was not in evidence here. One run was made on the first sample; the coefficient found for it was $+ 4.32 \times 10^{-7}$ for a tension of 1 kg/cm^2. Three runs were made on the second sample, giving 5.19, 4.80, and 4.84 for the coefficient. I take as the most probable value for the coefficient $+ 4.8 \times 10^{-7}$.

A piece was annealed by heating to redness. Readings were made of the resistance during a long series of applications of tension, through a maximum range of 320 gm. The breaking load of the annealed wire is about 350 gm.; that of the hard drawn is over 1 kg. The initial coefficient for small loads immediately after annealing was 4.22×10^{-7}. After seasoning as above, this coefficient had risen to 4.60×10^{-7}. There seemed to be considerably less hysteresis in the relation between tension and resistance than in the case of annealed manganin.

Young's modulus was measured in the same way as that of manganin. There was considerably greater departure from linearity and more hysteresis than was shown by manganin, and much more of both than was shown by the resistance measurements. Seasoning was by the application and removal of 500 gm., and the modulus was measured over a cycle of 400 gm.

The hysteresis loop for this range of tension had an extreme width of 12% of the extreme extension. If we take the initial slope of this loop as giving Young's modulus, we find 1.41×10^{12} Abs. C. G. S., or 1.46×10^{6} in kg/cm^2. As in the case of manganin, I shall assume that Poisson's ratio is $\frac{1}{3}$.

The behavior of the resistance beyond the elastic limit is compli-

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14 Reference 5, p. 135.

cated, but there seems to be nothing essentially unusual about it. The permanent effect is an increase of resistance. There are also slow changes with time, both on applying and removing the load, as is normal.

Discussion of Results, excepting Nickel.

The results obtained above are collected into Table I, in which is given the tension coefficient of observed resistance, the pressure coefficient of observed resistance, the cubic compressibility, the reciprocal of Young's modulus, that is, the extension under unit load, Poisson's

TABLE I.

Metal	Tension Coefficient of Observed Resistance	Pressure Coefficient of Observed Resistance	Cubic Compressibility	Reciprocal of Young's Modulus	Poisson's Ratio	Tension Coefficient of Specific Resistance
Li	$+48.\times10^{-6}$	$+6.8\times10^{-6}$	9.2×10^{-6}	$20.\times10^{-6}$	0.42	$+11.\times10^{-6}$
Ca	+8.37	+10.1	5.8	4.75	0.30	+0.8
Sr	−8.3	+48.0	6.4	7.5	0.36	−21.2
Sb	+5.0	+11.0	2.4	1.25	0.30(?)	+3.0
Bi	−29.2	+15.5	2.8	4.2	0.37	−3.65
Mang.	+0.59	+2.31	0.7(?)	0.72	0.33	−0.60
Therlo	+0.42	+2.37	0.7(?)	0.69	0.33	−0.73
Co	+0.994	−0.90	0.6(?)	0.5	0.30	+0.19

ratio, and the tension coefficient of specific resistance. The unit of stress, whether of tension or pressure, for which the various coefficients is given is the kg/cm². By the coefficient of "observed resistance" is meant the change of resistance, per kg. per cm², of the wire as actually measured in the experiments, with fixed electrodes. Such a wire increases in length and decreases in cross section under tension, and decreases both in length and section under pressure. The coefficient of "observed resistance," when corrected for the changes of dimensions, gives the coefficient of specific resistance, that is the change of resistance of a unit cube. The tension coefficient of specific resistance is obtained by subtracting $(1 + 2\sigma)/E$ from the coefficient of observed resistance.

Since the mechanism of conduction is an affair of atoms and electrons, and since the number of atoms and electrons in a unit cube both change when the material is subject to tension or pressure, it does not seem to me that there is a great deal of significance in the "coefficient of specific resistance." However, it is of interest to note in the table that the changes of dimensions of manganin and therlo are so large compared with the tension coefficient of observed resistance that the tension coefficient of specific resistance is negative, whereas the tension coefficient of observed resistance is positive. For the other metals the correction for change of figure is not large enough to change the sign of the coefficient of observed resistance.

It is in the first place to be remarked from the table that of the seven substances which are abnormal with respect to the sign of the pressure coefficient, only two, bismuth and strontium, are abnormal with respect to the sign of the tension coefficient. This would seem to indicate some essential difference between the conduction mechanism of these two substances and that of the others. Let us discuss what this difference may be in the light of the theory of metallic conduction which I have previously developed.

I have thought of conduction as due to a free path mechanism; the classical theory was a free path theory. The differences compared with the classical theory are these. In the first place, the free paths are thought of as long, because the free electrons are few in number. In normal metals, the paths of the electrons are to be thought of as through the substance of the atoms themselves. The path may be terminated when the electron makes the jump from one atom to the next. The chance of termination on making the jump will depend both on the amplitude of atomic vibration and the distance apart of the atoms. Now if the distance apart of the atoms varies little compared with the changes of amplitude, the variation of free path may be calculated in terms of the variation of amplitude only. The changes of amplitude, neglecting the effects due to changes of dimensions, may be calculated for changes of pressure and temperature, and so the change of path, and hence the changes of resistance may also be calculated. It is in throwing the entire burden of the variations on the free path, and in the method of computing the changes of the free path, that my theory differs mathematically from the classical theory. Now as a matter of fact, the changes of dimensions under changes of temperature are very small compared with the changes of amplitude, and the calculated changes of resistance agree well with the observed changes. Under changes of pressure the changes of dimensions are several fold larger, but still are so small compared with the changes of amplitude that an important part of the pressure coefficient may be computed. There is left an outstanding effect depending on the changes of dimensions. The precise value of this effect cannot be computed without a more detailed picture of the entire mechanism than we have at present, but we can at least see what its sign is; as the atoms are compressed more closely together there will be less difficulty for the electrons to make the leap, and the mean free path, and so the conductivity, will increase.

So much for the mechanism in the case of normal metals. This picture would lead us to expect a decrease of resistance with increasing pressure, as is normal. To explain the behavior of those abnormal metals whose resistance increases under pressure I believed that there might be two possibilities. In the first place there might be such abnormalities in the law of force between the atoms that the amplitude of vibration increases as the atoms are brought closer together, instead of decreasing as normal. I thought that this was probably the case with bismuth, and suggested that the same abnormality would explain the increase of volume on freezing. For such a metal the paths of the electrons are still to be thought of as through the substance of the atoms, and the interference with the free path to take place on making the jump from one atom to the next. A second possibility I thought to explain the behavior of lithium. Here the electrons occupy spaces in a lattice between the atoms, and conduction consists in motion of the electron lattice through the atomic lattice. This is similar to the view of Wien [15] and Lindemann [16] as to the general character of conduction; I believe that this can be the mechanism only in exceptional cases. The effect of pressure on such a mechanism is to constrict the channels between the atoms through which the electrons pass. A simple calculation will show that the constriction of the channels due to change of distance between atomic centers is much more than the opening of the channels due to decreased amplitude of atomic vibration. Hence the pressure coefficient of resistance of such a substance would be expected to be positive, as it actually is. With regard to the other abnormal metals,[17] I did not have any positive basis for deciding to which type calcium and strontium belong, although I expressed my belief that probably calcium belonged to the lithium type, and I believed that antimony belonged with bismuth on the basis of its expansion on freezing.

Since publishing my pressure data, the crystalline structure of cal-

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15 W. Wien, Columbia Lectures, 1913, 29–48.
16 F. A. Lindemann, Phil. Mag. 29, 127–140, 1915.
17 Reference 3, p. 183.

cium has been determined,[18] and this gives very strong probability to the view that its mechanism is also of the lithium type. It has been found that in metallic calcium the atoms of Ca occupy almost exactly the same positions as the Ca atoms in Ca F_2, the F atoms have merely dropped out of the structure. The inference is strongly suggested that the F atoms have been replaced by electrons, which do not give an X-ray photograph because of their small mass, and that therefore metallic calcium consists of interpenetrating lattices of atoms and electrons.

Let us now consider what these pictures of the mechanism lead us to expect for the tension coefficient. It is in the first place evident that we would expect the resistance of normal metals to increase in the direction of stretch, both because the distance between the atoms increases in this direction and because the amplitude of vibration in this direction increases to compensate for the decrease in period due to the weakening of the restoring force on the atoms due to their increased distance apart. A detailed working out of the theory must recognize in addition that changes in the positions of the atoms transversely may affect the period longitudinally. Now of course it is a fact that the resistance of normal metals increases in the direction of a tension. The same reasoning would lead us to expect a decrease of resistance in a direction transverse to the tension, and this also agrees with the facts in the few cases known.

Consider now bismuth. As tension is applied, the distance between the atoms increases longitudinally. The same abnormality in the force that compels an increase of amplitude when pressure is applied now compels a decrease of amplitude, and just as in the case of pressure the increase of amplitude causes a greater increase of resistance than can be overbalanced by the decrease due to the approach of the atoms, so now the decrease of resistance due to decreasing amplitude more than overbalances the increase due to the increasing separation of the atoms. The outstanding effect will be a decrease of resistance, which is actually the case.

It is otherwise, however, for metals of the lithium type. The resistance is here determined by the channels between the atoms. When a tension is applied, the channels are made narrower, because of the lateral contraction, just as they are made narrower when a hydrostatic pressure is applied, and we should expect an increase of resistance. This is actually the case for lithium.

The fact that the resistance of calcium increases under tension

18 A. W. Hull, Phys. Rev. 17, 42–44, 1921.

hence means that its mechanism is the same kind as that of lithium, and verifies the evidence from crystalline structure. The positive coefficient of antimony indicates the same thing. In this regard it may be said that recent work has cast considerable doubt on the reality of the supposed expansion when antimony freezes,[19] so that my former expectation would now lose its chief ground for support. Also against my former argument I may mention that there is a polymorphic transition of antimony at 135°, which I had previously failed to take into account. Even if the relations are abnormal for the modification which is stable up to the melting point, there seems no reason why we should expect the same abnormalities in the other modification, which is stable at room temperatures.

Strontium seems to require special consideration. Its tension coefficient is not abnormally high numerically; it is nearly the same as that of calcium, and much less than that of lithium or bismuth. On the other hand its pressure coefficient is unique in being at least three fold greater than that of any other abnormal metal. It seems reasonable to suggest that the mechanism of conduction in strontium may be a combination of both types. These would conspire to give an abnormally high pressure coefficient, and oppose each other, giving by difference a relatively small tension coefficient, of a sign which could not be predicted without further evidence.

These views of the conduction mechanism receive support from a numerical discussion. We confine ourselves to changes of resistance at constant temperature, that is to the changes under pressure and the longitudinal changes under tension. We assume that the change of resistance may be written down in terms of the changes of dimensions. The transverse and longitudinal changes of dimensions will affect the resistance in different ways, which are different for the two different types of mechanism. We write the equation

$$\frac{1}{R}\Delta R = k_l\frac{\Delta\delta_l}{\delta_l} + k_r\frac{\Delta\delta_r}{\delta_r}$$

where k_r denotes the change of resistance per unit strain transverse to the direction of the current, k_l denotes the change of resistance per unit strain longitudinally, and $\Delta\delta_r$ and $\Delta\delta_l$ are the change in the transverse and longitudinal distance of separation of the atoms. Now if the strains are produced by tension, we have

$$\frac{\Delta\delta_l}{\delta_l} = \frac{\Delta T}{E}, \quad \frac{\Delta\delta_r}{\delta_r} = -\frac{\sigma}{E}\Delta T$$

19 M. Toepler, Wied. Ann. 53, 343–378, 1894.

where ΔT is the tension; and if the strains are produced by pressure we have

$$\frac{\Delta \delta_l}{\delta_l} = \frac{\Delta \delta_r}{\delta_r} = \frac{1}{3} \cdot \frac{1}{v} \left(\frac{\partial v}{\partial p}\right)_\tau \Delta p\,.$$

Substituting these expressions for the strains gives

$$K_P = \frac{1}{3} \cdot \frac{1}{v} \left(\frac{\partial v}{\partial p}\right)_\tau [k_l + k_r]$$

and
$$K_T = \frac{1}{E} [k_l - \sigma k_r]$$

where K_T is the tension coefficient of resistance tabulated above, and K_P is the pressure coefficient of resistance above. But since these two coefficients are known experimentally, we have two equations to determine the two unknowns k_l and k_r. I have made the calculations and tabulated the results in Table II.

TABLE II.

Metal	k_r	k_l
Li	−3.2	+1.0
Ca	−5.4	+0.16
Sr	−15.8	−6.7
Sb	−13.6	−0.1
Bi	−7.0	−9.6
Mang.	−7.3	−1.6
Therlo	−8.1	−2.1
Co	+1.92	+2.6

Let us now consider what sort of numerical values our theory would lead us to expect. For a normal metal we expect k_l to be positive, and greater than unity, since in addition to the increase of resistance brought about by increasing the distance apart of the atoms, there is an increase due to the simultaneous increase of amplitude. Since most of the electron paths have a transverse as well as a longitudinal component, the same reasoning would lead us to expect that k_r would also be positive and less than k_l, but of the same order of magnitude, there being two transverse degrees of freedom against one longitudinal. In the same way, we would expect that for bismuth, where the effect of amplitude is abnormal in sign, and more than counterbalances the effect of changing distance between atoms, k_l and k_r should both be negative, and k_r less numerically than k_l. On the other hand, for those metals whose conduction mechanism is by the passage of electrons in channels between the atoms, we expect k_r to be negative, since increasing the transverse separation of the atoms decreases the resistance, and k_l to be relatively small. For strontium, which has a combination of both types of mechanism, we expect both k_l and k_r to be negative, k_r being numerically larger than k_l.

An inspection of the table shows that in every case these anticipations are strikingly verified, and the probable essential correctness of the theory receives strong support.

The values of the coefficients found for manganin and therlo indicate that for these the mechanism is for the most part like that of lithium, calcium, and antimony, but that there is in addition a small contribution by a mechanism of the bismuth type. Of course the phenomena for alloys are most complicated and varied in their types of behavior, but it is not difficult to picture to oneself that under some conditions when two different kinds of atoms crystallize side by side into the same space lattice that there should be channels left between the atoms for the passage of conduction electrons, or that the law of force between atoms of different kinds should show the same sort of abnormality that the atoms of bismuth show.

So far as I know there has been no previous attempt to make connection between any theory of conduction and the tension effects. In the light of the success of the above for abnormal metals it would now be of much interest to accurately determine the tension coefficients of the normal metals. If in addition the coefficient of transverse resistance could be determined, a most valuable check would be obtained, for we would then have three independent experimental coefficients, which must be expressible in terms of the two quantities k_l and k_r.

Discussion of the Effect in Nickel.

The peculiar nature of the phenomena for nickel makes it evident that there must be some unusual mechanism involved. It seems to me that there is probably an intimate connection with the polymorphic transition at 360°. Under a tension the transition temperature will be displaced by an amount proportional to the square of the tension, and if a certain function of the compressibility and Young's modulus has the right sign, the transition temperature will be depressed. (The compressibility and Young's modulus which enter this relation have not been determined experimentally.) To explain the failure of a single sharp transition point, we invoke the same sort of mechanism that we suppose to be responsible for elastic hysteresis. Modern theories of the structure of metals explain elastic after effects and hysteresis by the fact that the microscopic crystalline grains are unequally exposed to changes of stress. Certain grains are unfavorably situated, and the elastic limit of these is exceeded before that of the average. Just these grains would first reach the transition point under an increase of tension, and other grains not until later. Thus the transition and also the change of resistance would not be expected to take place discontinuously at a single tension, but to be spread over a range instead. If now the resistance of the new phase is less than that of the original phase, we have a reason for the sign of the observed effect. The minimum of resistance with increasing tension is reached at that point where the rate at which new grains are being transformed has been so decreased by exhaustion that the decrease of resistance brought about by the transition is equalled by the increase of resistance due to the normal tension effect on the transformed grains.

This view of the phenomenon demands in the first place that the effect of tension be normal on that phase which is stable above 360°. So far as I know this effect has never been measured, but it would be going out of one's way to assume that it is abnormal. It is also demanded that the resistance of the high temperature phase when subcooled into the low temperature region be less than that of the low temperature phase, at least under the tensions which are found inside the grains. This again is subject for further experimental investigation. So far as I know, no measurements have been made on nickel of high purity. The measurements of Werner [20] are the only ones which I know with respect to the behavior of resistance through the transition point. I find from his data that the temperature coefficient of his hard drawn wire between 31° and 110° was 0.0041 of its value at 0°, and for soft wire the corresponding coefficient between 18° and 111° was 0.0054. The purest nickel should give under the same conditions a coefficient of about 0.0062. Werner finds no discontinuity in the resistance at the transition point, but does find a change in the direction of the curve with temperature. It is highly probable that there was actually a discontinuity, which was masked by the effect of impurity. It has almost always turned out that a phenomenon which

[20] M. Werner, ZS. Anorg. Chem. 83, 275–321, 1913.

was at first thought to be continuous through a transition point is found, on increasing the purity, to be really discontinuous. It is true that the direction of the difference of temperature coefficients found by Werner is the reverse of that demanded by the above explanation, but in view of the extreme sensitiveness of the temperature coefficient to small impurities, I do not believe that a great deal of importance should be attached to this.

In support of the suggested explanation is in the first place the fact that neither iron nor cobalt show similar effects. These metals and nickel are similar in many respects, but are unlike in regard to their transitions. Cobalt does not have any polymorphic transitions, and the first transition of iron is at so much a higher temperature than that of nickel that it may well be without effect. The decrease of the tension of the minimum resistance of nickel with increasing temperature is also in accord with this view; at a higher temperature a smaller tension is necessary to make the transition take place. It is of course not possible to make any very exact numerical comparisons in view of the flatness of the minimum of the curves, but the data are at least not inconsistent with a depression of the transition point of a single homogeneous crystal by an amount proportional to the square of the tension. My explanation also demands that the resistance of the high temperature phase when it is subcooled be less than that of the low temperature phase. In view of Werner's failure to find a large discontinuity in the transition point, this probably means that the temperature coefficient of the high temperature phase is greater than that of the low temperature phase, and this is in accord with the fact that the initial rate of decrease of resistance with tension is less at high temperature than at low.

The abnormal character of the hysteresis loop on the falling branch is to be explained as follows. The primary effect of a tension is to force a transition from one phase to another. This transition would be expected to show hysteresis on reversing the direction of the change of tension. On the other hand, the sum of the pure tension effect in all the individual grains shows no hysteresis, because this is determined by the average tension, which is equal to the applied tension itself. Hence on decreasing the direction of the change of tension, the abnormal effects are decreased in magnitude, whereas the normal effects are unaltered. An application of this analysis to the upper end of the loop beyond the minimum of resistance (maximum on the diagrams) shows that on release of tension the resistance may drop to less than under any increasing tension.

Before this explanation can be finally accepted, experimental confirmation in several respects is required. Temperature measurements should be made on the high temperature phase above the temperature of transition. The resistance relations should be carefully studied in the neighborhood of the transition point for nickel of high purity. Finally it ought to be possible to recrystallize a wire by proper heat treatment so as to make it into a single large crystal. Under these conditions the character of the phenomena should entirely change. The initial effect of tension should be normal, and should continue so until a tension is reached so high as to force the transition to take place, when there should be a discontinuous drop of resistance greater in amount than the previous increase. Of course it is not obvious or necessary that the tension of this discontinuity be less than the elastic limit, and hence it may not be possible to realize it.

Summary.

The tension coefficient of resistance of lithium, calcium, strontium, antimony, bismuth, manganin, and therlo has been determined. These substances are all abnormal in that their pressure coefficients of resistance are positive. Young's modulus for these metals has also been determined. The tension coefficient for bismuth and strontium is found to be negative, but positive for the other five. In the discussion I have shown that these coefficients are in accord with my theory of resistance. The conduction mechanism of lithium, calcium, antimony, manganin, and therlo is on the whole "transverse," that of bismuth is longitudinal, and that of strontium is a combination.

The negative tension coefficient found by Tomlinson for nickel has been verified for nickel of high purity, and in addition the hysteresis effects and the effect of changes of temperature has been studied. Cobalt, on the other hand, is found to be entirely normal. It is suggested that the explanation of the abnormal behavior of nickel may be found in a depression by tension of the transition point normally at 360°.

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NOTES ON THE EARLY EVOLUTION OF THE REFLECTOR.

By Louis Bell.

NOTES ON THE EARLY EVOLUTION OF THE REFLECTOR.

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There is some tradition of reflecting telescopes in classical times, probably groundless save as the ancients had concave mirrors and these, if of fairly long focus, would present to the eye an enlarged image of a distant object.

The first suggestion that a mirror might be used instead of an objective seemed to have come from Mersenne about the middle of the seventeenth century in a letter to Descartes. The idea did not appeal to the latter and the matter was dropped. The first actual reflecting telescope was designed by James Gregory, who published an account of it in his "Optica Promota" in 1663. In this he described the rather elegant construction which still bears his name, a perforated parabolic mirror with an elliptical concave mirror placed beyond its focus and returning an erect image to the ocular through the perforation.

The next year Gregory started Reive, a well known London optician, at making a 6 foot telescope. This failed on account of inability to get the necessary figure, largely owing to the fact that Reive tried to polish his mirrors on cloth, never a satisfactory material for accurate figuring. It is possible that Gregory had better luck later, for there is a well defined tradition that he died from a stroke of apoplexy in 1675 while showing to a group of students Jupiter's moons through one of his own telescopes. Certainly Robert Hooke presented a passable example to the Royal Society at its meeting February 5, 1673–4.

Now enters upon the scene the biggest figure of the period in science, Isaac Newton, then a young man who had just discovered the dispersion of light. A great speculative philosopher and mathematician, he was neither a practical astronomer nor a particularly good experimenter, and failed to discover the difference of relative dispersion in refractive media by the inexcusable blunder of raising the refraction of the water with which he was comparing glass by loading it with sugar of lead, and then jumping at the conclusion that all substances varied equally both in refraction and dispersion.

In other words he assumed without real investigation that the quantity which we now know as ν was a constant for all substances.

Just how this singularly maladroit piece of experimentation leaked out is unknown, although the fact is stated in the most positive manner both by Sir David Brewster, Newton's admiring biographer, and by Sir John Herschel. It is not unlikely that the fact got abroad at Cambridge in Newton's later years, and was passed along to Sir William Herschel. Certainly, as one of Newton's later apologists naïvely suggests, the fact was not recorded in Newton's "Opticks."

Be that as it may, it was the kind of thing for which a second-year student in physics would get a wigging which would linger long in his memory, but a blunder with a great name behind it carries far, and in this case it put off for a couple of generations the discovery of the principle of achromatism.

Following his error, Newton gave up all idea of an achromatic lens, and turned his attention to reflectors, apparently being unacquainted with, or entirely disregarding, what James Gregory had done before him. Newton had taken to the country on account of the plague, and only about 1670 did he apparently begin to revolve in his mind a reflecting telescope. It was 1672, January 11th, when he presented to the Royal Society a small model of his preferred form of reflector, which is still in the possession of the Society.

This little model had an aperture of about 1″ and focal length of about 6″, and magnified some 38 diameters. Newton was able, he says, to detect with it the moons of Jupiter and the horns of Venus, both feats which can be accomplished with the very slightest of optical aid, providing the magnification is anywhere near that which Newton used. But here again Newton gave notable evidence of his unpractical experimenting, clever though he was.

He had firmly fixed in his mind the entirely erroneous idea that a spherical mirror was quite good enough for the purpose, even when an aperture of F/6 or F/8 was used, believing that the trouble with the long telescopes of the day was almost entirely their chromatic aberration. This was partly true, but the spherical aberrations would have been equally bad save for the very narrow aperture employed.

This error would certainly have brought Newton to grief had he attempted a telescope of any perceptible size and in fact there is some evidence that this actually happened. On January 25th, just two weeks after his model was displayed, the minutes of the Royal Society note that: "There was produced a reflecting telescope 4 feet long of Mr. Newton's invention which though the metaline concave was not duly polished yet did pretty well, but was under charged. It was ordered to be perfected against the next meeting."

At the next meeting the following: — "The 4 foot telescope of Mr. Newton's invention was produced again, being improved since the last meeting. It was recommended to Mr. Hooke to see it perfected as far as it was capable of being." So far as the annals record it never again appeared on the scene.

While it was not definitely stated by whom this telescope was made an entry of March 1st of the same year in the Journal de Sçavans definitely ascribes this 4 foot telescope to Newton, although the account is rather vague. It would look therefore as though Newton himself or some of his friends had tried out his invention on a larger scale, and had fallen into exactly the trouble that might have been expected.

Perhaps Newton's aversion to the paraboloid, of which he knew perfectly well the properties, may have been partly due to the fact, as stated in one of his letters, that there is no strictly geometrical method of grinding it. About the same time Hooke proposed to stamp up the specula out of silver and before the end of the year he was working on a 15 inch mould for a speculum of 10 feet focal length, of which nothing has since been heard.

Meanwhile enters upon the scene the personage known in the histories of science as "Cassegrain, a Frenchman." A communication from M. DeBercé to the Royal Society describes his invention and gives a very rough sketch of it. This was a translation of a letter sent by DeBercé to the Academy of Sciences from Chartres, and read at the séance of April 16, 1672. DeBercé says that this invention had been communicated by Cassegrain to him some three months previously, and how much before this time Cassegrain had been working on the problem will probably never be discovered. At all events the Cassegrainian telescope was disclosed to others than the inventor at substantially the same time as Newton's and was certainly an independent invention, although one letter to the Royal Society vigorously berates the Frenchman for stealing "Our Newton's" thunder, and explains how an ingenious friend of the writer is making a still further improvement on Cassegrain's form by using a flat secondary speculum.

Newton's comment on DeBercé's letter plainly shows that he did not enjoy a rival in the field, for it is somewhat discouraging in tone and incidentally his specific criticisms were consistently wrong, as for instance he had the erroneous idea that speculum metal reflects much better at 45° incidence than at normal incidence, whereas in fact the difference is too small for even its sign to be distinguished with certainty. It is the irony of time that the Cassegrainian form is the one which has survived in the greatest instruments of the present time.

The writer is glad to be able to drag "Cassegrain, a Frenchman," from something of the obscurity in which he has been veiled, introducing then, Sieur Guillaume Cassegrain, sculptor in the service of his glorious Majesty Louis Quatorze, modeller and founder of statues for the decoration of the king's gardens at Versailles.

In 1666 he cast a bust of the King, after Bertin's model, for which he received 1200 livres, and for the next twenty years or thereabouts made also many replicas from the antique, including groups like the Laocoön receiving payments from the Royal Treasury for his artistic services well into the year 1684, at which time we lose sight of him. He is believed to have died in the period between 1684 and 1686. Cassegrain like his friend DeBercé was of Chartres, a city long consecrated to the art of sculpture. At about the same time that DeBercé sent to the Academy of Sciences the little note on Cassegrain's telescope, Cassegrain himself wrote a long letter to the Academy concerning the speaking trumpet lately invented by Sir Samuel Morland.

It was in this letter that the writer got the first clew to Cassegrain's identity, since in it he displays beautiful draughtsmanship, as shown in the accompanying copper plate, in striking contrast with his friend's rough sketch, and also a notable familiarity with the art of bell founding, which very likely may have been practiced in his own atelier. In fact he writes like an educated and experienced artist, and was obviously regarded as a person of some consequence, for this letter formed the *pièce de resistance* at the meeting of the Academy of Sciences on the 2nd of May 1672. When and where Cassegrain was born one cannot tell with any certainty, although it is not unlikely to have been somewhere about 1630 to 1635, quite possibly in Chartres. At all events his profession and facilities were such as would very readily have led him toward a reflecting telescope if a hint of Mersenne's suggestion to Descartes, or of James Gregory's theory, had come to his notice.

Note that in the account of Newton's telescope in the Journal de Sçavans the word is used in its modern English sense instead of being, as now, confined in French to reflectors, while the Cassegrainian instrument is spoken of as a little "lunette d'approche." One does not generally suggest dimensions for a thing non-existent. Whether Cassegrain actually made and experimented with telescopes nobody actually knows, but as a founder, familiar with bronze and bell metal, it is not unlikely that he tried it.

It was bell metal, by the way, which was the basis of Newton's speculum metal. He merely whitened it a little with arsenic, as the alchemists had done before him, thinking the alloy one stage in the transmutation of copper to silver. Bell metal ranged in early times from one part tin to four of copper, to one part tin to two of copper. Newton's was very likely between the two, since he recommends one of tin to three of copper, a material which works and polishes well, but tarnishes with great rapidity, as Sir William Herschel found to his cost many years later.

Exit now the reflector, for more than about half a century. Newton made one more try at it, working on a 4 inch glass speculum to be silvered on the back after the plan early proposed by Gregory. The instrument was apparently never completed.

Another item often ascribed to Newton was the discovery of pitch polishing. That he found the process to work well is undeniable, but he did not disclose it for more than thirty years after his little telescope had been laid on the shelf of the Royal Society, in fact not until several years after there had been published, subsequent to Huygens' death, the fact that he had been in the habit of polishing his true tools for lens grinding in exactly this manner.

It was not until Newton was a venerable invalid, a half century after his telescope had been put away and forgotten, that the reflecting instrument was finally put upon the stage by John Hadley, the inventor of the reflecting octant.

Hadley knew enough to make his own speculum metal and in his struggles with it derived very little information from Newton's experimentation. He realized the importance of parabolizing his main mirror and of giving a hyperbolic figure to his small mirror. He polished them both not directly upon pitch, but upon pitch over-laid with the finest silk fabric, which apparently helped in distributing and holding the abrasive, and in 1722 presented to the Royal Society the first veritable reflecting telescope.

This instrument was of about 6 inches aperture and of about 5 feet focal length and proved on test to be better than the telescope of 123 feet focus belonging to the Royal Society and made by Huygens. The tests of the telescope show that it had a pretty good figure, and in fact Hadley hit upon the method of testing for figure used for many years thereafter, by examining the speculum at the centre of curvature and judging of the parabolic figure by the default of the image from the characteristics of a spherical mirror.

John Hadley in fact was the real inventor of the reflector in quite the same sense that Mr. Edison invented the incandescent lamp; from grasping the true principles of the matter and carrying them out to success while the only previous attempts had ended in dismal failure.

It was Hadley who taught the art to Molyneux and others and to whom James Short, the most celebrated of eighteenth century artists until the time of Herschel, was undoubtedly indebted for the start upon his brilliant career.

Short, however, worked to the Gregorian principle with success, doubtless since because this telescope rectifies the image, it is available both for terrestrial and celestial use. He made instruments from all accounts of beautiful figure, some of them up to 8 or 10 inches diameter, and of a speculum metal so nicely compounded and polished that some of his specula were still serviceable well toward the end of the nineteenth century.

There is good reason to suspect that Short may have been the inventor of the system of distributed grooves given to the tools, upon which successful figuring so much depends, since, dying, he ordered the destruction of his whole equipment, a quite needless precaution against the success of posterity had there not been very radical improvements, easily detected. James Short, too, was the first constructor of genuine equatorial mountings of which he executed several about the middle of the eighteenth century. He died leaving the methods which led him to success a mystery unsolved, for Herschel and his successors to puzzle over. The only change in the situation after Herschel's day was the independent invention of Steinheil and Foucault which gave to silver-on-glass its present supremacy. The earliest examples of Foucault's work show how little, otherwise than in the material and figuring of the speculum, the art of telescope construction had progressed since Hadley and Short were working a century before.

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THE EFFECT OF PRESSURE ON THE THERMAL CONDUCTIVITY OF METALS.

By P. W. Bridgman.

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Introduction.

It is known that there is an intimate connection between the electrical and thermal conductivities of metals. This connection finds expression in the constant value of the Wiedemann-Franz ratio for different metals; the meaning of this is supposed to be that the largest part of the heat transfer in metals is accomplished by the same electrons which transfer the electricity in current conduction. I have already measured the effect of pressure on the electrical resistance of a large number of metals, and have drawn certain conclusions from the results as to the mechanism of electrical conduction.[1] It seemed that it would be no less interesting to measure the effect of pressure on thermal conductivity. This paper contains the results of measurements of the effect of pressures to 12000 kg/cm^2 on the thermal conductivity of lead, tin, cadmium, zinc, iron, copper, silver, nickel, platinum, bismuth, and antimony at 30° C.

At the time that I undertook these measurements this was an entirely untouched field; not even the sign to be expected for the effect was known. Since starting this work, measurements have been published by Lussana [2] on the effect of pressures to 3000 atmospheres on the thermal conductivity of eight metals and four alloys. Except for two of the alloys, he finds that the thermal conductivity always increases under pressure, and the change is of such a magnitude that the Wiedemann-Franz ratio remains nearly constant. The classical conception of the mechanism of conduction would prepare one to expect this result. I have not been able to verify the results of Lussana, but find that the conductivity of more than half my metals decreases instead of increases under pressure. This is perhaps a result not to be expected, and must have considerable significance for our picture of the mechanism. With regard to the differences between Lussana's results and my own, I think that Lussana's work is open to serious criticism in several particulars, and I have gone into this in detail later.

Measurements of thermal conductivity are known to be among the most difficult in physics (witness the disagreement among different observers as to the sign of the temperature coefficient of thermal conductivity), and I was accordingly prepared for considerable difficulty in measuring changes, which might be expected to be of the order of a few per cent, in a quantity which is itself so hard to measure. The difficulties were of course enhanced by the technical necessity of making the apparatus so small that it could be enclosed in a pressure chamber. The anticipated difficulties were encountered; these measurements are by far the most difficult of any that I have attempted under pressure. The accuracy is accordingly much less than that possible in such measurements as of electrical resistance under pressure, for example. Certainly not more than two significant figures can be claimed for the pressure coefficient of thermal conductivity. If the work were to be repeated now it would be possible to improve considerably on the accuracy, or at least the presentability, of much of it. But I believe that the results which follow are essentially correct in their large features, and that the use which can at present be made of such data does not justify the expenditure of more time.

Description of Apparatus and Method.

The choice of methods was much restricted by the limitations of space imposed by the necessity of getting the specimen and all the attachments into a pressure cylinder, and by the necessity of using only four electrical leads to the heating element and thermo-couples. Two methods were used; these were both of the same general character in that they carry the determination of thermal conductivity back to the definition. A known steady supply of heat was put into the specimen, and by means of a thermo-couple the difference of temperature was measured between two points when equilibrium was reached. In terms of the geometry of the configuration the temperature gradient and the rate of heat flow across unit area can be immediately found, and hence the thermal conductivity may be found from its definition.

The first method was in theory the simplest, as there were practically no corrections to be applied. This was a radial flow method. The specimen was in the form of a massive cylinder, almost filling the bore of the pressure cylinder. Along the axis of the cylinder was a linear source of heat, and the difference of temperature was measured between two points at different radial distances from the axis. The formula for thermal conductivity in this case is

$$k = \frac{Q}{2\pi(\theta_1 - \theta_2)} \log \frac{r_2}{r_1},$$

where k is the conductivity, Q is the heat input per unit length of the axis, r_1 and r_2 are the radial distances of the two junctions of the thermo-couple, and θ_1 and θ_2 are the temperatures of these two junctions.

The flow of heat under these conditions is radial, except at points near the ends of the cylinder. By locating the thermo-couple midway between the ends of the specimen any end effect may be avoided. The method is therefore unusual in that there is no correction for heat leak. Further there is no correction for the change of dimensions of the specimen under pressure, for it is only the ratio of the two radial distances that enters the formula, and this is not changed by a hydrostatic pressure which uniformly changes the dimensions in every direction. There is a correction in the heat input due to the change of length of the heating unit under pressure, but this correction is equal to the linear compressibility, and is so small as to be almost negligible. The heating element was made of nichrome. Assuming the compressibility of nichrome to be calculable by the law of mixtures from that of nickel and chromium, its two constituents, the magnitude of this effect is 0.2% under 12000 kg/cm^2. The correction is an addition to the observed pressure effect. There is also to be considered the change of resistance of the heating unit under pressure, but by a suitable arrangement of the circuits this may be made to eliminate itself, as will be seen later.

I designed this method of radial flow to meet the needs of the high pressure measurements, and found only later that it had been previously proposed by Niven [3] in 1905, particularly for measuring the conductivity of comparatively poor conductors.

The method was used with many modifications, as will be described later. Its essential weakness is that it demands that the specimen be perfectly homogeneous; this is difficult to obtain in sufficient perfection in a casting, or even in commercial drawn metal. The method worked best for lead, which is fairly easy to cast, is a comparatively poor conductor, and is not crystalline, so that inhomogeneities due to crystallization during cooling were not important. The method failed for copper and nickel.

The second method was a longitudinal flow method. Heat is put into the end of a rod, the other end of which is connected with a massive copper block in intimate contact with the pressure cylinder, so that we have essentially a rod with a source at one end and a sink at the other. The difference of temperature between two points on the axis of the rod is measured with a thermo-couple. In contradistinction to the other method there is here an important correction due to the lateral loss of heat to the fluid by which pressure is transmitted. An elementary discussion shows that in order to reduce this correction to the smallest value it is necessary that the dimensions of the specimen be very small. The specimens actually used were only $\frac{1}{8}$ inch in diameter, a little over 1 cm. long, and the two thermal junctions were 2 mm. apart. The correction for lateral loss is evidently smallest for those metals with the greatest conductivity, so that this method supplements the other, working best for those metals for which the first method gave the poorest results. The magnitude of the correction for lateral loss may vary from 20% of the total effect for bismuth, to about 1% for copper and silver. The most disagreeable feature of this correction is that it changes with pressure because of the change of conductivity of the transmitting medium under pressure. This effect is large and has to be independently determined. The thermal conductivity of petroleum ether, which was the medium used in all the later part of this work, increases 2.2 fold under 12000 kg/cm^2. The determination of the pressure effect on petroleum ether will be described in detail later.

Discussion of further details of these two methods is reserved until later.

The electrical measurements involved in either method were essentially the same in character, and could be made with the same apparatus. The measurements necessary were of heat input, which involved a knowledge of current and resistance, and of temperature difference, which demanded a measurement of the electro-motive force of a thermo-couple. Since it was necessary to find only the relative changes in thermal conductivity, a source of heat of unknown magnitude would have been sufficient, provided that it remained perfectly constant. The arrangement of the circuits, to be described in the next paragraph, ensured approximate constancy of the heat input, so that the measurements of the heating current degenerated to check readings from which a small correction was determined. The measurements of both heating current and e.m.f. were made on the same potentiometer as was used in previous measurements of the effect of pressure on thermo-electromotive force.[4] This apparatus has already been described in sufficient detail. By an arrangement of suitably protected switches either the thermo-couple or a small potential tap from the heating circuit could be connected in place of the couple as previously used. The tap in the heating circuit was constructed of heavy manganin wire of a resistance of approximately 1/13000 ohm.

Correction for the change of resistance of the heating element with pressure was avoided by using a similar heating element in shunt with the one exposed to pressure. The shunt unit was mounted in the same thermostated bath as the pressure cylinder, but was not exposed to pressure, so that its resistance remained constant during changes of pressure. By writing down the equations for the divided circuit, one can see in an instant that if the total input of current into the two heating elements in parallel is maintained constant, the rate of generation of heat in that one of the two elements which is exposed to small fluctuations of resistance remains constant. The total input of current was maintained approximately constant by using for the source of supply a storage battery in series with a ballast lamp (iron filament in an atmosphere of hydrogen) obtained through the courtesy of the General Electric Company. The total fluctuation of resistance in series with the lamp due to changes of pressure was only 0.08%, and with constant e.m.f. the lamp reduces the fluctuations of current due to this to negligible proportions. However, the battery itself was not always sufficiently constant, and in order to determine the small correction due to its fluctuations, the total current delivered by the battery was measured by determining the current in the two branches of the shunt. This was done with the potentiometer and a low resistance tap, precisely as for the heating circuit. The total corrections due to fluctuations in the heat input were always a small fraction of 1%.

A correction also had to be applied for the change in thermal e.m.f. of the couple under pressure. The couple was of copper-constantan, and the corrections due to pressure may be taken from data already published.[4] The total correction at 12000 kg. due to this effect is 0.8%; the correction is to be subtracted from the apparent thermal conductivity.

It will be noticed that this method differs from those previously used in determining the effect of pressure on electrical conductivity or thermal e.m.f. in that it is not a differential method, but is direct. A number proportional to the total thermal conductivity is determined at different pressures, and from these data the effect of pressure is computed. As originally planned I had intended to make the method differential, measuring the difference of thermal conductivity between one sample exposed to the pressure and a similar one not exposed. This can be simply done by opposing the two thermo-couples of the two specimens. It soon appeared, however, that the regularity of the results was not sufficient to justify this refinement, and the simpler direct method was used. The direct method demands somewhat greater accuracy in the resistances and the comparison standard cell, which, however, was easy to attain.

Since the primary interest of these measurements was in the proportional changes of thermal conductivity produced by pressure, a highly accurate knowledge of some of the absolute data was not essential. For instance, it is much easier to compare within 0.1% the two potential taps for the heating unit and its shunt than to measure the resistance of either accurately to 10^{-7} ohms, which would have been demanded by 0.1% on the absolute resistance.

We return now to a discussion of the details of the two different methods. First the radial flow method will be described.

The difficulties were chiefly mechanical, involved in getting the specimen into proper shape, and ensuring the right boundary conditions. The method demands not only that the heat input take place accurately on the axis, but most of all that the exterior surface of the specimen be maintained at constant temperature. The difficulty of this last requirement can be seen when it is considered that the thermal conductivity of copper, for example, is 3000 times greater than that of the petroleum ether by means of which pressure is transmitted to the specimen. There must of necessity be some crack between the walls of the pressure cylinder and the specimen; even if this crack is made vanishingly small at atmospheric pressure, it assumes quite appreciable proportions under 12000 kg. owing to the elastic deformation of the cylinder under internal pressure. If the specimen does not remain accurately centered in the cylinder at all times the flow of heat becomes unsymmetrical and the method is vitiated. Originally I endeavored to meet this condition by surrounding the specimen with a massive cylindrical sheath of copper, approximately $\frac{1}{8}$ inch thick. The specimen itself was $\frac{5}{8}$ inch in diameter and 2.5 inches long, and the interior diameter of the pressure cylinder was about $\frac{7}{8}$ inch. The crack between specimen and pressure cylinder was of the order of 0.005 inch initially, and it might increase by double under the extreme pressure. The use of the copper sheath is applicable only to those substances with low conductivity compared with copper. It might be expected to work best for lead and bismuth, and to work less well for tin and zinc.

Many variations were tried on this method. The chief difficulty proved to be that of getting good thermal contact between the specimen and the copper sheath. I at first tried to cast the easily fusible metals into the sheath. Unexpected difficulties were encountered in getting good enough contact. Casting into the bright sheath in air always introduced a film of oxide. Casting into the sheath in vacuum was tried, and also casting after making a preliminary coating of the interior of the sheath with solder or tin or the particular metal in question or silver plating, both in vacuum and a protecting atmosphere of illuminating gas. Satisfactory results were not obtained with any metal except lead. It was a surprise that it was so difficult to make good contact with tin.

A difficulty anticipated with this method because of the unequal compressibility of the copper of the sheath and the metal of the core did not turn out to be formidable; this difficulty could be turned by drilling the sheath with a number of fine holes so as to allow direct access of the pressure to the interior.

The device was tried of splitting the sheath into two pieces at a plane passing through the axis. In this way the inside of the sheath could be thoroughly tinned, as also the outside of the specimen, and by squeezing the specimen between the two halves of the sheath while still hot, good thermal contact could be secured initially. But with this arrangement the unequal compressibility of sheath and core was prohibitive; there was a progressive change with each application of pressure, the two halves of the sheath separating and working loose from each other. With tin, for example, a progressive change in the apparent effect of pressure on the thermal conductivity of three fold was produced by three applications of pressure.

The attempt to maintain the external surface at constant temperature by using a sheath was finally abandoned, and the specimen was maintained concentric in the pressure cylinder by an arrangement of springs. The specimen was made larger than when the sheath was used, being now only 0.02 inch smaller than the internal diameter of the pressure cylinder. Between the surface of the specimen and the walls of the pressure chamber were placed six longitudinal strips of german silver 0.002 inch thick, and about 0.42 inch wide. A sectional view through the specimen and the pressure cylinder is shown in Figure 1. The strips are shown bent to a nearly circular figure be-

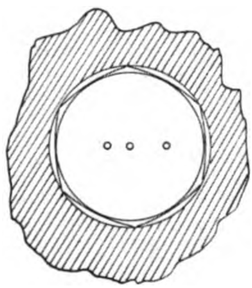

FIGURE 1. Section through the pressure cylinder and the radial flow specimen, showing the method of keeping the specimen concentric by springs.

tween the walls of the cylinder and the specimen. It is obvious that these springs will take up any stretch in the internal diameter of the cylinder, and maintain the specimen concentric at all pressures. The springs are so thin, and the thermal conductivity of german silver is so low that any error due to flow of heat away from the specimen through the springs is small, and need not be expected to change greatly with pressure. But as a precaution against any error from this effect the specimen was wrapped with a layer of paper between the springs and itself, thus minimizing any irregularity due to unequal heat loss at different parts of the surface of the specimen arising from inequalities in thermal contact between spring and specimen because of geometrical imperfections.

As far as I can judge, this method of maintaining the outer surface of the specimen as constant temperature was successful, but there were numerous other sources of irregularity. The chief outstanding mechanical problem now was that of getting the holes into the specimen for the heating element and the thermo-couple. The heating element must be of fine wire in order to have the requisite resistance, and the conditions of the problem demand that there be a cylindrical surface somewhere surrounding the heating element which is at constant temperature. That is, a surface at constant temperature is demanded both external and internal. I attempted to satisfy the requirement at the inner surface in the same way as at the outer surface by enclosing the heating element in a copper tube. The tube was small, usually about 0.040 inch outside diameter and 0.020 inch inside diameter. The outer surface of the tube was originally tinned and the specimen cast around it. I soon found the same difficulty in making good thermal contact as at the outer surface. If the specimen were melted after use and the copper tube examined it was very seldom that the surface was found completely wet with the metal of the specimen. But bad thermal contact at the center was not so serious as at the outer surface, because of the much smaller dimensions. Considerable improvement was made by using a tube of silver instead of copper. The difficulty here is due to the exceptional affinity between the silver and the metal of the specimen. If the mold is preheated, as is necessary to get a coherent casting, the alloying action between the metal and the silver is in many cases so great as to result in complete eating away of the silver tube. This effect was avoided by an arrangement of the mold by which the silver tube was drawn up into it from below after it was already filled with the molten metal, and the metal immediately chilled from the bottom up, so as to avoid the formation of blowholes. Specimens were made in this way and measured of lead, tin, and cadmium, but the method did not work with bismuth, the alloying here being so rapid that the silver was eaten away before the mold could be chilled.

The attempt to maintain internal equality of temperature by a tube of highly conducting material was finally abandoned, as it had been at the outer surface, and the heating element was placed directly in an axial hole made by casting the metal around a tungsten wire stretched along the axis during casting. The diameter of the nichrome heating element was 0.005 inch; this was enamelled, bringing the diameter to 0.006, and the diameter of the hole was 0.007, so that only a small amount of play was possible; furthermore the wire was always kept from direct contact with the specimen by at least the thickness of the enamel. This method of making the center hole could be used for all the metals that could be cast, including lead, tin, cadmium, zinc, bismuth, and antimony. With other metals of higher melting point which were used in the form of cylinders the heating element has to be put in an axial copper tube. In these cases the axial hole was 0.040 inch in diameter, drilled through the specimen, which was then turned concentric with the hole, and the copper tube, which was not less than 0.039 inch in outside diameter, was then sweated into place. After some practise it was possible to get fairly good sweated contact. Since the pressure penetrates the interior of the tube, error from unequal compressibility of the two metals at the interior surface must be small. It is obvious that the use of an axial copper tube to contain the heating element can be unobjectionable only for those metals whose thermal conductivity is small compared with that of copper. The method cannot be expected to give good results for copper itself, for example.

The thermo-couples as well as the heating element were at first placed in two copper tubes cast into the cylinder, but this was later abandoned for a hole cast into the metal itself. In making the hole for the thermo-couple there are two opposing tendencies that must be guarded against. In the first place the wire of the couple must be so good a fit for the hole in which it is placed that the thermal conduction from the ends of the cylinder along the wires of the couple can be neglected. In the second place, if the fit is too close, pressure is not transmitted freely throughout the interior of the hole when pressure is high and the transmitting medium has acquired a certain amount of viscosity. Such viscosity results in the introduction of stresses into the wire, with an effective change in its thermo-electric constants, so that readings of temperature difference are no longer reliable. The attempt was made to meet these two opposing conditions by making the hole larger at the two ends with a narrow neck in the middle where the junction is situated. The cylindrical specimen itself was 2.5 inches long. The narrow part of the hole to receive the junction was 0.014 inch diameter and 0.5 inch in length. The larger part of the hole at each end was 0.040 inch diameter, and joined to the smaller part by a conical part 0.25 inch long, thus leaving the 0.040 part at each end 0.75 inch long. The diameter of the wire of the thermo-couple was 0.010 inch, and this was brought to 0.012 by the enamel. An approximate calculation shows that these dimensions are amply sufficient to prevent any appreciable flow of heat along the thermo-couple wire to the region of the junction, even with the metal of lowest conductivity. The hole was cast in the shape required by filing down a wire to a neck of the dimensions given at the middle, and holding this stretched through the mold in the required location. After the casting had been made, the wire was removed by pulling out, the wire breaking at the narrow part, and allowing the two pieces to be pulled out separately from either end.

This method of casting the thermo-couple holes was of course not applicable to the metals with high melting points, and for these copper tubes sweated into place were used exactly as for the central heating element. The inside diameter of these tubes as used for this purpose was 0.016 inch. The junction of the couple was wrapped with fine wire in order to bring its diameter nearly up to the internal diameter of the tube, thereby preventing play in the tube and ensuring better thermal contact at the junction. In this case also the dimensions were such that any error due to flow of heat along the thermo-couple wire from one junction to the other was negligible. The results obtained with specimens made in this way are subject to a correction not easy to calculate for the disturbing effect of the copper tubes on the flow of heat. This correction should not be large because the

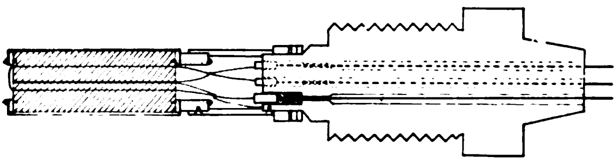

FIGURE 2. The radial flow specimen attached, with the electrical connections, to the insulating plug, ready to screw into the pressure cylinder.

copper tubes are comparatively small. The centers of the thermo-couple tubes were 0.098 and 0.201 inches respectively from the center of the specimen.

The electrical connections were got into the cylinder by means of a three-terminal plug of the same design as that used previously in determining the effect of pressure on electrical resistance by the potentiometer method. The only difference is that this plug could be made somewhat larger, because of the larger bore of the pressure cylinder, $\frac{7}{8}$ against $\frac{5}{8}$ inch. There is an advantage in this in that the thermo-couple leads can be got a greater distance from the heating element leads, and so there is less danger of heat leaking into the thermo-couple.

The manner of attaching the specimen to the plug ready for mounting in the cylinder is shown in Figure 2. The plug and specimen were attached together, and all electrical connections made before assembling so that the whole combination could be screwed into the cylinder as one self-contained unit. Attachments were made to the specimen itself by three steel pillars 0.062 inch in diameter at either end. These pillars passed through washers of transparent bakelite at either end. At the lower end the bakelite washer was in turn attached to a thin steel sleeve, which was attached by screws to the three terminal plug. The screws connecting the bakelite washer to the steel sleeve were made a loose fit in order to allow sufficient freedom for the german silver springs to keep the specimen concentric in the cylinder. The german silver springs were soldered to thin discs of german silver drilled to slip over the three steel pillars at either end and held in position by the same nuts which held the bakelite washers. The steel sleeve was cut away on two sides, allowing the ends of the thermo-couple and the heating unit to be brought through the bakelite washer and soldered to the three terminal plug. As an additional precaution against leakage of heat from the heating element into the thermo-couple, two massive pieces of copper in the form of a double wall were placed entirely across the pressure cylinder between the terminals of the plug connecting the thermo-couple and those connecting to the heating element.

Two forms of heating element were used for these cylindrical specimens, according to whether there was an axial tube, or the heating element was mounted in a fine hole cast along the axis of the specimen. When the tube was used, the heating element was in the form of a hairpin of 0.005 inch nichrome wire. By means of a jig the wire was made accurately 5 inches long when straightened out, so that it would exactly occupy the length of the tube when bent double. It was silver soldered at either end to a half-round piece of copper filed from a $\frac{1}{8}$ inch copper wire. The whole arrangement was covered with five or six coats of insulating enamel baked on, having a thickness of about 0.0008 inch. After enamelling, the wire was bent double into the hairpin form, thus bringing together the two halves of the copper wire so as to form a single completely round piece $\frac{1}{8}$ inch in diameter. As an additional precaution the two halves of copper were frequently insulated from each other by a thin layer of mica. The two halves were then bound together into a single piece by wrapping with fine silk thread. The resistance of the copper terminal pieces is so much less than that of the nichrome element itself that there is only a very small error introduced by any generation of heat outside the element. The massive copper pieces were attached to the terminals of the three terminal plug by a short length of flexible conductor twisted out of many fine strands of copper. The total resistance of the heating elements made in this way was nearly 10 ohms.

If the specimen were one of those with a small hole cast along the axis the heating element was a single length of 0.0035 inch nichrome wire, coated with enamel as the hairpin unit, silver soldered at one end to a massive copper wire, and grounded at the other to the specimen itself by soldering to a piece of fine german silver sheet, which was in turn soldered to the specimen. By using an intermediate member of fine german silver sheet the difficulty of attaching the fine wire to the massive specimen at exactly the required point was obviated, and it was easy to make the heating element the same length as the specimen itself, at least within 0.002 inch, which was close enough. The specimen itself, which was grounded to the pressure cylinder, was used as the return connection for the heating element. In order to eliminate any danger of bad contact between the specimen and the cylinder, a connection was soldered to one of the steel pillars and the ground connection of the three terminal plug.

The thermo-couple, as already mentioned, was constructed of copper-constantan. Each couple was made of three pieces of wire, first a length of copper, then one of constantan, and then one of copper, of the same length as the first. This was bent into a hairpin form, and the two legs of the pin slid through the two thermo-couple holes. The dimensions were so chosen as to bring the two junctions copper-constantan midway between the ends of the specimen. The two copper ends of the couple were attached by solder to the two copper thermo-couple terminals of the three terminal plug. It is thus seen that the e.m.f. of the couple is determined by the temperature difference of the two copper-constantan junctions.

The wires of which the couple were made were 0.010 inch diameter. It is essential that the junctions be absolutely smooth butt joints. It is a little trick to successfully make these, but the following procedure finally gave satisfactory results. The ends of the wires to be united were first coated thinly with silver solder and borax, precisely as ordinary objects are "tinned" for soft soldering, by dipping into a drop of molten silver solder covered with a little borax. The drop of silver solder was maintained molten at the desired temperature in a miniature electric furnace made by winding nichrome wire around a core (of course insulated from it) of $\frac{1}{4}$ inch nichrome rod, in the end of which was drilled a shallow depression to hold the drop of molten solder. After coating the ends of the wires, they were brought into exact axial alignment in a clamp with three adjustments. The wires were held in pieces of quartz tubing drawn down to the proper size, and projected perhaps $\frac{1}{4}$ inch or less from the tube. The solder on the ends of the wires was then melted and the wires fused together by a small blue gas flame. A flame of the required size may be obtained from a jet issuing from a piece of drawn out glass tubing. The gas to give the blue flame is supplied from a gasometer, which may be improvised from a couple of old tin cans, filled with a mixture in the right proportions of illuminating gas and air. The joint so formed is very strong mechanically. If the wires are not in satisfactory alignment they may be made so by hammering. The method is considerably superior to any simple electrical method of welding with which I am familiar. Of course there is no objection to smoothing the junction with a little fine emery paper; by this means a joint can be made that cannot be detected by the sense of touch, or of sight, after the wires are enamelled. The wires used were all cut to standard lengths by jigs, so that after enamelling the joints could be located by measurements. I have used this same method to make junctions as small as 0.004 inch, which are considerably more than two and one half times as hard to make by the ordinary method.

A great many measurements were made with specimens of the cylindrical form described above, and every effort was made to make the method give good results. The measurements with any one specimen were usually regular, and would repeat, and were apparently as good as could be asked. The trouble was that the numerical results obtained with different specimens of the same metal did not agree. The lack of agreement was much worse for some metals than others. I finally came to the conclusion that the method is essentially limited by the demands for homogeneity which it imposes on the specimen. Any slight flaw in the casting, resulting in cracks whose dimensions would change with pressure, must change the direction of the lines of flow with pressure. Further, any large scale crystal structure would give different results for different castings.

The second method was designed to obviate as much as possible the effect of inhomogeneity in the sample. As already mentioned, this is a longitudinal flow method, there being a source and a sink at the ends of the specimen, and the difference of temperature at two fixed points is measured. Since the heat input must cross each section of the specimen, any distortion of the flow by local inhomogeneities must have less effect than in the case of the cylinder, where as an extreme case it is possible that a flaw properly situated might force all the heat input to flow out of the cylinder at one side only. The much smaller dimensions of the longitudinal flow specimens would also seem to reduce the chance of errors from flaws.

A scale drawing of the longitudinal specimen mounted in the massive copper block ready for assembly in the pressure cylinder is shown in Figure 3.

The sink at one end of the specimen was formed by soldering it into a massive copper block of a diameter nearly the same as the interior of the pressure cylinder. Good thermal contact between the copper block and the cylinder was ensured by a centering arrangement of springs of strip metal, exactly as in the case of the cylindrical specimens, except that now the strips were made of heavy copper instead of thin german silver. In order to reduce mechanical distortion due to unequal compression of the specimen and the copper of the block, the bottom of the hole in the block into which the specimen was

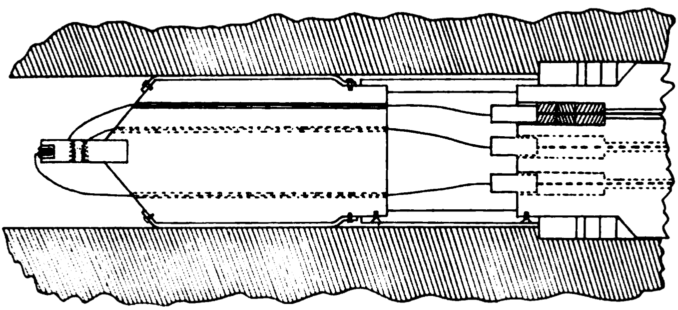

FIGURE 3. Longitudinal section of the pressure cylinder with the longitudinal flow specimen attached to the three terminal plug and in place.

soldered was brought into connection with the pressure transmitting medium by a small drilled hole.

The source of heat at the other end of the specimen was provided by a small coil of nichrome wire 0.005 inch diameter and exactly 1 inch long, silver soldered to copper leads of the same diameter. The resistance was of the order of two ohms. This wire was wound into a coil of small dimensions on a core of a fine piece of glass tubing, and was placed in a small copper capsule in the form of a square bottomed cup about 2.5 mm. deep. One end of the heating coil was grounded to the cup with solder, and the other end was connected to a lead from the three terminal plug. The copper capsule was let into a hole of the proper dimensions drilled in the end of the specimen. By the use of the capsule I hoped to more nearly realize a uniform heat input over the entire cross section of the specimen. To further realize these conditions, and obviate irregularity due to poor thermal conduction of any transmitting medium which might get into the cracks between specimen and capsule, the capsule was cemented into the specimen with deKhotinski cement, which has a considerably higher thermal conductivity than the liquid.

It will be noticed that the method of connection described above amounts essentially to grounding one end of the heating element on the sample itself, there being effectively a junction between the metal of the specimen and copper. When a heating current is passed there will be a generation of Peltier heat at the junction in addition to the Joulean heat in the element itself. This effect reverses in sign when the direction of the heating current reverses and so may be eliminated. However in the experiment as actually performed the independence of the reading of the thermo-couple of the direction of the heating current was used as a criterion for absence of electrical leakage between the heating and the thermal current circuit, so that elimination of the Peltier heat by reversal of the heating current was not desirable. For most of the metals the effect is very small and may be neglected, but for antimony and bismuth the effect becomes inconveniently large, rising to a change in the apparent magnitude of the thermal conductivity of 25% with reversal of heating current. For these two metals the ground to the capsule was discarded, and two independent leads to the heating element were used.

The thermo-couple was constructed in the same way as those for the cylindrical specimen; a length of copper, then a piece of constantan 1 inch long, and then copper again. This was bent into the form of a hairpin, whose two legs were thrust through two diametral holes 2 mm. apart in the specimen. The junctions were located on the axis of the specimen. After wrapping the wires around the specimen, as will be described in a later paragraph, the two copper ends of the hairpin were then soldered to the two copper thermo-couple terminals of the three terminal plug. The thermo-e.m.f. of the couple obviously gives the difference of temperature of the two junctions points.

In order to avoid error from heat leak along the wires of the couple it was necessary to make these of much smaller wire than in the case of the cylindrical specimens. The wire used was 0.004 inch diameter, and the hole in which the couple was placed was 0.007 inch. The couples were brought to a diameter of 0.005 inch by the coating of enamel. There is an outstanding possible play of the couples in the holes of 0.002 inch, which might lead to maximum errors in the apparent thermal conductivity of 5%. This is probably the chief source of error in this method, and it will be seen later that the results are consistent with this explanation. In some of the later measurements the attempt was made to cut down the motion possible in the thermo-couple by threading into the hole along with the couple a wire 0.002 inch diameter. In some cases this seemed to produce good effects, and in others apparently not. There is danger that too close a fit may introduce tensions into the thermo-couple, and so change its constants. It did not seem feasible to try for a much closer fit between the couple and the hole because of the difficulty of getting a perfectly smooth coating of enamel on the wire of the couple.

The specimens of the two soft metals lead and tin were cast, and the holes for the thermo-couple were cast with the specimens, using a 0.007 inch wire for the core, held in a proper position in the mold. For all the other metals the holes were drilled. A small jig was made and the holes were drilled in a jeweller's lathe. In spite of the use of the jig it was not possible to make all the specimens exactly alike, and the distance apart of the holes was independently measured in every specimen with a microscope. The extreme variation in their distance apart was from 0.190 to 0.210 cm. I found that a very convenient drill may be made of a piece of tungsten wire, pointed so as to form the conventional flat drill. The advantage of this over steel is that it is so tough that it is almost impossible to twist off, which may very easily happen to a steel drill in a clinging metal like gold, for example. While not as hard as steel, tungsten is hard enough to drill all the metals used here, which included iron, nickel, and platinum, all in the annealed condition.

Because the hole containing the couple is comparatively short, there is greater danger of error from heat leakage along the thermo-couple wire than in the first method. To eliminate this source of error, the wire of the couple was bent sharply at the entrance of the hole, at either end, and was wrapped once, or in some cases twice, about the specimen, and held in close contact with the external surface by a wrapping of fine silk thread. This ensures that the wire for some distance from each junction shall have the same temperature as the specimen at the junction, and so should eliminate error from leak. As an additional control with regard to heat leakage along the wire, the experiment was tried with two specimens of first using a couple constructed as above (constantan between two lengths of copper) and then a couple constructed of constantan between two lengths of "therlo." "Therlo" is an alloy made by the Driver Harris Co. with very nearly the same e.m.f. against constantan as copper, but with a thermal conductivity at least fifteen times less. The error from heat leak along the wires of the couple should therefore be very much less with the therlo couple. As a matter of fact, the same results were obtained with both couples, establishing freedom from heat leak.

A central band of one or two coats of enamel (thickness not over 0.0001 inch) was usually baked on the outside of the specimen. This was to avoid danger of short circuit where the thermo-couple wires were wrapped about the cylinder. Baking on this band of enamel, which required about 210° C, served the additional purpose of annealing the specimen. All the specimens were further heated to 150° or so when soldered into the massive copper block.

As already explained, the dimensions were forced by compromise between various opposing tendencies. If it were not for lateral loss, the specimens would have been made much longer, and the source and sink would have been situated further from the couple. This would have allowed the lines of flow to more completely straighten out in the vicinity of the couple. It is possible by a rough calculation to get an approximate idea of the extent of the failure of the lines of flow to be entirely straight. The actual distribution of temperature over the section at the source end cannot of course be accurately determined, but it seems fair to assume that the temperature is higher at the center of the rod than at the outside surface. A solution corresponding to this state of affairs may be obtained by assuming a distribution of temperature according to a Bessel's function over the source end. I carried through an approximate examination in this way, and convinced myself that any errors from this effect were not important.

The largest correction to be applied in the longitudinal flow method is for the lateral loss of heat through the transmitting medium. The lateral loss is a small part of the total input, so that we may assume the distribution of temperature approximately linear along the specimen. The temperature gradient at the mean point between the two junctions is the difference of temperature divided by the distance between the junctions. The total flow of heat long the rod at this point under this gradient is not the total input, because there has been some lateral loss between the source and the junction. This loss takes place partly from the lateral curved surface, and partly from the end. The loss from the lateral curved surface of the bar was computed on the basis of the formulas for the radial flow of heat through a cylinder between inner and outer surfaces maintained at constant difference of temperature. The temperature of the inner surface, that is, the temperature of the bar, varies along the axis. The total lateral loss was taken as the integral of the losses of short elementary cylinders. The loss from the upper end was calculated from the formula for the flow between two concentric hemispheres, one having the radius of the specimen, and the other the internal radius of the pressure cylinder. The value obtained in this way is obviously too large, as most of the pressure cylinder is situated further from the end of the specimen than its internal radius. The lateral flow and the flow from the ends as computed in this way involve the difference of temperature between the source end of the specimen and the outer cylinder. This temperature difference may be expressed approximately in terms of the heat input into the rod and the thermal conductivity of the rod. The temperature difference may now be eliminated, giving a result in terms of heat input and the conductivities of the liquid, k_2, and the metal of the bar, k_1. For the lateral loss of heat I obtained the formula $11.7\frac{k_2}{k_1}Q$, and for the loss from the end $12.\frac{k_2}{k_1}Q$ (this last is too high). As an approximate value for the total loss through the transmitting medium the value $20\frac{k_2}{k_1}Q$ was used in the computations.

The value of k_2, the thermal conductivity of petroleum ether, was determined by direct experiment, as will be described later. At atmospheric pressure the value found was 0.0004. At a pressure of 12000 kg/cm^2 the conductivity has increased 2.2 fold. The change in the value of the lateral flow correction with pressure determines the correction that must be applied to the apparent change of conductivity of the metal under pressure in order to obtain the true change. The magnitude of this correction evidently changes greatly with the metal used. The details with regard to this correction will be discussed under the different metals separately.

In the longitudinal flow method there is no correction to be applied for the change in dimensions of the heating element, as there was in the radial flow method. The same arrangement of a shunt circuit which eliminated the necessity for a correction for the changing resistance of the heating element with the radial flow method was used here also. There is a correction to be applied here, not necessary in the radial flow method, for the change of dimensions of the specimen under pressure. Under pressure the cross section becomes less and the distance between the thermal junctions also becomes less, the first by twice the linear compressibility and the second by the linear compressibility. The sum of the two effects is a correction equal to the linear compressibility in the direction of an increase to the apparent conductivity. In the absence of direct determinations of compressibility at high pressures, the corrections applied in the following were then taken from Richard's determinations at low pressures, neglecting the changes of compressibility with pressure. The correction is in any case small; the value for bismuth is a maximum at 1.2%. In addition to these corrections there was an effect not present appreciably with the radial flow method due to loss of heat along the leads to the heating element. The leads were of copper and the heating element was of nichrome. The thermal conductivity of copper is about 30 times and the electrical conductivity about 50 times that of nichrome. Hence most of the heat input is confined to the nichrome and there cannot be much heat conduction out along the leads; any cooling of the copper leads can affect only the extreme ends of the nichrome coil. The correction for this effect must be small, but may affect the absolute conductivity. Only the change of this correction with pressure can affect the final results, and I have neglected it.

There is one source of error that might be anticipated to be serious with the longitudinal flow method, namely convection effects in the liquid. In any event the convection effects may be expected to vanish at high pressures because of the known large increase of viscosity of the transmitting medium. I did expect the effect to be appreciable at atmospheric pressure, however, and at first made no reading at atmospheric pressure, but used 2000 kg. as the zero. It later appeared, however, that points obtained at atmospheric pressure lay on a smooth curve with those at higher pressures, so that no error from this effect is to be expected. It is in any event to be remarked that the error from convection may be made vanishingly small by cutting down the heat input. Assuming that the convection loss is only a small part of the conduction loss, we see that if the heat input is halved the temperature differences in the liquid are halved, the rate of convective flow of the liquid is halved, and the rate of heat flow into the liquid is halved, so that the total heat carried away convectively is quartered. (Of course the loss of heat by conduction to the liquid is only halved). In the actual experiment the magnitude of the heating current was so chosen that the temperature of the source end of the specimen was about 5° higher than that of the pressure cylinder. In one or two cases parallel runs under pressure were made with changes in the heat input by a factor of 2, with no change in the results. Also at atmospheric pressure I verified that the temperature difference measured on the thermo-couple was proportional to the square of the heating current. Both of these checks prove freedom from convective effects.

The longitudinal flow method showed itself a great improvement over the radial flow method with regard to the agreement of the results obtained with different specimens of the same metal. The individual readings for a single specimen, however, were very much more irregular, and until the interpretation of the irregularities was found, I was in doubt whether this method was actually any improvement over the previous one or not. After readings had been made on a number of specimens, it appeared that in almost all cases the readings for a single specimen would lie on three or perhaps four straight lines. The extreme separation of these lines amounted to a difference of not more than 5% in the total thermal conductivity. It has already been mentioned that this is the variation which might be caused by a shift of position of the thermo-couples in their holes. The natural interpretation is that the couple may take one of two extreme positions, like the bottom of an oil squirt, bent so as to touch either the top or the bottom of the hole. The couple may change from one to the other of these positions with change of pressure. The two couples together can thus occupy any one of three different positions, so that the readings will lie on one of three lines. If the initial curvature of the wires of the couples is such that one of the positions of equilibrium is not exactly at the top or bottom of the hole, then there will be four relative positions of the couples, and the results will lie on four lines, two of them comparatively close together. Figure 9, given later in the detailed discussion of the data for nickel, is a good example of the tendency to lie on discrete lines.

After the reason for the irregularities had been found it was possible to a considerable extent to control the location of the readings on one or the other of the possible lines. It was found, as might be expected, that rapid changes of pressure were favorable to a jump of the readings from one line to another, but that by changing the pressure very slowly it was in many cases possible to obtain long successions of points on a single straight line. It is easy to see that the nature of the irregularity is such that it was quite possible, before its true nature was recognized, to obtain results which might be erroneous even as to the sign of the effect. Suppose for example that the true effect gives points lying on a falling curve, but that as pressure increases the thermo-couples snap over in the holes in such a way as to pass from the low lying to the high lying curves. If only a few readings are taken, and if the snapping about of the couples is a fairly regular matter, as it often is, it can be seen that a regular succession of points may be obtained apparently lying on a rising curve. To avoid error from this effect each individual set-up demanded individual study, as the way in which the wires snap about is an individual matter. The true curve can be found by taking a larger number of readings than at first I thought necessary, and by studying the possibilities of obtaining successions of points on single straight lines under very slow changes of pressure. In one or two cases, after the effect had been recognized, it was found that the results originally obtained were in fact of the wrong sign, and that on setting up the specimen again and carefully analyzing the situation the correct result indubitably appeared of the opposite sign. Examples of this will be mentioned later.

Experimental Procedure.

The procedure for either the radial or the longitudinal flow method was essentially the same. The specimen was first attached to the three terminal plug, connections to the interior end of the plug being made with solder. The plug with the specimen in place was then screwed into the pressure cylinder, and connections made to the outer end of the plug by soldering. The temperature bath was then put in place, and temperature adjusted to 30°, which was the universal temperature of these experiments. (The accuracy was not great enough to justify an attempt to determine the temperature coefficient of the pressure coefficient, such as had been possible in the case of the measurements of electrical resistance, for example.)

The heating current was now turned on and equilibrium waited for, as shown by the constancy of readings of the thermo-couple. An interval of 20 or 30 minutes was necessary to reach initial equilibrium after adjusting the temperature bath. This time was nearly all consumed in acquiring equilibrium throughout the interior of the pressure cylinder, and particularly between the outer and inner ends of the three-terminal plug. The leads through the plug were of necessity of steel in order to obtain sufficient strength, whereas the metal at either end of the plug was copper. Hence any temperature inequality between different parts of the plug introduces spurious e.m.f.'s into the circuit, so that it is essential to wait for complete equilibrium. As far as the specimen itself and the heat flow through it from the heating element is concerned, equilibrium was attained very quickly. If the heating current was turned on or off after the bath and cylinder had reached equilibrium, the thermo-couple reached equilibrium in a fraction of a minute in the radial flow method, and in at most two or three minutes in the longitudinal flow method.

While the bath was coming to temperature equilibrium, I usually made a preliminary application of 12000 kg. to be sure that the joints were tight, and that the electrical insulation resistance between the heating element and the thermo-couple circuit was sufficiently high. The insulation resistance was measured independently of the potentiometer, and had in all cases to be as high as 100 megohms if satisfactory results were to be obtained. The pressure transmitting medium in all these experiments was petroleum ether. The use of this was necessary to avoid stresses in the couples introduced by viscosity effects. The mechanical difficulties of retaining this liquid under pressure without leak are very materially greater than for the mixture of ether and kerosene which I had used before. Slight mechanical imperfections in the bearing surfaces become much more important, so that the preliminary test for leak on the whole saved time. There did not seem to be any seasoning effect proper produced in the specimen itself by the initial application of pressure, as indeed there ought not to be if the specimen is homogeneous and well annealed. In a number of cases when the radial flow method was used and the specimen was mounted in a copper sheath, the specimen was seasoned by several applications of the maximum pressure before any of the electrical connections were made. Seasoning in this case was necessary because of unequal compressibility of the specimen and the copper of the sheath.

After reaching equilibrium at atmospheric pressure, the pressure zero was read. The magnitude of the current in the heating element and its shunt was then read on the potentiometer, both for the direct and the reversed direction of the heating current. The reversal of the heating current was accomplished with a double pole double throw switch and took place so quickly that there was no appreciable interruption in the heat input. The mean of readings with direct and reverse heating current eliminates the effect of any parasitic thermal e.m.f.'s in the heating circuit. The final reading made was that of temperature difference on the thermo-couple. This reading was also made with both direct and reverse direction of the heating current. Any error from electrical leakage from the heating circuit (in which the current is of the order of 0.5 amperes) into the thermo-couple circuit (in which the e.m.f. is of the order of a few microvolts) because of defective insulation, is thereby eliminated. The reversal of the heating current does not eliminate the effect of spurious e.m.f.'s in the thermo-couple circuit. There were no such spurious e.m.f.'s of detectible magnitude in the circuit in the absence of the heating current, as could be checked by the complete vanishing of the thermal e.m.f. on breaking the heating current. The only assurance against spurious e.m.f.'s introduced by the heating current itself was in the careful design of the apparatus. I could think of no way of making a direct check of this point without being able to introduce a cooling equal to the heating. In addition to the measurements above, as a rough check, the total current output from the storage battery into the ballast lamp was read on a commercial ammeter.

The pressure was then changed, usually by a step of 2000 kg., and after attaining equilibrium, the same succession of readings was again made. An interval of about 15 minutes was usually sufficient for equilibrium. This time is required to dissipate the heat of compression in the liquid and to again attain complete equality of temperature at the outer and inner ends of the three terminal plug. The usual routine consisted of two complete sets of readings to 12000 kg. and back. The results were plotted roughly as soon as obtained, making correction for any slight changes in the heating current and for fluctuation in the temperature of the copper coils of the potentiometer. This last correction was usually not necessary. When using the longitudinal flow method, after the cause of the irregularity of the readings was discovered, the plotted results were used to best direct the subsequent changes of pressure. The final computation of the results and the application of the other corrections which have been described above was made later.

The detailed data for the different metals follows.

Detailed Data.

Lead. More readings were made on this metal than on any other, and in general the results are better and the final result is probably more accurate than for any other. Nearly all the readings were made by the radial flow method, to which this metal is best adapted, because of its comparatively low conductivity.

Three grades of metal were used. A first rough trial was made with ordinary commercial lead. Several of the early measurements, in which a copper sheath was used, were made with so-called test lead from Eimer and Amend. The majority of the measurements, however, were made on lead of unusually high purity provided by the Bureau of Standards for the calibration of thermo-couples by the melting point. The analysis of this lead was as follows:

Ag .0002 – .0003%, Sb .0019 – .0028%, Sn .0008 – .0011%, Cu .0003 – .0004%, Fe. .0004 – .0006%, As, Bi, Zn, Co, Ni, trace, Cd, Mn, none; Lead (by difference) 99.9948 + %.

The melting point according to the certificate of the Bureau was 327°.3.

The accidental errors in the measurements were greater than differences due to slight amount of impurity, since all varieties of metal gave essentially the same results. The results are summarized in the following table.

TABLE I.

SUMMARY OF RESULTS FOR LEAD.

Description of Specimen	Percentage Change of Conductivity for 12000 kg/cm²
Solid Cu sheath, 3 copper tubes, commercial	21.5
" " " " " " "test"	20.2 19.2
" " " central Cu tube, "test"	23.6
Split Cu sheath, central Ag tube, Bur. Stds.	23.6 20.2
Large cylinder, no sheath, central Ag tube, Bur. Stds.	21.8 23.8
" " " " " " " " "	19.7 20.2 (12.1)
" " " " 0.007″ central hole, Bur. Stds.	18.7 20.0
" " " " " " " " "	16.5
Longitudinal flow, Bur. Stds. holes cast in specimen	20.9
" " " " " " " "	(8.1)
Average	20.7

In the above table, those specimens for which the nature of the central tube only is specified had the thermo-couple holes cast in the body of the metal in the isthmus shape already described. The figures in parentheses in the table were omitted in taking the average, as evidently they are affected by some large error.

Of these measurements the best was the second run with the first specimen with the 0.007″ central hole. It is indeed to be expected that this manner of mounting the specimen would give the best results. The individual readings of this run are reproduced in Figure 4.

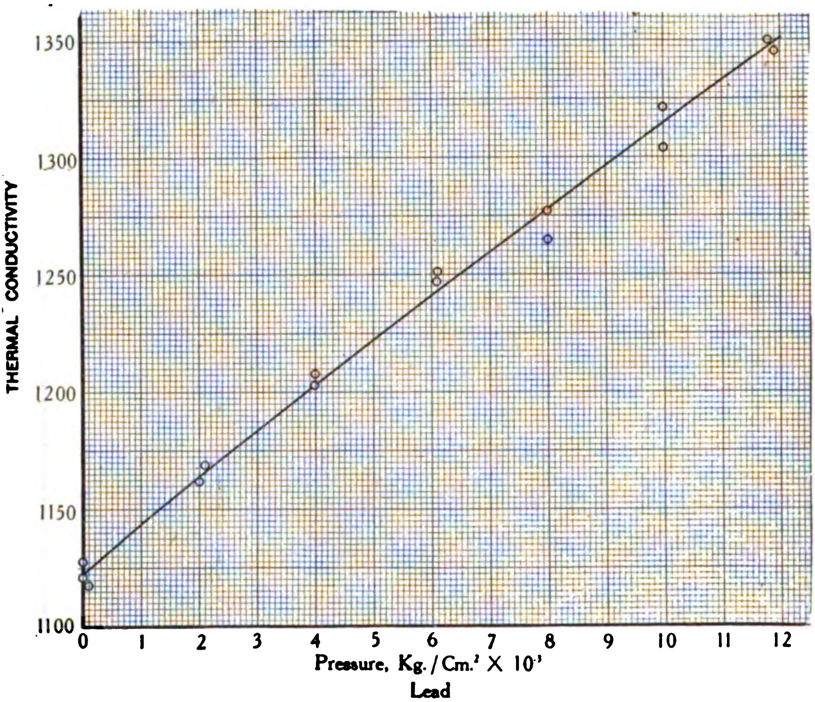

FIGURE 4. Lead. Thermal conductivity on an arbitrary scale against pressure in thousands of kg/cm². These results were obtained with a radial flow specimen.

The first of the two sets by the longitudinal flow method comes next in order of consistency, the individual points of this run lying even more nearly on a smooth curve than those shown in Figure 4, except for the initial point at atmospheric pressure, which lay off the curve by a large amount. The readings on the second longitudinal sample were not good for some unknown reason; possibly there may have been flaws in the casting, as was indeed found to be the case one or two other times when the longitudinal specimen was cast. For lead, the correction to be applied to readings on the longitudinal specimens for the lateral flow through the transmitting medium was unusually large, reducing the observed effect from 30% to 20%.

The readings on the two best samples were sufficiently good to show a departure from linearity with pressure, the slope becoming less at the higher pressures. This is as would be expected. Part of the departure from linearity is without doubt to be ascribed to the effect of the transmitting medium, and since for most of the other metals the accuracy was not great enough to show this departure from linearity, it did not seem worth while to attempt to establish it more exactly for lead.

It will be seen that the two best runs agree pretty closely with each other and with the mean of all the measurements, so that considerable confidence may be given to the average value, 20.7%, as the most probable effect of 12000 kg. It is to be remarked that this change of thermal conductivity is markedly greater than that of electrical conductivity under the same pressure, which is only 14.6%.

The average pressure coefficient of thermal conductivity per kg. change of pressure is 0.0000173. Lussana[2] found as the average coefficient between atmospheric pressure and 2600 kg., 0.0000134. His initial coefficient was 0.0000164, and the final 0.0_4116, showing departure from linearity in the same direction as I found, but much greater in amount. His results do not show, however, any departure from linearity in this range in the relation between pressure and electrical conductivity.

Tin. A few preliminary measurements were made on tin with "chemically pure" metal from Eimer and Amend, but all the results of any value were obtained with Bureau of Standards tin, prepared for the calibration of thermo-couples by means of the melting point. The analysis of the tin was as follows: Pb .007%, Cu .003%, Fe .002%; As, S trace; Sb, none; Tin (by difference) 99.988%, and the melting point given by the Bureau's certificate was 231°.9.

Twelve different samples of tin were used, ten by the radial and two by the longitudinal flow method. None of those specimens in which a copper sheath was used gave results of any value whatever. The reason for this has already been discussed; the effect is due to lack of good thermal contact between the tin and the sheath. Neither was it found possible to get any results of value with those specimens in which the heating element was enclosed in a central silver tube. For some reason it was not possible to make such good contact between silver and tin as between silver and lead. It may be that the greater ease of alloying between silver and tin resulted in inhomogeneity of the casting in the neighborhood of the central tube. Fairly consistent results were obtained, however, from those castings in which the heating element was placed in an axial hole 0.007 inch in diameter cast with the specimen. The results with the poor specimens which were discarded varied from 9 to 45% (the latter was for the split sheath which showed a progressive change with successive applications of pressure). The results with the better specimens are shown in the table.

TABLE II.

SUMMARY OF RESULTS FOR TIN.

Description of Specimen	Effect of 12000 kg/cm²
Large cylinder, no sheath, 0.007" central hole	15.4%
" " " " " " "	{14.2 / 13.6
" " " " " " "	{18.4 / 18.1
Longitudinal flow, all holes cast in specimen	11.1
" " " " " " "	11.9
Average	14.7

It is to be noticed that the results with the longitudinal flow specimens are lower than those for the radial flow specimens. The individual readings were not so good by the longitudinal method. It would perhaps have been better to have used a piece of extruded tin wire rather than the casting, in order that the specimen might be more homogeneous. The correction for lateral flow in the longitudinal method was 4.3 %, about a quarter of the total effect as measured. It is further to be noticed that the readings on the radial flow specimens which were repeated agree with each other more closely than they do with the mean, so that apparently the differences between the specimens are real. This may perhaps be partly ascribed to the effect of crystalline structure. Tin does not crystallize in the regular system, and is distinctly more crystalline in character than lead, for example, so that it is not surprising that there should be differences.

The best results were obtained with the first of the radial flow specimens listed above. The results with this specimen are reproduced in Figure 5. The same departure from linearity which was shown by lead is shown here also.

The average of the results listed above gives a mean pressure coefficient to 12000 kg. of thermal conductivity of $+ 0.0_4122$ per kg/cm². The corresponding coefficient of electrical conductivity is $+ 0.0_5929$.

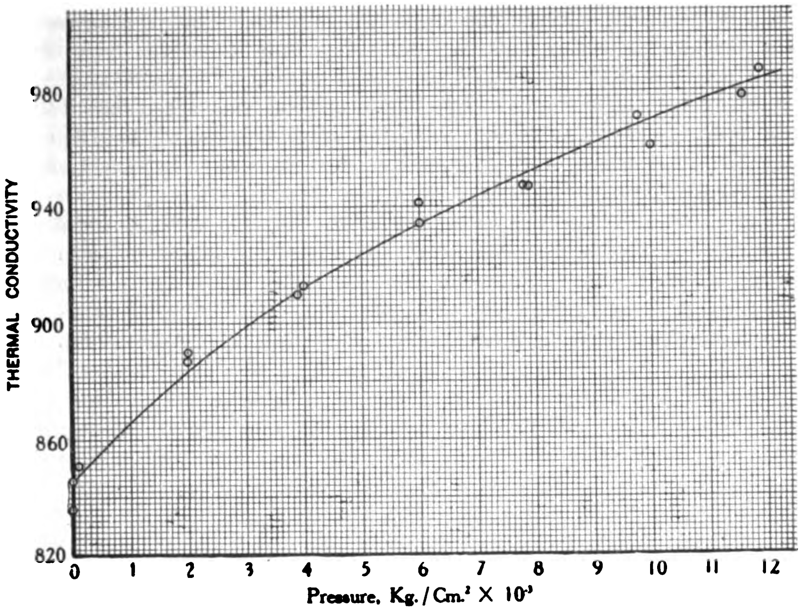

FIGURE 5. Tin. Thermal conductivity on an arbitrary scale against pressure in thousands of kg/cm². Results obtained with a radial flow specimen.

Lussana [2] found for the average coefficient of thermal conductivity to 2600 atmospheres 0.0_5719 (pressure expressed in kg/cm²). Lussana's relation was not linear, but there was a falling off of the coefficient of 19% between 0 and 2600 atmospheres. Up to 2600 atmospheres Lussana found the relation between pressure and the electrical resistance of tin to be linear, and the value of the coefficient 0.0_5820. In my previous work I have found the relation to be sensibly not linear, and the initial value of the coefficient to be 0.0_41055.

Cadmium. The material used for the experiments was "chemically pure" cadmium from Eimer and Amend. I have no analysis. In all, eight sets of readings were made, four by the radial flow method, and four by the longitudinal flow. In general the results were not good. A number of smooth curves were obtained by the radial flow method, but the numerical values for the different specimens varied greatly, the extremes being 5.0 and 31.5%. One of the radial flow specimens had a central silver tube to contain the heating element; the results with this were very irregular. The three other specimens had a 0.007 inch central hole, and the results with them were not so irregular. The best mean values with the four radial flow specimens were respectively 5.8, 26.7, 4.2, and 14.8%. The mean of these is 12.9%, or discarding the highest as differing too much from the others, the mean of the three remaining is 8.3%. Doubtless much of the irregularity in the results by the radial flow method was due to imperfections in the castings.

The individual readings for any one specimen by the longitudinal flow method were widely scattered, and would not give much confidence in the correctness of the result if it were not for the fact that the different specimens gave results in agreement. The longitudinal flow specimens were two of them cast and two extruded. One of the castings was annealed at 130° for 30 minutes, the other not. The extruded specimens were seasoned by a preliminary application of 12000 kg. These differences in treatment seemed not to affect the results. The last of the longitudinal flow specimens did not give results of value; the other three specimens gave changes under 12000 kg. of 9.0, 8.7, and 8.7% respectively. The magnitude of the correction for the pressure effect on the petroleum ether was 3.8%. The best mean coefficient from these measurements is taken to be 8.9%.

The pressure coefficient of thermal conductivity given by the above mean value is $+0.0_574$. For the mean pressure coefficient of electrical resistance between 0 and 12000 kg. I found 0.0_5932. The results for the effect of pressure on thermal conductivity were not sufficiently accurate to establish any departure from linearity with pressure. The effect of pressure on electrical resistance becomes less at the higher pressures; I found the initial coefficient to be 0.0_41085, and the mean 0.0_5912. These results may be compared with those of Lussana. Up to 3000 he found the effect of pressure on both thermal and electrical conductivity to be linear; the coefficient of thermal conductivity he found to be 0.0_4122, and of electrical conductivity 0.0_592. The first is higher than my value, and the second lower. Lussana's value for the pressure coefficient of thermal conductivity would be lowered by about 5% if my correction for the effect of pressure on the thermo-couple is applied. Lussana applied no such correction.

Zinc. Runs were made on three samples by the radial flow method, and two by the longitudinal method. The radial flow results were valueless; there is considerable difficulty in making homogeneous castings. The longitudinal flow specimens were extruded hot, and should be sufficiently homogeneous. The material was obtained from the Bureau of Standards, one of their melting point samples, and had the following analysis: Fe .005%, Pb .0004%, Cd .0018%, As, S trace; Sb, Sn none; Zinc (by difference) 99.993%. Melting point 419°.4. The results obtained with the two longitudinal samples were scattering, the points lying on several distinct lines, as already explained, but the correct slope could be picked out with some assurance. The change produced by 12000 kg. was 2.4% for one sample, and 2.7% for the other. Take as the mean 2.5%. The correction for the pressure effect on the transmitting medium was 3.0%, reducing a directly observed effect of 5.5% to 2.5%.

The pressure coefficient of thermal conductivity given by the above is $+0.0_521$. I had previously found the pressure coefficient of electrical resistance for a pressure range of 12000 kg. to be -0.0_5463. Lussana, over a pressure range of 2600 atmospheres, found the effect of pressure on thermal conductivity to fall off slightly from linearity, and the mean coefficient to be 0.0_541. The pressure coefficient of electrical resistance he found to be -0.0_5602; this relation he found to be sensibly linear.

Iron. Two samples were made for the radial flow method, and four for the longitudinal. The radial flow results were unsatisfactory. The thermo-couples and heating element were placed in copper tubes sweated into larger holes drilled in the specimen. The results were irregular, and the irregularities repeated themselves, showing that the effect is real, and denotes some defect in the specimen, not in the accuracy of the measurements.

Two of the longitudinal flow specimens were made from American Ingot Iron, from the same piece as the wires whose pressure coefficient of resistance and thermal e.m.f. have been previously measured.[4] The total impurity in this iron was not over 0.03%. The other two samples were made from a small sample of electrolytic iron of high purity for which I am indebted to the Bureau of Standards. The analysis of this iron is as follows:

Carbon	0.005	per cent.
Silicon	0.007	"
Sulfur	0.011	"

For some unknown reason the readings on one of these four samples were not satisfactory. The readings on the other three were quite typical of the results by this method, lying on one of several discrete lines. The results obtained with one of these are reproduced in Figure 6. The observed effect for iron is an increase of apparent conductivity of between 5 and 6% under 12000 kg., but the correction for the effect of pressure on the transmitting medium is so large, 5.3%, as to wipe out nearly all the observed effect. The results obtained

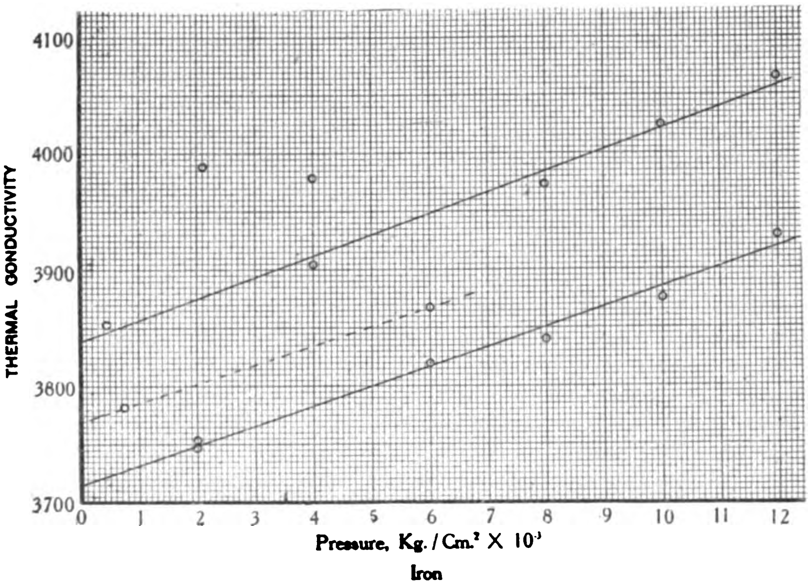

FIGURE 6. Iron. Thermal conductivity on an arbitrary scale against pressure in thousands of kg/cm². Results obtained with a longitudinal flow specimen. The points lie on several lines of the same slope; the reason for this is explained in the text.

with the three good samples, after applying all corrections, were −0.6, −0.2, and −0.2% respectively. The discarded data were not inconsistent with these values. We take as the mean −0.3%.

The pressure coefficient of thermal conductivity given by the above results is -0.0_63. I found for the pressure coefficient of electrical conductivity between 0 and 12000 kg. $+0.0_5229$. Iron is not one of the metals measured by Lussana, so there are no previous values for comparison.

Copper. Two specimens were made for the radial flow method, and four for the longitudinal. As in the case of iron, the thermo-couples and heating element of the radial flow specimens were placed in copper tubes sweated into place, and the results were not at all satisfactory. The points were not regular, and the irregularities repeated, showing some real effect. Furthermore, the two radial flow specimens were made from contiguous lengths from the same piece of commercial drawn rod, and the irregularities were much the same in character for each specimen, showing a real effect of inhomogeneities in the metal.

The four longitudinal specimens were made from electrolytic copper which I obtained a number of years ago from the Bureau of Standards.

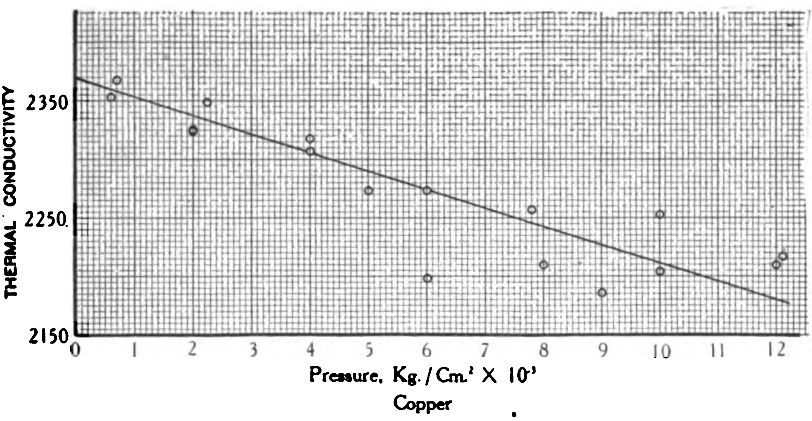

FIGURE 7. Copper. Thermal conductivity on an arbitrary scale against pressure in thousands of kg/cm². Results obtained with a longitudinal flow specimen.

Their analysis is as follows: Cu 99.995 per cent, trace S, no Ag, Cu_2O, As, or Sb. It is to be noticed that the purity is unusually high.

Measurements were made on some of these samples in the annealed condition, and others not; there seemed to be no difference in the results.

These four samples gave fairly good results, the points lying on discrete lines, as usual. The least scattering of these is reproduced in Figure 7. The effect is seen to be fairly large, and *negative*, the mean effects shown by the four samples were −9.7, −8.2, −8.3, and −7.5% change respectively under 12000 kg/cm². The mean is −8.4%, but we will take as the best result −9.0% instead, because the best of the specimens gave the higher results. Because of the high conductivity of copper, the correction for the transmitting medium is small, being only 1.2%.

The pressure coefficient of thermal conductivity deduced from the data above is -0.0_575. No departure from linearity could be detected. I have found for the effect of pressure on electrical conductivity over this range the mean value $+0.0_5181$. Lussana's results for copper run only to 1000 kg. He found the effect to be linear, and the conductivity to increase under pressure, the opposite sign from my results. His coefficient is $+0.0_510$. The relation between pressure and electrical resistance he also found to be linear, and the coefficient to be 0.0_5212.

Silver. Only the longitudinal flow method was used on this metal. The material was obtained from Baker, and was said to be of high purity, but I have no analysis. Three different samples were used. The two runs made on the first two samples apparently indicated an increase of conductivity under pressure. The reason for this has been discussed in detail previously; not enough readings were taken, and there was a tendency for the points to shift from a lower lying to a higher lying curve with increase of pressure, thus simulating a false effect. More readings were made on the third sample, and an effort made to control the position of the points on one or another of the possible lines. The results with this sample are reproduced in Figure 8. The tendency of the points to lie on one or another of three distinct lines is obvious, as also the fact that the effect is negative, the conductivity decreasing with increasing pressure, instead of increasing, as was indicated by the results first obtained. The two first samples were now set up again, and the measurements repeated, with the precautions which had been gained from the intervening experience. The results shown by these two samples were now also unmistakably negative, and of nearly the same numerical value as shown by the third sample. The numerical magnitude of the total decrease of conductivity under 12000 kg/cm^2 shown by the three samples was 4.1, 4.4, and 4.6% respectively. The correction for the pressure effect on the transmitting medium was 1%.

The pressure coefficient of thermal conductivity given by the above data is -0.0_537. The average pressure coefficient of electrical conductivity between 0 and 12000 kg. I have found to be $+0.0_5334$. This metal was not investigated by Lussana, so there are no previous results for comparison as to the thermal conductivity.

Nickel. Both methods were used for this metal, and material from two different sources. I am indebted to the International Nickel Co. for a piece of $\frac{7}{8}$ inch round rod of high commercial purity (99%). From this two samples were made for the radial flow method, large cylinders without the sheath. The thermo-couples and heating element were placed in fine copper tubes sweated into place. The results obtained with these samples were very irregular, and it was evident that the thermal contact between the copper and the nickel was not sufficiently good. These measurements did little more than establish a strong probability that the effect was negative. After the radial flow measurements had been made, two small pieces were

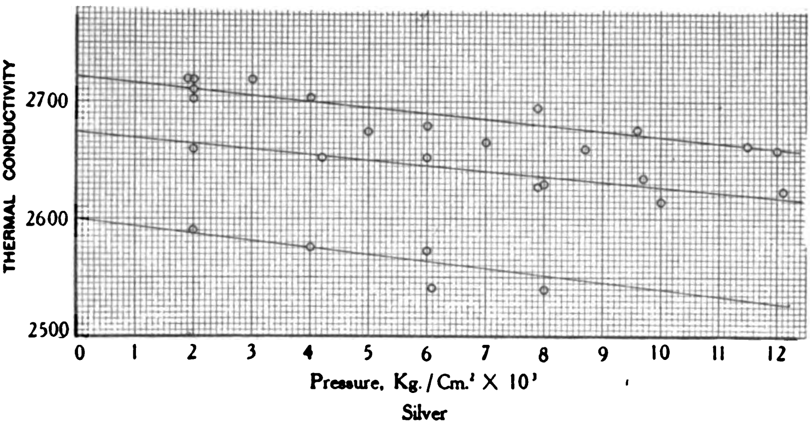

FIGURE 8. Silver. Thermal conductivity on an arbitrary scale against pressure in thousands of kg/cm². Results obtained with a longitudinal flow specimen. The points lie on several lines of the same slope; the reason for this is explained in the text.

cut from one of the cylinders for longitudinal flow specimens, and a few readings were made with them. This was before the explanation of the scattering of the points by this method had been found. The results were very scattering, and repetition would have been necessary to make sure of even the sign of the effect.

After completing the measurements on commercial nickel I was fortunate enough to obtain through the kindness of Mr. I. B. Smith of the Research Laboratory of the Leeds and Northrup Co. several samples of exceedingly pure nickel. I have no analysis of the nickel, but its high purity is vouched for by the unusually high value of the temperature coefficient of electrical resistance, which between 0° and

100° was 0.00634, against 0.0049 for commercial nickel over the same range. Two longitudinal flow samples were made from this pure material.

The first of these samples gave points lying on three different lines separated by the usual 5%. The slope corresponded to a decrease of conductivity of 13.5% for 12000 kg. In setting up the second sample I made the attempt to prevent motion of the thermo-couple wires in the holes with a piece of 0.002 inch wire laid beside them, as has been

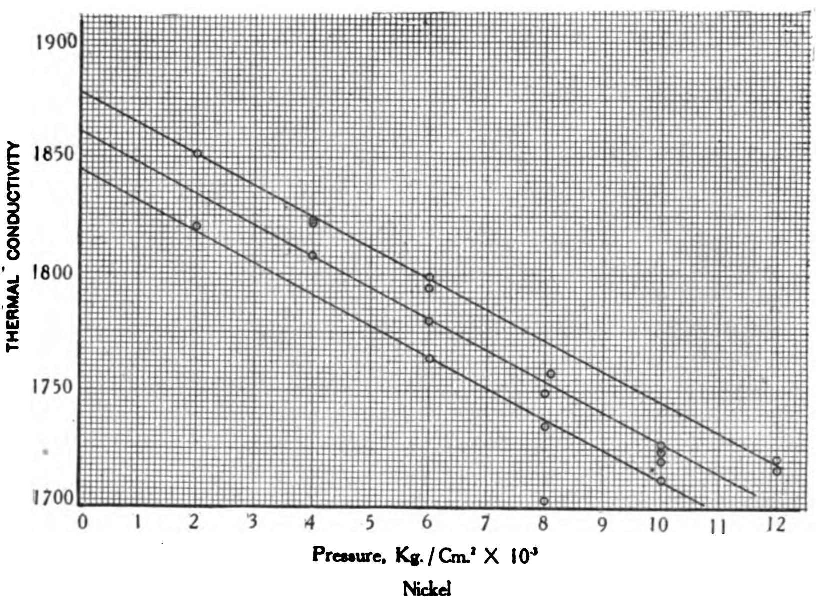

Figure 9. Nickel. Thermal conductivity in arbitrary units against pressure in thousands of kg/cm². Results obtained with a longitudinal flow specimen. The points lie on several lines of the same slope; the reason for this is explained in the text.

mentioned in the description of the method. This was the first attempt, and as often happens, succeeded better than subsequent attempts. For some reason I was fortunate enough to get the wire into place without introducing strains into the thermo-couples, and the results showed a gratifying regularity. The results of the final run with this second specimen are shown in Figure 9. It is seen that the readings still lie on three different lines, but these lines are now separated by much less than 5%, as is to be expected. A partial run with this same specimen, which had to be discontinued because of leak and also because of short circuit in the three-terminal plug, gave exactly the same slope for those readings which could be obtained as the final run. The change shown by this second sample was a decrease of conductivity of 14.5% for 12000 kg., agreeing fairly well with the first sample. Since the second sample gave somewhat more regular results, it is given greater weight in the mean, which I take as 14.1%. The total correction for the effect of pressure on the transmitting medium was 5% of the total conductivity, amounting to about 33% of the observed change under 12000 kg.

The results found above give for the pressure coefficient of thermal conductivity $-0.0_4 12$. There are no previous results for comparison.

An incidental result obtained from the measurements with commercial and pure nickel was a comparison of the absolute thermal conductivities. The longitudinal flow method is not well adapted to give the absolute conductivity because of the uncertainty in the corrections for loss through the leads, etc. (the absolute conductivities directly calculated average about 5% higher than the values of Jaeger and Diesselhorst), but the comparative values of absolute conductivity of different materials should be nearly correct. The thermal conductivity of the two samples of pure nickel was found to be 37% higher than that of the two samples of commercial nickel. Considerable confidence may be put in these values, as the individual readings were very consistent; the two samples of pure nickel gave results differing by less than 0.5%, and the results on the two samples of commercial nickel were identical to three significant figures.

Platinum. Measurements were made on two different samples by the longitudinal flow method. The material was obtained from Baker, the purest which they could supply. I have no analysis, but have the statement of Baker that the impurity is guaranteed to be less than 0.1% and is probably not greater than 0.01%.

The measurements on platinum were scattered on three lines of a maximum separation of about 5%, as is usual with this method. The two specimens gave identical results, a decrease of conductivity of 1.9% for a pressure change of 12000 kg/cm^2. The observed effect was positive, but the correction for the effect of the transmitting medium is so large, 5.2%, as to alter the sign of the result.

The pressure coefficient of thermal conductivity as given by the above measurements is $-0.0_5 16$. The pressure coefficient of electrical conductivity at 30° between 0° and 12000 kg. is $+0.0_5 186$.

Bismuth. An attempt was made to obtain measurements on this metal by the radial flow method, but without success. It did not prove possible to get sufficiently good thermal contact with the fine copper tubing, and I have already mentioned that the attempt to use silver tubing failed because of the extremely rapid alloying action between silver and bismuth. Measurements were finally made on three different specimens by the longitudinal flow method.

A great deal of time was spent in the endeavor to obtain pure material. In my previous work on the effect of pressure on electrical resistance I had purified the metal by electrolysis. I now endeavored to repeat this, but without success; I could not make the bismuth form a coherent deposit. The procedure of the previous work was exactly repeated as far as I could tell. Previously the hydrosilicofluoric acid had been obtained from a German source; this was no longer available and acid from the J. T. Baker Chemical Co. was used instead. The acid was of high purity as indicated by the analysis on the label, but there is a possibility that some impurity not covered by the analysis might have been responsible. I then obtained some bismuth from the U. S. S. Metals Refining Co. I have to thank them for supplying me with six pounds of the metal, in two lots. Their product is prepared electrolytically, in distinction from the ordinary commercial product, and is guaranteed by them to have less than 0.1% impurity. Ordinary commercial bismuth has about 3% impurity. My test for purity has been the temperature coefficient of electrical resistance. This electrolytic bismuth showed a very low coefficient, only 0.0022 between 0° and 30°. Ordinary commercial bismuth is higher. Professor F. A. Saunders was kind enough to make a spectroscopic analysis; he found a very strong silver line, which seemed to indicate a rather considerable impurity. I consulted the U. S. S. Metals Refining Co. again, and they were so kind as to send me a second sample, which they had submitted to chemical analysis, and found to contain only silver in detectible quantity, and this was less than 0.06%. But the temperature coefficient of this new sample was again very low. I attempted a purification by slow crystallization from the melt, with the result of bringing the coefficient up to only 0.0025. Ordinary commercial material, purified in the same manner, showed a coefficient of 0.0034. Professor Saunders was again kind enough to make a spectroscopic analysis and found again the strong silver line, which seemed to him could only arise from a rather large amount of impurity. He found traces of Cu and Pb, and no traces of Sn, Cd, Zn, Li, As, or Sb. Professor G. P. Baxter was now so kind as to make a quantitative determination of the silver, and found 0.03%, confirming the conservative estimate of the U. S. S. Metals Refining Co. I now succeeded in finding a small residue of my original electrolytic bismuth, and Professor Saunders made a spectroscopic analysis of this. He could find only traces of Cu and Pb, the Cu being stronger than in the commercial electrolytic bismuth, and some silver, evidently considerably less than in the commercial. The conclusion seems forced that a quantity of silver as small as 0.03% can depress the temperature coefficient to half the normal value, thus exerting an effect very much greater than such impurities as Pb and Sn, which are present in ordinary commercial bismuth. That difficulty would be expected in removing the silver by recrystallization is evident on an inspection of the mixed crystals diagram for these two metals. This would also be indicated by the energetic alloying of silver and bismuth, which made impossible the preparation of the radial flow specimens.

Under the circumstances it seemed that the best thing to do was to use the commercial electrolytic bismuth, with its known analysis of 0.03% of silver, in the expectation that the effect of this small impurity is abnormally high on the temperature coefficient of resistance. I had previously found that the effect of impurity on the pressure coefficient of resistance is much less than on the temperature coefficient of resistance.

The samples were made from $\frac{1}{8}$ inch wire which had been formed by hot extrusion in the regular way. One advantage of forming the specimen by extrusion is that the crystalline structure is very much finer than when the specimen is cast, and so the results are much more likely to give the average for all the directions of a single crystal.

The thermo-couple holes were drilled in these specimens in the regular way, but a modification was necessary in mounting the heating element. Previously the heating element was mounted in a copper capsule, which was cemented into a hole drilled in the end of the specimen. This was no longer possible, because the capsule was so large that it was not possible to drill a hole to receive it without breaking out the walls in so brittle a material as bismuth. The heating element was accordingly placed in a smaller hole drilled directly in the end of the specimen. This has the disadvantage that the terminal conditions of temperature are not so accurately defined as with the other metals, and the motion of the heating element in its receptacle may produce other irregularities. This was indeed the fact; the points were more scattered than with the other metals, and the width of the band of scattering was greater than the 5% usually found. The magnitude of the scattering varied with the specimen, as it might be expected to.

In setting up the first sample, the ground of the heating element was made to the sample itself. It has been previously explained that this introduced an effect due to the Peltier heat, which is unusually large for this metal. The mean of readings with two directions of the heating current should eliminate this effect. With the other two samples an independent ground was used, and the effect disappeared.

It is to be expected that if there is any error due to heat leak along the thermo-couple wires that it will be especially large for this metal, whose own thermal conductivity is so small. In order to test this point, duplicate runs were made on the third sample, first with the ordinary copper-constantan couple, and then with a couple of "therlo" constantan. Very nearly the same results were found, showing the adequacy of the precautions taken to prevent leak along the couple wires.

The third sample gave the most regular results; probably some accidental twist or bend in the wires made them less likely to be displaced under pressure than those of the other samples. The results with this sample are reproduced in Figure 10.

The results found with the different samples are as follows: #1, −38.8% for 12000 kg.; #2, −38.8%; #3 (copper-constantan couple) −37.3%, and #3, (therlo-constantan couple) − 35.5%. Mean −37.8 %. Because of the low conductivity of bismuth the correction for the transmitting medium is large, amounting to 13.8%, and is in the direction to make the true effect more negative than the observed effect.

The pressure coefficient of thermal conductivity to be deduced from the above measurements is -0.0_431. It is to be noticed that this is negative, and also that it is abnormally high. The abnormal sign agrees with the pressure effect on electrical conductivity, which also decreases under pressure. I found at 30° the average pressure coefficient of electrical resistance up to 12000 kg. to be -0.0_4212. Bismuth was not among the metals measured by Lussana, so there are no previous results for comparison.

Antimony. Measurements were made on four samples, all by the longitudinal flow method. The material was obtained from the J. T. Baker Chemical Co. It was supposed to be especially pure, although I have no analysis. Antimony from the same source was formerly used by the Bureau of Standards to give a fixed melting point, but their experience was that although the chemical analysis might show very little impurity, there was nevertheless present in all antimony from American sources some slight impurity which was sufficient to displace the melting point by several degrees. Presumably my antimony suffered from the same impurity.

Two of my samples were made from cast antimony and two from extruded metal. The metal was cast by pouring it into a groove in a massive iron block to a thickness of a trifle over $\frac{1}{8}$ inch. The chilling was hence very rapid, and the crystalline structure very fine. From

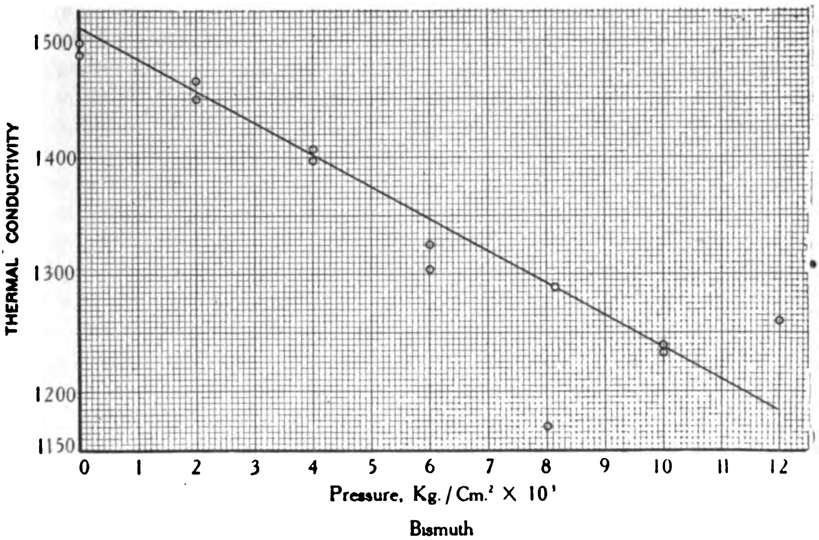

FIGURE 10. Bismuth. Thermal conductivity in arbitrary units against pressure in thousands of kg/cm². Results obtained with a longitudinal flow specimen.

this casting two pieces were machined for the longitudinal flow method. The metal is so brittle that it was not possible to cut it with the tool in the ordinary way, but the machining had to be by grinding. Even then extreme caution was necessary, and there were many failures before success was attained. The wire for the other two specimens was extruded at a red heat through a high speed steel die of the required dimensions. Some little practice was needed before the proper manipulation was found. It is possible to extrude antimony at a considerably lower than a red heat, but with a wire as large as $\frac{1}{8}$

inch the product is likely to be very brittle, or break spontaneously into small pieces. If the temperature is raised very close to the melting point, however, it is possible to get by extrusion a uniform straight wire with no apparent flaws, and not as brittle as the casting.

The thermo-couple holes were drilled in the four pieces in the regular way. The heating elements, as in the case of bismuth, were mounted directly in small holes drilled in the ends, it not being feasible to drill so large a hole as the use of the copper capsule would have demanded. This manner of attaching the heating element was responsible for the greater irregularity of the points, as also in the case of bismuth. In one case the scattering was such and the accidental

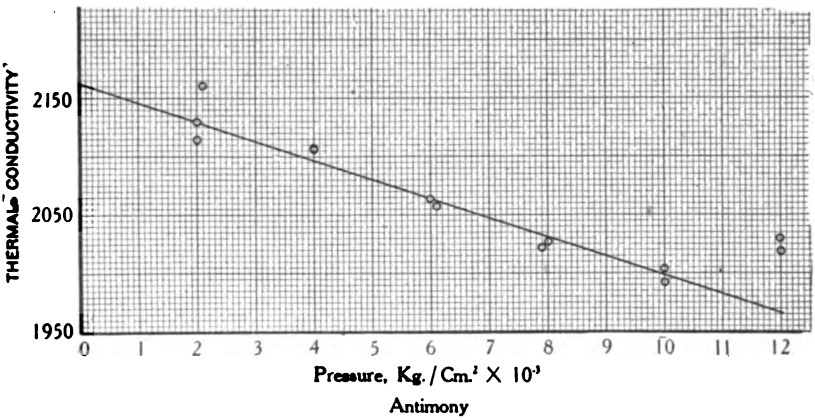

FIGURE 11. Antimony. Thermal conductivity in arbitrary units against pressure in thousands of kg/cm². Results obtained with a longitudinal flow specimen.

distribution such that a positive sign for the effect might have been suspected.

The readings obtained with the first of the cast specimens were the most regular; these are reproduced in Figure 11. The thermo-couple used with the second of the extruded specimens was therlo-constantan, instead of copper-constantan. The readings with this were essentially the same as with the others, thus again showing freedom from heat leak along the wires of the couple.

The thermal conductivity decreases with rising pressure; the two cast specimens gave respectively −23.9 and −26.3%, and the two extruded specimens −24.8 and −23.9%. The mean of all four is

-24.7%. It is to be noticed that within the limits of error no difference is to be detected between the cast and the extruded specimens.

The pressure correction for the transmitting medium was 15.3% on the total conductivity; this means that the final corrected result was three times as large as the observed pressure effect.

The above results give for the pressure coefficient of thermal conductivity -0.0_421, larger than for any other metal except bismuth. I have previously found that the electrical conductivity of antimony also decreases with rising pressure, and at 30° the average coefficient to 12000 kg. is -0.0_4108.

Lussana has also measured the effect of pressure on the electrical and thermal conductivity of antimony, and his results are in precise disagreement with mine. He finds that the electrical conductivity increases under pressure, as it does for normal metals. At 25° his coefficient, presumably to 3000 atmospheres, is 0.0_5874. There must be something vitally wrong here; measurement of electrical resistance under pressure should offer none of the difficulties of thermal conductivity, and there should be no reason for a disagreement as to sign between different observers. The relation between pressure and thermal conductivity Lussana finds to be distinctly not linear. The initial change is at a rate corresponding to a coefficient of 0.0_5251, and at 3000 the rate corresponds to a coefficient of only 0.0_5164. So large a departure from linearity in a metal with as high a melting point as antimony is without precedent. It does not seem improbable that the sign that Lussana found for the coefficients of both electrical and thermal conductivities may be due to a closing of minute fissures between the crystalline grains under pressure, such as Borelius and Lindh found for bismuth.[5] The departure of his thermal conductivity from linearity is in accord with this suggestion.

Petroleum Ether. It has already been explained that it was necessary to determine the absolute conductivity and pressure coefficient of this substance in order to obtain the correction due to the effect of pressure on the transmitting medium in the longitudinal flow method. The method adopted for determining these constants for petroleum ether was a radial flow method, and demanded very little change in the apparatus already used for metals. The apparatus is shown in Figure 12. It consists of an inner cylinder of copper held concentrically within an outer hollow cylinder, which in turn fits closely inside the pressure cylinder, and is maintained concentric with it by the same spring device that was used for the metals. The petroleum ether fills the annular space between the two copper cylinders. The axis of the central cylinder contains a linear source of heat, that is a wire carrying a current, precisely as for the metals. The difference of temperature between the outer surface of the inner cylinder and the inner surface of the outer cylinder is measured by thermo-couples. These were of the same construction as for the metals, and were mounted in fine copper tubes, which were sweated into small holes drilled lengthwise of the cylinders. Of course with this construction the couples could not be located exactly on the surface of either cylinder, but the thermal conductivity of the copper is so much higher than that of the petroleum ether that practically all the temperature drop takes place across the annular space of the liquid, and a rough computation shows that with the dimensions used any error from this cause is negligible. As a precaution against failure of perfect geometri-

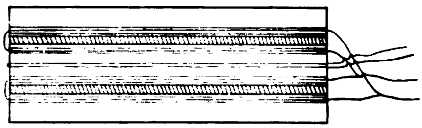

FIGURE 12. Longitudinal section of the apparatus for measuring the thermal conductivity of petroleum ether. The liquid is shown shaded between copper cylinders, with a central heating unit and two sets of thermal junctions bridging the liquid.

cal centering of the inner cylinder in the outer one, three couples instead of one were used, spaced at even angular intervals around the cylinders, and these were connected in parallel, so that the reading obtained gave the mean of the temperature differences around the cylinder, and any geometrical imperfection is eliminated. The annular space between the cylinders was only 1.3 mm. wide. This is so narrow as to remove any error from convection in the liquid, even at atmospheric pressure, and it has already been explained that such error vanishes at higher pressures because of the rapidly increasing viscosity. No effects were found in the measurements to suggest error from such a source.

Because of the substantial equality of temperature throughout the copper cylinders, it is to be expected that errors from slight changes in position of the thermo-couples, which played so large a part in the measurements of the metals, would vanish. This is indeed the fact, and the measurements showed a high degree of regularity, much beyond that obtained for any metal.

In making the readings, the entire interior of the apparatus was filled with petroleum ether. The same method would serve for any other liquid which does not absorb impurity, or become conducting under pressure. Unfortunately most of the liquids whose other properties are best known, and which it would be most interesting to measure, such as the alcohols, are not of this kind. To determine the thermal conductivity of these under pressure it will be necessary to so modify the apparatus that the liquid can be insolated from the electrical leads.

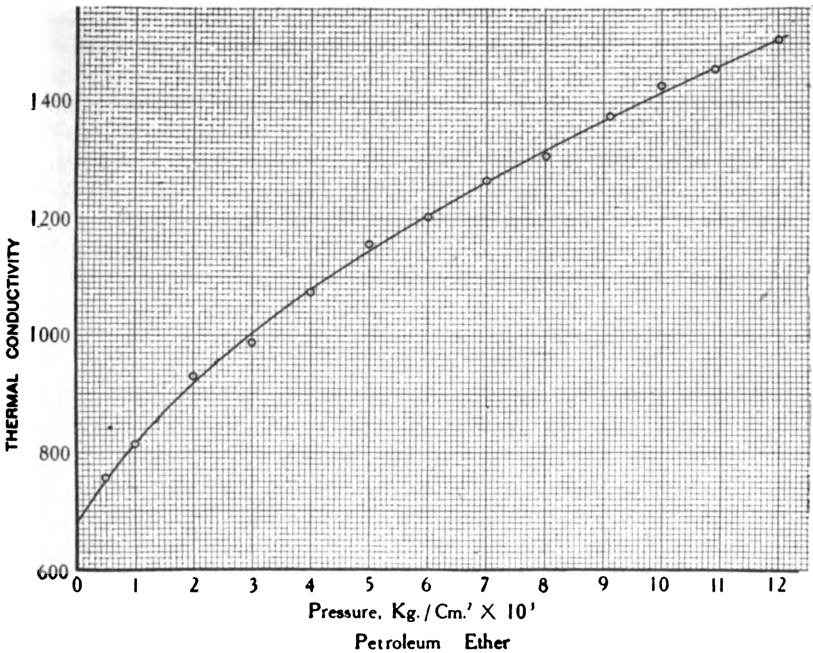

FIGURE 13. Petroleum Ether. Thermal conductivity in arbitrary units against pressure in thousands of kg/cm². Results were obtained with the apparatus of Figure 12.

The observed results with petroleum ether are shown in Figure 13. The greater regularity of the results as compared with the metals is manifest. The lowest pressure of these readings was 500 kg. The reason for not going to atmospheric pressure was not error from convection currents, but because at this pressure the heating effect would have been sufficient to vaporize the ether, and so introduce error.

The effect of pressure is seen to be a large increase of conductivity, amounting at 12000 kg. to an increase of 2.22 fold. The increase is not linear with pressure, but there is a departure from linearity in the normal direction, in that the change becomes proportionally less at the higher pressures.

The initial rate corresponds to an increase of conductivity of about 20% per thousand kg. So far as order of magnitude goes, this agrees with Lussana, who found the correction for his transmitting medium to be at the rate of 30% per thousand kg. He did not find a departure from linearity. Of course there is no reason to expect more than agreement as to order of magnitude, because his transmitting medium was a comparatively heavy oil, quite different in properties from mine.

The measurements of the effect of pressure on the thermal conductivity of liquid is a thing worth doing for its own sake, and I hope to get the chance to make measurements on a number of others. In fact I already have results for two alcohols and kerosene. Suffice it here to mention that there seems to be an intimate connection between the pressure effect on thermal conductivity and the pressure effect on the velocity of propagation of sound.

General Comment on Lussana's Results.

The only previous measurements of the effect of pressure on thermal conductivity are those of Lussana. Since his results often differ essentially from mine, even as to sign, and since this is a matter of considerable importance for theoretical considerations, some critical survey of his results seems called for. In general, Lussana finds that the thermal conductivity of all metals increases under pressure, and this increase is nearly the same as that of the electrical conductivity, so that the Wiedemann-Franz ratio remains nearly constant under changes of pressure.

Lussana's method was an adaptation to high pressures of one originally due to Depretz and Biot. A long bar of metal has a source of heat at one end and is immersed in a medium through which the heat may flow away laterally. The temperature of the bar, which is assumed constant across the section, is measured at three equi-distant points along it, and in terms of the two differences of temperature thus obtained, a relation can be found between a certain geometrical factor and the ratio between the thermal conductivity and the lateral conductivity into the surrounding medium. The essential difference between this method and mine is that in mine there is a source at one end of the bar and a sink at the other, so that nearly all the heat input flows through the bar and out at the other end, and only a comparatively small part is lost laterally to the surroundings; whereas with Lussana all the heat input flows out laterally. In Lussana's method the correction for the effect of pressure on the transmitting medium affects directly the entire heat input, whereas in my method the pressure correction is to be applied only to that part of the heat input which escapes laterally.

My most serious criticism of Lussana's method concerns this correction for the transmitting medium. The magnitude of the correction is about 30% per thousand kg., whereas the order of magnitude of the changes of thermal conductivity of the metals is at most only 3%, or one tenth of this. This demands that the effect of pressure on the transmitting medium be known ten times well as the final result for the metal. Nevertheless, Lussana determined the correction for the liquid to only one significant figure; as a matter of fact there is a misprint in his paper, which made the correction appear to be at the rate of 300% for one thousand kg. I inquired about this in a letter to Lussana, and he told me that the decimal point had been displaced one figure, and that the correct result was 30%, agreeing with my own results as far as order of magnitude goes. Having determined the correction to one significant figure, not even noticing the departure of the effect from linearity with pressure, Lussana gives his coefficient for metals to three significant figures. Three significant figures for the metal would have demanded at least four significant figures in the correction.

Lussana states that his results were computed from the observations by the method of least squares; he does not anywhere reproduce a single set of observations, nor does he state the probable error of his results, surely a significant omission considering the method of computation. There is no clue in his paper to the accuracy to be attached to his results.

There seems to be almost no correlation between Lussana's results and my own. In only one case, that of zinc, do we find the same sign for the change produced by pressure in the Wiedemann-Franz ratio. It seems to me that for the present we are justified in assuming that there are large errors in Lussana's results.

Discussion.

Probably the most significant theoretical conclusions from the above data may be derived from the pressure coefficient of the Wiedemann-Franz ratio. The classical electron theory would lead us to expect that the coefficient would be zero, since the ratio is the same for all metals at the same temperature, and the same metal under different pressures at the same temperature is merely a special case of two different metals. As a matter of fact the ratio is not constant, but may either increase or decrease with increasing pressure; in the majority of cases it decreases. The average values of the coefficient between 0 and 12000 kg. are shown in Table III.

TABLE III.

Metal	Pressure Coefficient of Wiedemann-Franz Ratio
Pb	$+0.0_5 6$
Sn	$+0.0_5 3$
Cd	$-0.0_5 17$
Zn	$-0.0_5 25$
Fe	$-0.0_5 26$
Pt	$-0.0_5 35$
Ag	$-0.0_5 70$
Cu	$-0.0_5 93$
Ni	$-0.0_4 13$
Sb	$-0.0_4 10$
Bi	$-0.0_4 10$

My own theory of electrical conduction attempted to explain the Wiedemann-Franz ratio,[6] and to do this, I imagined the same sort of mechanism of conduction as the classical theory. I still can see no reason to suppose that the most important part of heat conduction is not as imagined by the classical theory; the success of the theory in accounting for the numerical value of the ratio, which is approximately constant for the different metals, (it may vary from 6.38×10^{10} for aluminum to 9.14×10^{10} for bismuth), is too striking to be put aside with no substitute. At the same time it is evident that the account given by the classical theory cannot be complete; no account has been taken of the conduction by the atoms, and the agreement of the theoretical with the experimental value is not as close as we must demand of a finished theory.

It is natural to look to the still unexplained part of thermal conductivity to account for the departures from constancy of the Wiedemann Franz ratio under pressure. The part of the conductivity which is due to the electrons would be expected to have the same pressure coefficient as the electrical resistance (except for a possibility to be mentioned later); the remaining part must be capable of either positive or negative variation under pressure, and must be of the right order of magnitude.

In the first place, let us consider the possible magnitude of the non-electronic part of heat conduction. The first deduction of the theoretical value of the Wiedemann-Franz ratio, given by Drude, was an elementary one, in which certain simplifying assumptions were made, particularly that the velocities of all the electrons were the same. Later Lorentz gave a more exact discussion, taking account of the Maxwell distribution of velocities among the electrons, and obtained a value for the ratio only two thirds of that of Drude. The discussion of Lorentz has later been verified by Bohr and others. The elementary value for the ratio is much closer to the experimental values than the more rigorous one, but still lies somewhat low. The failure of the more exact value to agree more closely with the facts has been regarded by some as a blot on the classical theory, but by others is regarded rather as to the credit of the theory, because the Wiedemann-Franz ratio as calculated by the elementary theory was felt to be too close to the experimental value to sufficiently allow for the atomic part of the conduction.

I shall take this latter point of view, and consider that the value for the Wiedemann-Franz ratio calculated by Lorentz represents the part due to electronic conduction, and that the difference between this theoretical value and the actual value represents the part of the heat conduction that must be accounted for in other ways. This point of view at once imposes certain numerical limits on the changes under pressure that it should be possible to obtain experimentally. For the total change of thermal conductivity under any pressure must never be so great as to more than wipe out the part of the conductivity which was initially ascribed to the non-electronic part. This means that the total decrease of thermal conductivity, after allowing for a change equal to the change of electrical conductivity, must not be greater than the difference between the total initial thermal conductivity, and the part given by Lorentz's expression. In practise this imposes a restriction only when the thermal conductivity increases under pressure less rapidly than the electrical conductivity. An examination of the results obtained in this paper will show that this condition is met in all cases. The condition imposed is most restrictive in the case of nickel. Under 12000 kg. its thermal conductivity decreases by 14.5%, and the electrical conductivity increases by 1.8%. The sum of these, 16.3%, is an upper limit which the fractional part of the total conductivity due to the non-electronic part must not exceed. Now the theoretical value for the Wiedemann-Franz ratio is 4.3×10^{10} (Lorentz's value), and the experimental value for nickel is 6.99×10^{10}. This allows the possibility of 39% of the thermal conductivity initially being of non-electronic origin, which is more than twice the extreme set experimentally. It would seem that under these conditions, when so comparatively large a part of the atomic conductivity has been wiped out by pressure, that the relation between conductivity and pressure must depart from linearity. The experimental accuracy was not great enough, however, to show such a departure.

Further consideration of the theoretical significance of these results is reserved for a forthcoming paper in the Physical Review.

Summary.

Two methods are described for measuring the thermal conductivity of metals under pressure. The first of these is a radial flow method, which has many theoretical points of advantage, but is of limited applicability in practise because of the difficulty of getting metals in a condition of sufficient homogeneity. The second is a longitudinal flow method, the essential of which is the small size of the specimen. The irregularities of the individual readings by the second method are greater than by the first method, but the effect of inhomogeneities is less and different specimens of the same metal will give the same result.

Measurements of the effect of pressures to 12000 kg/cm^2 on the thermal conductivity of 11 metals have been made by one or the other of these methods. The effect may be either positive or negative, and is more often negative than positive. In only two cases, lead and tin, does the Wiedemann-Franz ratio increase under pressure; for the other metals it decreases, and sometimes by large amounts. In addition to the metals, the pressure coefficient of thermal conductivity of petroleum ether has been measured. The conductivity increases by a factor of about 2.2.

The only previous measurements have been by Lussana, who obtained results entirely different from those found here. His method is criticised in some detail, chiefly on the basis of the uncertain correction for the lateral loss of heat to the pressure transmitting medium.

These results indicate that a fairly large part of thermal conduction

in a metal is performed by the atoms. Theoretical reasons are given for estimating the atomic contribution to the thermal conductivity as 50 per cent of the electronic contribution.

I am much indebted to the skill of my assistant Mr. J. C. Slater, who made a large number of the readings. It is also a pleasure to acknowledge my indebtedness to the Rumford Fund of the American Academy of Arts and Sciences for a grant with which a part of the expenses were defrayed.

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THE FAILURE OF OHM'S LAW IN GOLD AND SILVER AT HIGH CURRENT DENSITIES.

BY P. W. BRIDGMAN.

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Introduction.

It is to be expected that at high current densities the usual linear relation between current and E.M.F. in a metal will break down, that is, that Ohm's law will fail. J. J. Thomson,[1] for instance, has shown that on the basis of the classical free electron theory of metallic conduction the current will eventually increase only as the square root of the E.M.F. at very high values, which means that the resistance will increase at high current densities. On the usual assumptions the current densities at which this effect will become important are of the order of 10^{11} amp/cm^2. Many attempts have been made to detect the existence of this effect experimentally, but hitherto without success. The chief obstacle to success is that the changes of resistance due to heating by the heavy current are sufficient to mask the changes of resistance due to a possible departure from Ohm's law.

By the employment of a new method of attack, which avoids errors due to temperature rise, I have, I believe, not only succeeded in establishing the existence of the effect, but in measuring the departures from Ohm's law with some exactness in gold and silver. These results are described in the following paper. I find a departure from Ohm's law in the direction of an increase of resistance at high current densities, the maximum effect being of the order of 1% at a current density of 5×10^6 amp/cm^2.

Historical Survey.

A few attempts were made to detect the existence of the effect before Maxwell, but we may pass these over as not approaching in sensitiveness the method of Maxwell.[2] Maxwell was the chairman of a committee, the other members of which were J. D. Everett and A. Schuster, appointed by the British Association in 1876, to investigate the accuracy of Ohm's law. Two experimental methods were used, both apparently proposed by Maxwell; the experiments were performed by Chrystal.

The first was a substitution method, by which the resistances of five similar coils were compared in various combinations of two in parallel and two in series against a single one. The current density in the single coil was thus double that in the multiple arrangement, and if there is an effect of the kind sought, the resistances should be different. A small positive result was found, but was ascribed to errors in view of the fact that the second method, which was much more delicate, gave negative results.

The current densities employed in the first method were so low as not to cause appreciable heating of the wires. The second method was such that currents large enough to heat the wires to incandescence could be used. A heavy and weaker current were passed alternately in rapid succession through a fine wire, which was made one of the arms of an ordinary bridge. The period of alternation was so high that there were no appreciable cooling effects. Observations were made with a galvanometer of period long compared with that of the alternations. The apparent resistance was read first with the large and the small currents flowing in the same direction, and then with the small current reversed. Let us suppose that there is an effect of the kind sought, which means that the resistance for the large and the small current is not the same. If the galvanometer indicates balance, it must be because it is really off balance for both currents, to the one side for the large current, and to the other side for the small current. If now the small current is reversed the galvanometer will be off balance to the same side for both currents, and there will be a steady deflection. Hence if there is an effect, the balance will be altered by changing the direction of the small current. There were difficulties in the method. The period of alternation had to be chosen as high as 60 per second in order to avoid appreciable cooling effects in the short intervals of time when the small current replaces the larger one. There were considerable mechanical difficulties in designing an alternator of the requisite constancy, for it is obvious that the durations of the large and the small currents must be absolutely constant. A platinum contact dipping into a mercury cup and driven by a tuning fork was used, but always gave trouble, and the limits of accuracy were set by this part of the apparatus.

The conclusion as usually quoted which was drawn from these experiments was that any deviation from Ohm's law must be less than one part in 10^{12}. This statement needs some expansion; it is obvious that no measurements can be made directly to this degree of accuracy. For one thing, changes of temperature of the surroundings absolutely preclude the direct attainment of any such accuracy as this. The meaning of the statement is as follows. Maxwell remarked that any departure from Ohm's law must involve only even powers of the current; it is obvious that the first power cannot enter, for if so there will be a dependence of resistance on the direction of the current, which cannot be the case in an isotropic material. The initial departures from the law may be supposed to be proportional to the square of the current density. The maximum current density employed by Maxwell was 5.6×10^4 amp/cm². At this density the resistance was found to have changed by not as much as 0.3%. Assuming the square law, this means that at a current density of 1 amp/cm² the departure from Ohm's law cannot be more than 1 part in 10^{12}. The original paper contains the careful statement of the conclusion in this form.

The metals used by Maxwell were cylindrical wires of platinum (0.042 mm. diameter), German silver (0.051 mm.) and iron (0.14 mm.). The maximum current densities were respectively 3.4, 1.2, and 5.6×10^4 amp/cm².

Recently Wenner[3] of the Bureau of Standards has objected to the second form of Maxwell's experiment. He has repeated a modification of the first method with very much higher accuracy, and finds no deviation of as much as 3 parts in 10^7. His objection to the second experiment is that negative results might be obtained even if there is a departure from Ohm's law. Thus if the potential difference across each arm of the bridge is proportional to the square of the current flowing through it, negative results will be obtained, because the bridge will stay in balance for any current, large or small. More generally, negative results will be found if the potential difference across each arm is the same function of the current for each arm. It is to be noticed, however, that this is not the manner of departure from Ohm's law which is to be expected. The departure sought for is not a function of the total current flowing through the resistance, but is a function of the current density. Furthermore, this function is known to be nearly independent of current density for low values. Doubling the total current in a conductor of small section will produce a much greater departure from Ohm's law than doubling the same current in a conductor of larger section. This is the only sort of departure from Ohm's law which we are looking for, or indeed which seems at all likely, and in my opinion Maxwell's experiment is entirely competent to answer this question up to its limits of error. As regards the other sorts of departures from Ohm's law, I believe that Wenner's position is sound.

Since Maxwell, very few attempts have been made to detect the effect, probably because of the appalling sensitiveness of Maxwell's experiment as usually quoted. Lecher [4] in 1906 made measurements on platinum and silver wires. He attempted to correct for the temperature effect in the platinum by observing the thermal expansion of the wire when carrying a very heavy current, and comparing the resistance to this heavy current with the resistance to a feeble current passing through the wire when heated artificially to have the same thermal expansion as when carrying the heavy current. The diameter of the wire was 1 mm., and the current density was 3.8×10^3 amp/cm^2; there was no effect greater than 0.1%. The silver wire was 0.03 mm. in diameter. It was placed in a rapid stream of water, and the resistance measured under a current increasing until it fused. No temperature correction was applied. The apparent temperature when the wire burned out, using the ordinary temperature coefficient of resistance, and assuming that Ohm's law is true, was 130°. It is to be expected that this temperature would be somewhere between 100° and the melting point of silver. Hence within limits of error which may be several hundred percent, the experiment is consistent with Ohm's law. The accuracy is very much less than that of Maxwell, but on the other hand, the current density is very much higher, reaching a maximum of 1.4×10^6 amp/cm^2, 25 fold greater.

H. Rausch von Traubenberg [5] has attempted to avoid the temperature difficulty by employing a condenser discharge of very short duration. The current densities that may be reached are higher than previously realized, attaining in one case a maximum of 10^7 amp/cm^2. But the measurements of potential are inaccurate, being estimated by the break-down of a spark gap in air, and there are other sources of error arising from distributed capacities and inductances and the necessity of a long range extrapolation. I estimate that the error may certainly be as high as 10 or 15%. Within these limits, no effect was found. The material used was constantan, of a diameter of 0.1 mm. The maximum current density reached produced a potential gradient in the wire of 400 volts per cm. With "Kruppin" wire, a gradient of 1500 volts/cm. was reached, although with not so high a current density. In spite of the very considerable error, these experiments have pushed the boundary of the validity of Ohm's law considerably farther back than the previous limit. With regard to the materials used, it should be said that the maximum departure from the law is to be expected in those materials which, other things being equal, have a long free electronic path, and which are presumably the best conductors. From this point of view the most promising place to look for the effect is in silver, copper, and gold. Of course, on the other hand, it is to be said that in poor conductors it is possible to reach much higher potential gradients for the same current density, so that this advantage may outweigh the disadvantage.

Outline of Method.

In the method which I have used, the specimen exposed to the high current density is made one arm of a bridge, and its resistance is measured simultaneously for a heavy direct current and a small superposed sinusoidal current of acoustical frequency. If there is a deviation from Ohm's law the resistances to the two currents will not be the same. The following considerations show why this is. It must in the first place be remembered that a bridge is an instrument for testing the equality of potential of two points in a net work. In Figure 1, x denotes the arm of the bridge which contains the fine wire in which the

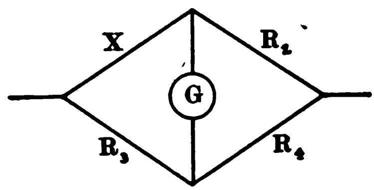

Figure 1. Skeleton of the bridge connections.

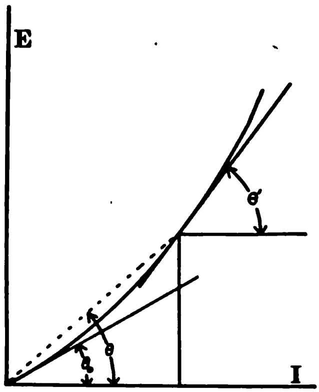

Figure 2. Hypothetical relation between current (I) and e.m.f. (E) not satisfying Ohm's law.

current density is to be high. In this branch we suppose that Ohm's law does not hold, but the relation between current and E.M.F. is given by a curve of the form shown in Figure 2. The other arms of the bridge, R_2, R_3, and R_4 are made of larger wire, in which the current density is always small, and hence their resistance is ohmic. Assume for the moment that it is possible to balance the bridge simultaneously for D.C. and A.C. Now let a heavy direct current I_1 flow through x and R_2 and a direct current I_3 through R_3 and R_4. Also let a small alternating current $i_1 \sin \omega t$ flow through x and R_2 and $i_3 \sin \omega t$ flow through R_3 and R_4. The difference of potential between the ends of x is

$$I_1 \tan \theta + i_1 \tan \theta' \sin \omega t.$$

The potential difference across the galvanometer is

$$(I_1 \tan \theta + i_1 \tan \theta' \sin \omega t) - (I_3 + i_3 \sin \omega t) R_3.$$

Since the extremities of R_2 and R_4 join, their potentials are equal, or

$$I_1 (\tan \theta + R_2) + i_1 (\tan \theta' + R_2) \sin \omega t = (I_3 + i_3 \sin \omega t) (R_3 + R_4).$$

This splits into two equations

$$I_1 \tan \theta + I_1 R_2 = I_3 (R_3 + R_4) \qquad (1)$$

$$i_1 \tan \theta' + i_1 R_2 = i_3 (R_3 + R_4) \qquad (2)$$

Now if the galvanometer is balanced for D.C. the constant part of the potential difference across its terminals must vanish, or

$$I_1 \tan \theta - I_3 R_3 = 0.$$

Combined with (1) above we get

$$I_1 R_2 - I_3 R_4 = 0,$$

or, dividing

$$\frac{\tan \theta}{R_2} = \frac{R_3}{R_4}. \qquad (3)$$

In the same way, if the bridge is balanced for A.C. the alternating part of the potential difference across the galvanometer vanishes, or

$$i_1 \tan \theta' = i_3 R_3.$$

Combined with (2)

$$i_1 R_2 = i_3 R_4$$

and dividing

$$\frac{\tan \theta'}{R_2} = \frac{R_3}{R_4}. \qquad (4)$$

Now conditions (3) and (4) are incompatible unless $\tan\theta' = \tan\theta$. Except for singular points, this means that the relation between current and E.M.F. in the arm x must be linear, or Ohm's law is satisfied. Conversely if Ohm's law is not satisfied, the setting for balance will not be the same for D.C. and small A.C., $\tan\theta$ may be called the direct current resistance, and $\tan\theta'$ the alternating resistance. They may both be determined by the ordinary bridge formulas by first adjusting R_3 and R_4 for D.C. balance, and then readjusting them for A.C. balance. The departure from Ohm's law at a given current density, which I denote by D, is the fractional difference between $\tan\theta$ and $\tan\theta_0$, the tangent to the curve at the origin, that is, the resistance under small currents. This definition gives the equation for D:

$$D = \frac{\tan\theta - \tan\theta_0}{\tan\theta_0}.$$

It is now obvious that if we measure θ and θ' at all points of the curve we can find the curve itself by an integration, hence the tangent at the origin, and so the departure from Ohm's law at any given current density. The mathematical details of this deduction will be given later.

It is evident that the method in simple outline, as given above, avoids the difficulty of the unknown temperature correction because both currents are flowing simultaneously, and hence the temperature of the wire is the same to both. There is, however, a temperature effect of a different kind from that usually met in this sort of experiment which arises as follows. The total rate of heat input under the current is proportional to the square of the total current, that is to $(I_1 + i_1 \sin \omega t)^2$. The $2I_1 i_1 \sin \omega t$ term in this expression denotes an alternate heating and cooling, so that superposed on the large steady temperature increase there is a small sinusoidal fluctuation of temperature whose average is zero. But this small fluctuation of temperature produces a small fluctuation of resistance, and a heavy current flowing through a fluctuating resistance gives rise to a fluctuating difference of potential at the terminals of the resistance. There is, therefore, effectively introduced into the x arm of the bridge a spurious additional sinusoidal E.M.F. which changes the A.C. balance. The action is similar to that of a microphone.

We now discuss mathematically this spurious E.M.F. and the experimental means taken to eliminate its effects. We are for the present concerned solely with this effect, and in the following treat the resistance as ohmic. Any residual effect left after the elimination of this "microphone action" constitutes the departure from Ohm's law for which we are searching. Various tacit assumptions will be made in the course of this discussion which will be justified later.

Return to Figure 1 for the bridge, and consider the heating effect in the arm x, treating its resistance as ohmic. The heat input is proportional to $(I_1 + i_1 \cos \omega t)^2$, where i_1 is small compared with I_1. Expanding this, neglecting the term in i_1^2, the rate of heat input is proportional to $I_1^2 + 2I_1 i_1 \cos \omega t$, that is, there is a constant input proportional to I_1^2, independent of the presence of the A.C., and there is a sinusoidal heating and cooling of the same period as the A.C. which is proportional in intensity to both the D.C. and the A.C. Under this heat input the conductor experiences a change of temperature, which may be analyzed into a constant change dependent only on the D.C., and a small alternating rise and fall, of the same period as the A.C., but not necessarily in phase with it. The factor of proportionality which determines the amplitude of the alternating part is not the same as that which determines the amplitude of the steady part, but is a function of the period, becoming less for higher frequencies. Let us call the steady change of temperature τ_0, the amplitude of the in-phase part of the alternating part τ_1, and that of the out-of-phase part τ_2. If the heat input is removed rapidly, τ_2 will be small compared with τ_1. The increase of temperature above that of the surroundings is therefore $\tau_0 + \tau_1 \cos \omega t + \tau_2 \sin \omega t$. Now if R_0 is the initial resistance at the temperature of the surroundings, α the temperature coefficient of resistance, and R the actual resistance when the current is passing, we have

$$R = R_0[1 + \alpha(\tau_0 + \tau_1 \cos \omega t + \tau_2 \sin \omega t)].$$

The potential difference across the terminals of x is

$$R(I_1 + i_1 \cos \omega t).$$

Expanding this by substituting the value of R above, and using the relations

$$\cos^2 \theta = \tfrac{1}{2}(1 + \cos 2\theta), \quad 2 \sin \theta \cos \theta = \sin 2\theta,$$

we get:

$$\begin{aligned} \text{Potential difference} = \; & R_0\{I_1(1 + \alpha\tau_0) + \tfrac{1}{2} i_1\alpha\tau_1\} \\ & + R_0\{I_1\alpha\tau_1 + i_1(1 + \alpha\tau_0)\} \cos \omega t \\ & + R_0\{I_1\alpha\tau_2\} \sin \omega t \\ & + R_0\{\tfrac{1}{2} i_1\alpha\tau_1\} \cos 2\omega t \\ & + R_0\{\tfrac{1}{2} i_1\alpha\tau_2\} \sin 2\omega t. \end{aligned}$$

Using the same notation as before for the current in the other arms of the bridge, we have at D.C. balance

$$R_0 I_1 \left\{ 1 + \alpha\tau_0 + \tfrac{1}{2}\alpha\tau_1 \frac{i_1}{I_1} \right\} = I_3 R_3,$$

and

$$I_1 R_2 = I_3 R_4.$$

Dividing to eliminate the currents, we obtain

$$R_0 \left\{ 1 + \alpha\tau_0 + \tfrac{1}{2}\alpha\tau_1 \frac{i_1}{I_1} \right\} = R_2 \frac{R_3}{R_4}.$$

This shows that in general, even neglecting the $\cos^2$ term in the heat input as we have above, the D.C. balance will depend on the A.C. But this effect is doubly small, since τ_1 is small compared with τ_0 and i_1 small compared with I_1, and hence the effect may be neglected. The correctness of this assumption was checked experimentally.

With regard to the A.C., the expression above shows that there cannot be complete balance. There will always be higher harmonics, and there will be an out-of-phase component (in $\sin \omega t$). These terms are small, as examination of the coefficients shows, but may nevertheless be perceptible. The ear can set on the fundamental alone, and so eliminate the higher harmonics. The out-of-phase component gives rise to a smearing out of the sharpness of the minimum. This can be corrected by introducing another out-of-phase component to neutralize it by a variable mutual inductance between input and detecting circuits.

The equilibrium conditions for the in-phase component are

$$R_0 i_1 \left[1 + \alpha\left(\tau_0 + \tau_1 \frac{I_1}{i_1} \right) \right] = i_3 R_3$$

and

$$i_1 R_2 = i_3 R_4.$$

Eliminating the current,

$$R_0 \left[1 + \alpha\left(\tau_0 + \tau_1 \frac{I_1}{i_1} \right) \right] = R_2 \frac{R_3}{R_4}.$$

The condition for A.C. balance is therefore different from that for D.C. balance, the large term I_1/i_1 occurring in the expression for A.C. balance against the small term i_1/I_1 in the expression for D.C. balance.

As the experiment was actually performed, R_2 was kept constant, and R_3 and R_4 were varied. R_3 and R_4 consisted of extension coils connected to a bridge wire, which was tapped by a moving slider. Hence adjustment was made by adding to R_3 an appropriate resistance, and at the same time subtracting the same resistance from R_4.

Let us call the initial resistance for balance with no heating effect (small D.C. or A.C. only) R_3 and R_4. To maintain balance under the heavy D.C. with no A.C., R_3 must be increased by ΔR and R_4 diminished by ΔR. A.C. balance with the heavy D.C. flowing is now maintained by an additional increase of $\Delta R'$ to R_3 and decrease of R_4 by $\Delta R'$. The conditions for balance under these three states of current flow are:

$$R_0 = R_2 \frac{R_3}{R_4} \tag{5}$$

$$R_0\left[1 + \alpha\left(\tau_0 + \tfrac{1}{2}\tau_1 \frac{i_1}{I_1}\right)\right] = R_2 \frac{R_3 + \Delta R}{R_4 - \Delta R} \tag{6}$$

$$R_0\left[1 + \alpha\left(\tau_0 + \tau_1 \frac{I_1}{i_1}\right)\right] = R_2 \frac{R_3 + \Delta R + \Delta R'}{R_4 - \Delta R - \Delta R'} \tag{7}$$

Subtracting (6) from (7) and discarding squares and products of ΔR and $\Delta R'$ gives

$$R_0\alpha\tau_1\left[\frac{I_1}{i_1} - \frac{1}{2}\frac{i_1}{I_1}\right] = R_2 \frac{\Delta R'(R_3 + R_4)}{R_4^2 - R_4(2\Delta R + \Delta R')}.$$

Also neglecting $\frac{1}{2}i_1/I_1$ compared with I_1/i_1, and substituting for R_0 its value from (5) gives

$$\alpha\tau_1 = \frac{i_1}{I_1} \frac{\Delta R'\left(1 + \frac{R_4}{R_3}\right)}{R_4 - (\Delta R' + 2\Delta R)},$$

which gives again approximately

$$\alpha\tau_1 = \frac{i_1}{I_1} \Delta R' \left(\frac{1}{R_3} + \frac{1}{R_4}\right). \tag{8}$$

From (6), for the D.C. setting, we get approximately

$$\alpha\tau_0 = \Delta R\left(\frac{1}{R_3} + \frac{1}{R_4}\right) \tag{9}$$

Hence finally we have

$$\frac{\tau_1}{\tau_0} = \frac{i_1}{I_1} \frac{\Delta R'}{\Delta R}. \tag{10}$$

If now the alternations are slow, so that at every moment the wire is approximately in thermal equilibrium, the factor of proportionality connecting τ_1 with the alternating heat input is the same as that connecting τ_0 with the steady heat input, so that we would have

$$\left.\begin{aligned} \tau_0 &= \text{const } I_1^2 R \\ \tau_1 &= \text{const } 2 I_1 i_1 R \end{aligned}\right\}$$

which gives, substituting above in (10),

$$\frac{\Delta R'}{\Delta R} = 2.$$

That is, for slow alternations, the difference between A.C. and D.C. settings due to the microphone action alone is twice the D.C. shift due to temperature rise under the steady current, and this relation holds no matter how feeble the alternating current. Since the steady rise of temperature is high, because the current density has to be pushed to the limit that the conductor will carry without burning out, it is obvious that at slow alternations the microphone action will entirely mask any sought for deviation from Ohm's law. The acoustical frequencies used in these experiments were not low enough to reach the extreme value 2 for the ratio $\Delta R'/\Delta R$. At the lowest frequency, 320 cycles, the ratio had reached about 1.2.

At rapid rates of alternation, however, the conditions of heat transfer change. At low frequencies the thermal conductivity of the surroundings alone determines the equilibrium; at higher frequencies part of the heat input is used in raising the temperature of the surroundings and a term enters proportional to the specific heat, and at still higher frequencies this term preponderates, and the factor of proportionality between amplitude of rate of heat input and amplitude of temperature alternation becomes proportional to the specific heat and inversely proportional to the frequency. We shall later apply a dimensional analysis to obtain more information about τ_0 and τ_1, but for the present we may write, for any frequency

$$\tau_1 = \text{const } f(\omega)\, 2 I_1 i_1 R,$$

and as before

$$\tau_0 = \text{const } I_1^2 R.$$

This now gives

$$\frac{\Delta R'}{\Delta R} = 2f(\omega).$$

At high frequencies $f(\omega) \propto \frac{1}{\omega}$, so that at high frequencies the microphone effect is proportional to the reciprocal of the frequency, and it may be eliminated by proceeding to infinite frequencies (or zero reciprocal frequency).

The procedure suggested by this analysis was that followed in the experiment. For a fixed D.C. the difference between the settings at D.C. and A.C. balance (that is $\Delta R'$) was observed over a range of frequencies, $\Delta R'$ was plotted against the reciprocal of frequency, and extrapolated to zero. The residual, if there is one, is the effect due to deviation from Ohm's law at the particular D.C. density in question.

This procedure was repeated for a number of currents, and so the departure from Ohm's law obtained as a function of current density.

Before proceeding further with the theoretical discussion it will pay now to describe the experimental details, in order that we may have an idea of the order of magnitude of the quantities involved.

Experimental Details.

The bridge was an ordinary four gap alternating current bridge, so constructed that inductive and capacity effects in the bridge were negligible. The resistance R_2, which was kept constant during the measurements on any single specimen, was a coil of heavy manganin wire immersed in an oil bath to carry away the Joulean heat. This resistance was approximately equal in magnitude to the resistance x which carried the high current density. The resistances R_3 and R_4 consisted of heavy manganin coils connected by a slide wire, which was tapped by a slider. The wire was about 1 meter long, with a total resistance of about 3 ohms. The resistance of the extension coils was five or ten times greater than that of the specimen x, and the generation of heat in them was so small that it was not necessary to immerse them in an oil bath.

The method of connecting the D.C. and the A.C. sources and of tapping across with the detectors for D.C. and A.C. balance is shown in Figure 3. The direct and alternating current sources are connected to the same terminals of the bridge, with a large inductance L in the D.C. line to prevent the A.C. backing into the battery, and a large condenser C in the A.C. line to prevent the D.C. backing into the A.C. source. D.C. balance was shown by a Leeds and Northrup high sensitivity galvanometer of about 8 ohms internal resistance connected as shown with a high resistance R_7 in series and another small resistance in shunt to cut down the sensitivity. In the latter part of this work this galvanometer was replaced by another of less sensitiveness. The A.C. detector was a telephone tapped between the same points as the D.C. detector, but with a large condenser in series to prevent D.C. getting into the telephone circuit. The telephone was tapped across a transformer placed in this circuit. In this circuit is also one of the coils of a mutual inductance, *M*, the other coil of which is in series with the A.C. source, and is not shown. This makes possible

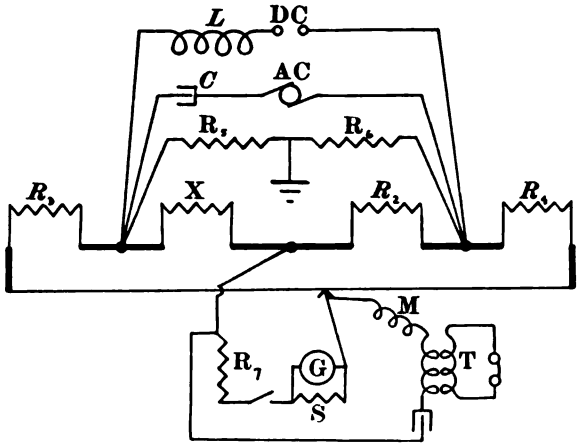

FIGURE 3. Details of the bridge connections.

the elimination of the out-of-phase component by suitable adjustment. The A.C. was prevented from entering the galvanometer circuit by the high resistance in series with it, and by an open key when the galvanometer was not in use. The condenser in the telephone circuit proved an unnecessary precaution, the resistance of the transformer and mutual inductance being sufficient to prevent enough diversion of the D.C. into the telephone line to introduce appreciable error. The condenser was used in most of the work, but in some of the later readings it was omitted. The telephone was one of 1100 ohms resistance, made by the Western Electric Co., type 509 W. The transformer was one of the small ones of the General Radio Co. made for this purpose, type 166.

In Figure 3, the resistances R_5 and R_6 which are connected to the same points as the sources of the current constitute an auxiliary bridge.

The intermediate point was put to ground. This is the regular method of avoiding capacity effects in the telephone. R_5 and R_6 were so large that there was no serious diversion of current from the bridge.

The D.C. source was a storage battery connected in series with a General Electric Co. ballast lamp (iron filament in hydrogen) of 1.4 amp. capacity. In series with the lamp was a commercial ammeter with which the constancy of the input current was checked. Any desired fraction of the output of the battery could be diverted from the bridge by a variable shunt between the lamp and the bridge. The actual current into the bridge or through the sample was not measured directly, but was computed from the ammeter reading, and the resistances, which were measured with the requisite accuracy. If a heavier current than 1.4 amp. were needed, two ballast lamps could be used in parallel. The dimensions of the sample were such that in almost all cases the maximum current that it could carry without burning out was not over 1 amp.

The source of A.C. was a vacuum tube oscillator. The methods of connecting this were the canonical methods, and need not be gone into here. I am much indebted to Professor L. E. Chaffee and Mr. S. Ballantine for assistance and advice in setting up this circuit. For the first readings a Western Electric Co., hot lime transmitting tube Type VT2 was used, but this soon was burned out, and for most of the work a G. E. transmitting tube, type T Pliotron, was used.

Not only does the present differ from preceding attempts in the method of measurement, but also in the form and dimensions given to the metallic resistance carrying the high current density. In all preceding work the metal has been in the form of a fine wire, of diameter of the order of 0.001 inch or more. An elementary discussion will show that a wire of these dimensions will carry only a limited current of the order of 10^7 amp/cm^2 in the most favorable case. The limit is reached at a rate of heat input so high that the interior of the wire is at the melting point while the outer surface is at 0°, the thermal conductivity of the metal just sufficing to carry off the heat input under the temperature gradient so produced. It is possible to gain somewhat by rolling the wire flat as a galvanometer suspension, but not a great deal. An elementary dimensional discussion will show that the only way to gain on the upper limit of current density is by decreasing the thickness of the specimen, so that a given difference of temperature between interior and exterior will give a larger temperature gradient, and therefore a greater heat dissipation. The thinnest metal that can be obtained is in the form of beaten leaf, and it was with gold and silver leaf that I made the measurements. I was afraid to try films deposited by cathode spattering because it did not seem to me that the condition of the metal was sufficiently like that of ordinary metals, whereas the leaf may be supposed to be more like the massive aggregates of metal of ordinary dimensions. However, a few experiments at the end with spattered films of gold gave the same results as beaten leaf of the same thickness, and my fears are probably ill founded. The next work that suggests itself in this connection is an extension of the results found here for silver and gold to other metals, using spattered films.

The thickness of the gold and silver leaf was determined by weighing a known area, assuming in the calculation that the density is the same as that of ordinary metal. The thickness of the silver leaf was 2.0×10^{-5} cm. Three thicknesses of gold were used, 8×10^{-6}, 1.67×10^{-5}, and 5×10^{-5}.

It was with some difficulty that I obtained the intermediate thickness of gold. Gold is beaten out in comparatively large quantities at a time in books of gold beaters skin, a great many thicknesses together. The last stage of the beating reduces the thickness by a factor of 6, from 5×10^{-5} to 8×10^{-6}, and only these thicknesses can be obtained commercially. I am indebted to Mr. Drew of Province Court, Boston, for his kindness in interrupting the last stage of the beating, and at some trouble removing a few of the partially beaten leaves from a large book. The sheets so obtained from the partially completed process were not nearly as perfect as those from the normally completed beating.

The state of the metal in a thin film differs in some unknown respects from that in larger masses. It has long been known that the specific resistance of spattered films is several times higher than that of the massive metal.[6] The specific resistance of gold films has been shown to be very high for very small thicknesses, to decrease rapidly as the thickness increases up to a certain point where the resistance is about five times normal, from here on to remain nearly constant over a range of thickness of about 20 fold, and beyond this point to decrease to the normal value. The temperature coefficient of resistance of spattered films has been frequently observed to be negative. The films of leaf metal used in this work did not show such great abnormalities as the usual spattered films, but nevertheless the resistance was very different from that of the massive metal. The temperature coefficient of my gold leaf was about 0.0015 between 0° and 30°, and was the same for the two thicknesses with which most of the measurements were made, namely 8×10^{-6} and 1.67×10^{-5} cm. The temperature coefficient of the silver was much more nearly normal, and was 0.0032, its thickness being 2.0×10^{-5}. The specific resistance of these metals was much higher than normal. That of the thinnest gold varied from 8.4 to 19.7×10^{-6} ohms per cm. cube, average 11.6. The spattered films of the same thickness varied from 15.4 to 23.8×10^{-6}, average 19.2. The normal value for massive gold is 2.42×10^{-6}. The thicker gold had a higher resistance than the thinner, varying from 9.75 to 18.5, average 13.3×10^{-6}. The cause of the high specific resistance of the spattered gold is doubtless to be found partly in the lack of crystalline structure and perfect coherence, due to its manner of formation. The high resistance of the leaf metal, on the other hand, is doubtless in large part due to mechanical imperfections. Examination of the thinnest leaf under a microscope shows a large number of folds and creases; it is practically impossible to spread the leaf on a surface so that it will lie smoothly in a single unwrinkled layer. The mechanical imperfections in the thicker gold were even greater than in the thinner, as already mentioned, doubtless partly due to the interruption of the beating process at a disadvantageous stage. Two samples of gold 5×10^{-5} thick had resistances of 15.0 and 10.0×10^{-6}, not essentially different from the other pieces.

The specific resistance of the silver leaf varied from 3.5 to 5.1×10^{-6}, average 4.1×10^{-6}. The normal value for silver is 1.63×10^{-6}. It is seen that silver lies much closer to the normal than does gold. Under the microscope it too was full or minute imperfections, but of a different character from the gold. There were no folds, but a number of minute round perforations through the leaf.

It would doubtless have been most desirable if these experiments could have been performed on more massive samples with the normal electric constants, but the necessity of conducting away the heat seems absolutely to preclude such a possibility.

The resistance has to be artificially cooled if current densities high enough to obtain an appreciable effect are to be reached, and the problem of mechanical support had to be solved. For this purpose the leaf metal was mounted on a piece of glass. The glass was covered with a very thin coat of insulating enamel by dipping it in a very dilute solution of the enamel in chloroform, the leaf metal was blown or otherwise spread over the surface, and was then baked at 210° until the enamel was hard. Gold or silver leaf so attached to the surface of glass is full of minute flaws, but by a search under the microscope parts can usually be found of sufficient homogeneity. The leaf was cut so as to leave a narrow isthmus of the shape shown in Figure 4. It is the isthmus that carries the high current density. Connection to the leaf on either side of the isthmus was by means of fine leads of copper caught to the leaf with a touch of solder. A special tool had to be made for cutting the isthmus. The point of a very fine needle was made to travel in any desired direction across the surface of the foil, scratching through to the glass, by an arrangement of two screws

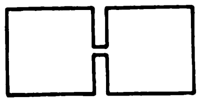

FIGURE 4. The isthmus form of the specimen.

at right angles to each other. With this device, under the lens of a microscope, the isthmus could be cut to a high degree of precision. The dimensions of the isthmus varied somewhat from specimen to specimen, but the length was of the order of 1 mm. and the width of the order of 0.1 mm. The dimensions of each specimen were measured with a microscope.

The specimen was cooled by a stream of water flowing across the isthmus at right angles to its length. This water was delivered from a small glass nozzle suitably held and directed. At first kerosene was used as a cooling liquid, in order to avoid danger of short circuit, but the cooling was not sufficiently rapid and the desired current densities could not be reached. I also tried currents of compressed air and hydrogen, with results very much inferior to those even for kerosene. In order to protect the specimen from the short circuiting action of the water, it was covered on the upper surface, except over the isthmus itself, with an additional coating of enamel. Any enamel on the isthmus itself is fatal. At first I used tap water, but this was too conducting. Ordinary distilled water, however, proved to be sufficiently insulating so that no short circuiting effects from it could be detected. After the distilled water had been used for some time slight irregularities began to appear due to increasing conductivity from miscellaneous impurities picked up from the air of the room; these irregularities could be made to disappear by replacing the water with fresh.

It is necessary that the velocity of the cooling water be maintained constant. For small streams, a syphon arrangement was satisfactory, but for more rapid delivery the proper head was maintained by air pressure obtained from a large compressed air bottle, and was regulated to any desired value with a safety valve of special construction.

In addition to the gold and silver leaf, I made attempts to detect the effect in manganin wire 0.001 inch thick rolled flat, and with Wollaston wire of platinum about 0.00006 inch thick. The attempt with manganin failed because the heating effects were too large, due to the dimension of the specimen. The attempt with platinum failed because of mechanical difficulties in mounting the wire and subjecting it to a stream of water. It is possible that with more pains it might be feasible to obtain results with platinum in this way.

The thickness of the leaf metals used in this experiment was more than sufficiently small to ensure conduction of the Joulean heat developed by the heavy current without excessive rise of temperature. To illustrate the order of magnitudes involved let us consider an example. One specimen of silver that gave good results had the following dimensions: Length 0.536 mm., width 0.072 mm., and thickness 2×10^{-5} cm. The maximum current before the specimen burned out was 0.745 amp., and the initial resistance was 1.30 ohms. The heat input into this specimen was therefore:

$$\text{Heat input} = \frac{(.745)^2 \times 1.30}{4.18} \text{ gm cal/sec.}$$
$$= 0.173 \text{ gm cal/sec.}$$

This heat flows out through the area of one face, which is $0.0536 \times 0.0072 = 3.87 \times 10^{-4}$ cm². The heat outflow per unit area is therefore $(0.173)/(3.87 \times 10^{-4}) = 4.5 \times 10^2$ cal/sec cm². Since the thermal conductivity of silver is approximately unity, the temperature gradient required to drive this thermal stream is 4.5×10^2 degrees per cm. But the total thickness of the film is 2×10^{-5}, so that the extreme temperature difference in the specimen between front and back face is $4.5 \times 10^2 \times 2 \times 10^{-5} = 0.009°$.

It is of interest to compare the heat input with the heat capacity of the specimen. Its volume is $3.87 \times 10^{-4} \times 2 \times 10^{-5} = 7.8 \times 10^{-9}$ cm³. Taking for the specific heat of silver 0.056, and the density as 10.5, we find the heat capacity to be $10.5 \times 0.056 \times 7.8 \times 10^{-9} = 4.6 \times 10^{-9}$. If there were no heat outflow the temperature would rise at the rate of $(0.173)/(4.6 \times 10^{-9}) = 0.038 \times 10^9 = 38{,}000{,}000$ degrees per second.

The magnitude of the steady temperature rise actually observed in this specimen was about 50°, or 5000 times more than the mean rise of temperature required to procure conduction of the heat input out of the metal itself. It is obvious, therefore, that practically all the resistance to heat out flow is in the thin layer of cooling water immediately in contact with the surface of the metal. Our previous estimate of the maximum current density that a wire can carry must be cut down many fold. A 0.001 inch wire cannot carry 10^7 amp/cm^2 under practical conditions. It still remains true under these new conditions, however, that the only change of dimensions of the specimen which will increase the maximum obtainable density is a decrease of diameter.

Our numerical example shows that the body of the metal can be regarded as approximately at a single constant temperature, both for the steady rise of temperature and for the alternating fluctuations. In the mathematical discussion above it was assumed that the temperature of the metal could be specified by a single number; the numerical discussion just given constitutes the justification of this.

Dimensional Discussion of the Cooling Process.

In order to get further in our understanding of the phenomena we must now consider in some detail the steady and alternating changes of temperature τ_0 and τ_1, remembering that practically all the resistance to heat outflow is in the cooling water. It is of course not possible to give an exact solution; the best that we can do is to give a dimensional discussion. Let us consider in the first place the equations of heat transfer in a fluid that is in motion. The equations may be obtained by a slight generalization of the process by which the equation of heat transfer is deduced for a medium at rest. Let us suppose that the medium is homogeneous except for temperature differences, that its specific heat per unit volume is c and its velocity of motion at any point v. Consider a small closed surface S at any point in the liquid. The rate of rise of temperature of the matter within this surface is the total heat input divided by the heat capacity. The heat input consists of two parts. The first is the ordinary conduction across the boundary, and is $\int\int k \frac{\partial \tau}{\partial n} dS$, where k is the thermal conductivity. This assumes that the velocity v is so small compared with the velocities of molecular motion within the liquid that the ordinary process of conduction takes place independent of the motion. The second part of the heat input is that which is convected, and is $-\int\int c\tau v_n dS$. From these two expressions we get the equation

$$\int\int\int c \frac{\partial \tau}{\partial t} dv = \int\int \left(k \frac{\partial \tau}{\partial n} - cv_n\right) dS.$$

Applying Green's theorem to the surface S, transforming the surface to volume integrals, using the condition that Div $v = 0$ because the liquid is to be considered as incompressible, and removing the integral sign, gives for the differential equation of heat transfer

$$\frac{\partial \tau}{\partial t} = \frac{k}{c} \nabla^2 \tau - v \cdot \text{Grad}\, \tau .$$

This equation applies at points inside the liquid. There will also be an equation to fix the boundary conditions. This equation is of the ordinary type, independent of the motion of the liquid, and is merely the statement that the heat input across the boundary is equal to the conductivity of the fluid multiplied by the normal temperature gradient.

Apply these equations now to the present problem. If the motion of the liquid is not turbulent, and if the lines of flow are not altered by the heat input, then at the surface of the metal the liquid flows in planes parallel to the surface, the velocity increasing from the surface. The determination of the velocity distribution is a problem of hydrodynamics, and involves the viscosity of the liquid and the variables which describe the mechanical roughness of the surface, but as far as we are interested in the problem the elements which enter our heat equations are determined if we can specify the velocity gradient at the surface. The other elements which enter the equation of heat transfer are the thermal conductivity of the liquid and its specific heat per unit volume.

Subject now to the restrictions mentioned, we may make a dimensional analysis of the situation. Notice in the first place that since the flow of water is transverse to the specimen the rise of temperature etc. is independent of the length, provided only that the specimen is long enough for us to neglect end effects.

We now enumerate the elements with which we are concerned and their dimensions.

Name of Quantity	Symbol	Dimensional Formula
Average rise of temperature	τ	τ
Rate of heat input per unit length	Q	$H\,L^{-1}\,T^{-1}$
Velocity gradient in liquid	g	T^{-1}
Thermal conductivity of liquid	k	$HL^{-1}\,T^{-1}\,\tau^{-1}$
Specific heat of liquid per unit volume	c	$HL^{-3}\,\tau^{-1}$
Breadth of specimen	b	L
Frequency of impressed heat input	ω	T^{-1}

Consistently with our numerical discussion we have not tabulated the thermal properties nor the thickness of the specimen itself, since the effect of these is vanishingly small.

We have now two cases to consider; first the steady temperature rise. The period of the impressed heat input does not enter, and we have to find all the dimensionless products of the first six quantities of the list above. Since there are four fundamental kinds of quantity (instead of unit quantity of heat H, we might have expressed heat in mechanical units, thus replacing H by M, with no change in the final result), and hence two dimensionless products. Inspection shows these products to be $k\tau/Q$ and k/gcb^2. Hence the relation which we want may be expressed as

$$\tau = \frac{Q}{k} f\left(\frac{cgb^2}{k}\right),$$

where f is some unknown arbitrary function. This relation can be tested by experiment, and so some idea obtained of the correctness of the assumptions underlying the discussion. For instance, at constant rate of flow of cooling water, the above equation shows that the steady temperature rise should be proportional to the rate of heat input, or to I^2. We can obtain an additional check for low rates of flow. For low rates, but not too low, it seems natural to assume that an important part of the rise of temperature is inversely proportional to the rate of flow, or inversely as g. This means that in f there is an important term which is the reciprocal of its argument, and we obtain as a partial expression

$$\tau = \text{Const } Q/gcb^2.$$

Some experimental information may be obtained here by varying the breadth of the sample at approximately equal rates of flow. That the average rise of temperature should be less for the greater breadth seems somewhat paradoxical, and affords a more drastic test than the proportionality of temperature rise to the rate of heat input.

Now let us consider the alternating fluctuations of temperature. To distinguish from the steady case, and consistently with the previous notation, we denote the amplitude of the alternating heat input by Q_1, and the amplitude of the alternating temperature change by τ_1. We now have seven quantities, and hence three dimensionless products. The additional product, beside the two already obtained, is g/ω. The relation between the variables now takes the form

$$\tau_1 = \frac{Q_1}{k}\varphi\left(\frac{cgb^2}{k}, \frac{g}{\omega}\right),$$

where φ is an arbitrary unknown function of its two arguments. For high frequencies g/ω is a small quantity. It is obvious in the first place that at high frequencies the temperature amplitude cannot approach a constant value, but must vanish. Hence developing the function φ for small values of the argument g/ω, putting the constant term zero, and retaining only the first order terms, we have for high frequencies the approximate relation

$$\tau_1 = \frac{Q_1}{k} \cdot \frac{g}{\omega} \cdot \psi\left(\frac{cgb^2}{k}\right).$$

If now we make the further assumption that the conductivity cannot enter at high frequencies, the effect being determined by the specific heat alone, we get the approximate relation

$$\tau_1 = \text{Const}\ \frac{Q_1}{c\omega b^2}.$$

The striking thing about this relation is that the velocity of flow has disappeared, the amplitude of temperature fluctuation being proportional to the heat input, and inversely as the frequency. This again is a result that can be tested by experiment, and its verification would go far toward making probable the assumptions underlying the discussion.

These values which we have found by dimensional analysis for the steady and alternating changes of temperature may now be substituted back in the relation $\tau_1/\tau_0 = i_1\Delta R'/I_1\Delta R$ which we obtained from the equations for the bridge. The rate of heat input entering the dimensional formulas for τ_0 and τ_1 may be written down at once in terms of the currents, namely

$$Q = I^2_1 R'$$
$$Q_1 = 2I_1 i_1 R',$$

where R' is the resistance per unit length. Substituting these values in the dimensional formulas for τ_0 and τ_1, and these again in the value of τ_1/τ_0 above, gives

$$\frac{\Delta R'}{\Delta R} = 2\,\frac{\varphi\left(\frac{cgb^2}{k}, \frac{g}{\omega}\right)}{f\left(\frac{cgb^2}{k}\right)}.$$

We now discuss the various experimental checks which were made of the adequacy of the apparatus and the correctness of the assumptions underlying the mathematical discussion. One may perhaps feel that I have gone too much into detail here, but in view of the importance of the result, and the feeling of disquiet which must be produced by the fact that the final result has to be obtained by an extrapolation I do not believe that multiplication of precaution is superfluous.

Various Experimental Checks and Precautions.

Relative Magnitude of D.C. and A.C. A fundamental requirement of the experiment has been that the A.C. be small compared with the D.C. If it is not, the $\cos^2$ term in the heat input becomes appreciable, and the steady temperature rise is different when the A.C. is flowing than when it is absent. This is easy to check experimentally. If the A.C. is sufficiently small, the D.C. setting should be the same whether the A.C. is flowing or not. This condition was checked repeatedly. Before the experiments were begun the constants of the oscillating circuit were so chosen for the different frequencies that the condition should be met. In general the accuracy of setting for D.C. zero was much greater than for A.C. zero. The latter could not usually be set much closer than 0.5 mm. of bridge wire, whereas the D.C. balance could be determined to 0.1 mm. Sometimes at the lower frequencies there was a displacement of the D.C. balancing point by an appreciable amount when the A.C. was turned on, but less than the error in the A.C. reading. Error at low frequencies does not affect the extrapolation. At the high frequencies, however, any shift of D.C. balance in the presence of the A.C. was less than 0.1 mm. An error of 0.1 mm. in the slider setting means on the average an error of 1/20000 in the resistance.

An additional check was afforded by the requirement that the difference between A.C. and D.C. settings (that is $\Delta R'$) should be independent of the A.C. intensity over a considerable range of A.C. intensity in the neighborhood of that used in the measurements. This requirement is shown at once by the formula on page 152. At constant D.C., i.e. constant ΔR, $\Delta R'$ is independent of the alternating current. This requirement obviously continues to be demanded if part of the $\Delta R'$ is due to a departure from Ohm's law of the kind for which we are searching, for this departure depends only on the D.C. density and not on the small A.C. This again was checked over the range of frequencies, with particular care at the three highest frequencies, as error here will affect the extrapolation. The check is not as good as that above because of the greater error of the A.C. readings, but the requirements were met within the errors of reading, which were 0.5 mm. The A.C. intensity was varied over a range of 10 or 20 fold, and its intensity determined by means of a thermo-couple. At the greater intensities of this range, the difference between A.C. and D.C. settings might be of the order of 1 cm.

For the actual measurements the A.C. was made as large as possible in conformity with these requirements, in order that the sensitiveness might be as high as possible. The requisite intensity varied somewhat at the different frequencies, (there was resonance at about 1000 cycles in the telephone) but as a rough average was about 0.02 amp. This is of the order of 1/30 of the D.C. current near its maximum, so that the $\cos^2$ term, which must be negligible, is of the order of 5×10^{-4} of the steady heating term. The D.C. shift of zero near the maximum was of the order of 15 cm. of bridge wire. $5 \times 10^{-4} \times 15$ is 0.075 mm., which is thus beyond the possibility of detection or error.

Equality of A.C. and D.C. Zeroes. If the bridge is properly set up and free from capacity and inductive effects, the D.C. zero for small direct currents and the A.C. zero (that is, the A.C. setting in the absence of D.C.) should be the same. This condition was satisfied within the limits of reading at the three highest frequencies. The settings could be made and read to 0.1 mm. for both A.C. and D.C. It was only when the A.C. settings were made in the presence of D.C. that there was room for as much uncertainty as 0.5 mm. At the lower frequencies the two zeroes might sometimes differ by as much as several tenths of a mm., but this does not affect the extrapolation or the final results.

Independence of Rate of Cooling. If the extrapolated difference between A.C. and D.C. readings is due to a real departure from Ohm's law, and is not in some way connected with the microphone action, which depends on the heating, then for a fixed D.C. density the extrapolated difference of readings must be independent of the steady rise of temperature, which may be made to vary by changing the speed of the cooling water. That this condition is satisfied is shown in Figure 5, in which the difference between D.C. and A.C. settings is plotted against the reciprocal of the frequency. The D.C. heating current was the same for the two curves, but the rate of flow of the cooling water was different, the steady rise of temperature of the upper curve being 1.4 times as great as that of the lower.

Effect of Speed of Cooling Water on Steady Temperature Rise. For low speeds of cooling water with a constant D.C. the rise of temperature decreases rapidly with increase of speed. At higher rates of flow, however, the effect becomes much less, until above a certain point there is very little dependence of steady temperature rise on the rate of flow. In the actual experiments the speed was chosen high enough to be within this insensitive range. This ensures that conditions on different days with different samples were rather closely the same. There were slight outstanding differences in the cooling conditions

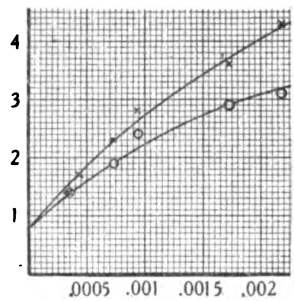

FIGURE 5. Results obtained with gold 8×10^{-6} cm. thick. The abscissae are reciprocals of the frequency and the ordinates difference between A.C. and D.C. settings in cm. of bridge wire. The two curves were obtained with the same D.C. but with different heating effects, produced by varying the rate of flow of the cooling water. The two curves should extrapolate to the same point if there is a genuine departure from Ohm's law.

of the different samples, however, due to slight changes in the position of the sample in the holder, changes in the angle of presentation of the nozzle of the cooling stream, etc.

Dependence of Steady Temperature Rise on Rate of Heat Input. Our dimensional analysis, subject to the restrictions assumed for the manner of flow of the cooling water, has shown that the steady rise of temperature does not depend on the metal, but only on its breadth, the rate of heat input, and several other factors which do not change if the cooling fluid is always water, and its velocity is in the range where it does not much affect the cooling. Now the rate of heat input is proportional to the square of the D.C. and may be computed in terms of the resistance and the dimensions of the specimen. This computation was made, and the results plotted for a dozen specimens, comprising gold of the three thicknesses and silver.

In the first place the steady temperature rise for all these specimens is a linear function of the rate of heat input up to rates which are near the burning out point. When this point is approached there is a break in the curve, and the temperature rise increases more rapidly than the heat input. The probable explanation is that the lines of flow of the cooling water are changed, it not being unlikely that there are localities where the water is even vaporized. The general magnitude of the temperature rise observed is consistent with this idea.

In the second place, the steady temperature rise shows no correlation whatever with the thickness of the sample or its material, but the rise for the thickest and the thinnest gold or for the silver is approximately the same for the same breadth of sample. This establishes the correctness of the assumption that the dissipation takes place in the layer of water in contact with the specimen, and does not depend at all on the properties of the metal, always provided of course, that the metal is sufficiently thin. The rise of temperature for specimens of different breadths is less at the same input per cm^2 of surface for the wider specimens. The total variation was by a factor of 2, the rise of temperature of the narrowest specimen per unit heat input per cm^2 being about twice that of the widest. The breadths varied from 0.007 to 0.022 cm. At the same time it was a matter of experiment that it was possible to reach higher current densities in the narrow samples without burning out than in the wider ones. The reason for this is that the break in the curve at which the linear relation between temperature rise and heat input ceases is reached much sooner for the wide than for the narrow samples.

Effect of Breadth of Sample on Heating. In order to test more exactly the precise dependence of rise of temperature on breadth, a series of runs was made on the same piece of gold, cutting down the breadth after each run so as to make it successively narrower. The sample was initially 1.18 mm. broad and 1.33 mm. long. End effects are important at these dimensions, and agreement with the results of the dimensional analysis is to be expected only for the narrower samples. Readings were made at six breadths. In the first place, as the breadth became less the heating effect became continually greater for the same current. Now if ρ is the two dimensional resistance of the thin metal film, and b its breadth, its resistance per unit length is ρ/b, and the heat input Q per unit length is $I^2\rho/b$. Our dimensional analysis shows that

$$\tau = \frac{I^2}{k} \cdot \frac{\rho}{b} \cdot f\left(\frac{cgb^2}{k}\right).$$

Now f at one extreme, for very slow rates of flow, contains an important term which is the reciprocal of its argument, whereas at high rates of flow we have seen that experiment suggests that it tends to become constant. Hence, keeping all the other arguments constant except b, the variation of τ may be expected to be between $1/b$ and $1/b^3$.

Experimentally the three narrowest breadths were 0.090, 0.155, and 0.305 mm. The shift of D.C. setting (proportional to temperature rise) for 0.2 amp. was respectively 8.5, 3.4, and 0.2 cm. Variation as $1/b^3$ would have given 8.5, 1.7, 0.3; as $1/b^2$, 8.5, 2.9, 0.9; and as $1/b$, 8.5, 5.0, and 2.7. The variation is within the limits set and probably nearly as $1/b^2$, within the limits of error.

This check is very rough, but does bear out the paradoxical variation with b given by the dimensional argument. The difficulties of the experiment are great, it being impossible to exactly reproduce the conditions of flow when the sample is removed from position, its breadth cut down, and then replaced. In the actual measurements of the departures from Ohm's law, this source of irregularity was of course not present, a complete series of measurements at different D.C. strengths being made on the sample unchanged in position and with the same flow.

These measurements also showed the same fact as that mentioned in the preceding section, namely that the relation of proportionality between heat input and temperature rise ceases at high rates of heat input, and the point of break in the linear relation comes at lower values of heat input per unit area for the large than the small breadths. This set of measurements with changing breadth of the same sample showed that the current at the point of break is roughly proportional to the square root of the width, which means that the break occurs when the generation of heat per unit length reaches a fixed value, independent of the breadth.

Dependence of Results on Position of Electrodes. There is another sort of check of entirely different character which may be made by changing the position of the electrodes on the sheet of metal leaf. There is an essential difference between the resistance of a two and a three dimensional mass of conducting material. If two electrodes are immersed in a three dimensional conducting medium, and the distance between them is increased indefinitely, the resistances between them will approach a finite value, the sum of the so-called electrode resistances. On the other hand in a two dimensional medium, the total resistance between electrodes of definite shape increases indefinitely as the distance between the electrodes is increased indefinitely. One cannot speak of the electrode resistance of a two dimensional conductor. Applied to the present case, this means that if the electrodes are not situated near to each other on opposite sides of the isthmus, the total resistance will not be merely the resistance of the isthmus and practically nothing else, but the rest of the sheet will make a finite contribution. But the rest of the sheet will not make a finite contribution to the departure from Ohm's law, since the departure increases very rapidly with increasing current density, and it is only in the isthmus that the density is high.

The experiment was tried of making a specimen with two sets of electrodes, one pair close to the isthmus, and the other pair much

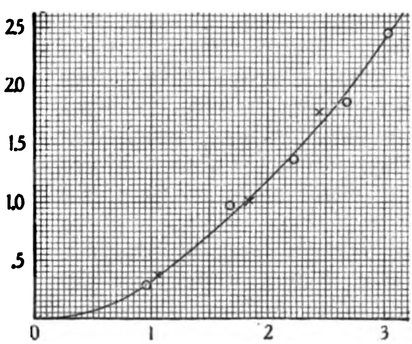

Figure 6. Results obtained with gold 8×10^{-6} cm. thick. Abscissae, current density in 10^6 amp/cm^2, ordinates, extrapolated difference between A.C. and D.C. settings in cm. of bridge wire. The two sets of points were obtained with the same specimen, but with different positions of the electrodes. The points shown by the crosses are the observed points corrected by the ratio of the total resistance to the resistance of the isthmus. If the effect measured is a genuine departure from Ohm's law, the corrected points should fall on the same curve with the others.

more remote. The total resistance between the second pair of electrodes was 1.27 times that between the first. Now if the effect found is genuine, the extrapolated value of $\Delta R'$ for the same value of the current density in the isthmus should have the same absolute value irrespective of the position of the electrodes, and should not merely be the same fraction of the total resistance between electrodes. That this demand is met is shown in Figure 6. The points for the two positions of the electrodes lie on the same smooth curve within the errors of the measurements, and this error is evidently much less than a factor of 1.27.

Experimental Procedure and Data.

After the preparation of the sample, and measurement of its dimensions, it was adjusted in the holder, the fine copper wires which were soldered to the metal leaf were in turn attached to heavier leads, and these were connected to the proper gap in the bridge, and the nozzle for the cooling water was adjusted to give the most efficient stream over the specimen. As already mentioned, the cooling water was distilled water. This was contained in one of two large carboys, which were at the temperature of the room. On starting the water, there was no change in the zero due to change of temperature of the specimen. Because of the limited capacity of the carboys, it was necessary several times in the course of the experiment to interrupt the readings to turn water from one carboy into the other.

Readings were begun at some small value of the direct current, obtained by insertion of the proper shunt in the feeding circuit. An initial reading was made of the A.C. zero, with no D.C. flowing. Readings were now made of the D.C. balancing point with A.C. flowing, and the A.C. balancing point with D.C. flowing, and the two readings were repeated with D.C. reversed. This set of four readings was repeated for each of several frequencies, beginning at the lowest. The frequencies used were 320, 460, 680, 1140, 1530, 2450, and 3750. The frequencies were calibrated with tuning forks. At each frequency the mutual inductance was readjusted to give the sharpest setting.

Originally more readings were taken. These were the balancing point for small D.C., A.C. zero with no D.C., D.C. balancing point for large D.C. with no A.C. flowing, balancing point for large D.C. with A.C. flowing, and balancing point for A.C. with large D.C. flowing. After a number of runs had been made, taking all these readings, I was satisfied that these precautions were not necessary, and the readings mentioned above proved sufficient.

The A.C. zero, which gives the actual resistance of the specimen itself, may change slightly during a run at a single D.C. intensity and a series of frequencies. This may be due to some mechanical change in the specimen, because of friction by the cooling water, or at high current densities may be due to incipient burning out. Any such change during a run was always small however, and could be disregarded in the computations.

The series of readings at different frequencies was now repeated at some higher value of D.C., and the series continued until so high a

D.C. was reached that the specimen burned out. It was usual to make readings at five to seven different D.C. intensities.

The difference between D.C. and A.C. settings for each constant D.C. were now plotted against the reciprocal of the frequency, and the results extrapolated to zero (that is, infinite frequency). Of course the chief point of the experiment is this extrapolation. It is in the first place to be remarked that the extrapolation is not a wide one, the observed frequencies covering a range of ten fold, so that the distance of extrapolation is only one tenth of the distance covered by the readings. It is in the second place to be noticed that we have theoretical evidence as to the form of the curve in the neighborhood of zero. The part of the plotted difference which involves the heating effect, and which we are trying to eliminate varies linearly as the reciprocal of the frequency. The remainder, which is the departure from Ohm's law, is constant, independent of the frequency. The curves therefore extrapolate as straight lines, their curvature disappearing as they approach the axis. Given this condition, the extrapolation can be made with very little uncertainty. The curve is already so nearly straight at the last two or three frequencies that in most cases a straight line could be drawn through these points without error. However, this is not quite satisfactory. In the early part of the work I made the extrapolation by plotting the results on a large scale, and drawing free hand the curve which seemed to best satisfy the conditions. Later, however, in order to make the process of extrapolation more mechanical and less subject to error by personal bias, I adopted the following mechanical device. A steel spring was made to conform as closely as possible to the observed points by imposing on it three restrictions. It was made to go through, or near, the observed point at the lowest frequency, another point near the middle of the frequency range, and it was made to go through such a point on the axis that it coincided as closely as possible with the points at the upper end of the frequency range. This process will be more fully described in the next paragraph.

Three typical sets of readings, giving differences between A.C. and D.C. settings in terms of cm. of bridge wire plotted against reciprocal of frequency, are shown in Figures 7, 8, and 9. These figures are for the two thicknesses of gold and the one of silver. It will be seen that there can be little doubt about the extrapolation. The fact that the extrapolated curve does not pass through the origin, but strikes the axis at a point higher up, constitutes the evidence for the departure from Ohm's law. The mechanical extrapolation was made as follows.

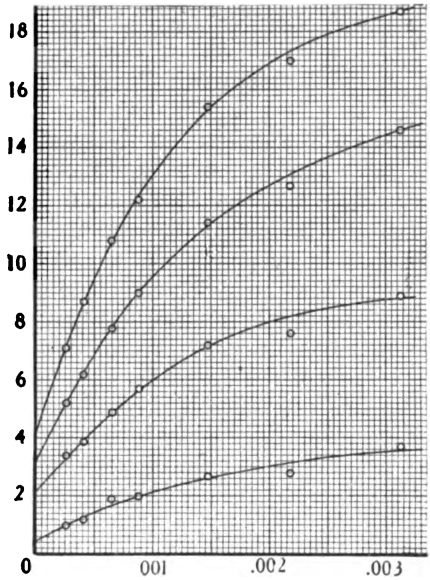

Fig. 7.

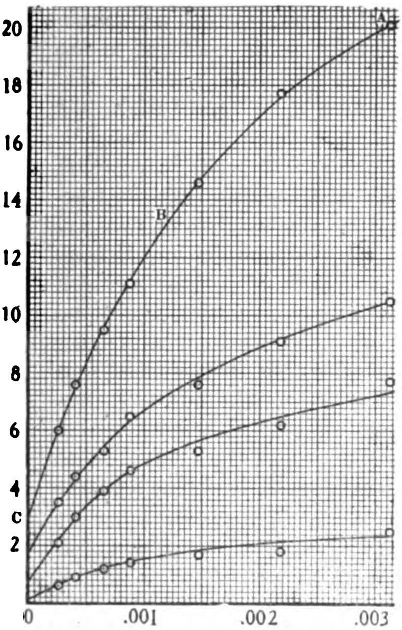

Fig. 8.

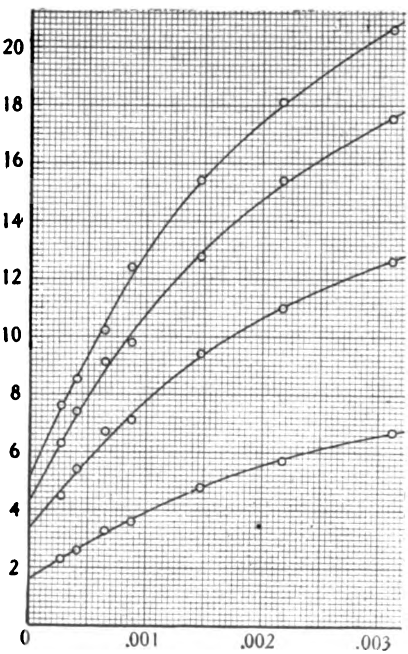

Fig. 9.

Figure 7. Results obtained with one specimen of gold 8×10^{-6} cm. thick. Abscissae, reciprocal of the frequency; ordinates, difference between A.C. and D.C. settings in cm. of bridge wire.

Figure 8. Results obtained with one specimen of gold 1.67×10^{-6} cm. thick. Abscissae, reciprocal of the frequency; ordinates, difference between A.C. and D.C. settings in cm. of bridge wire.

Figure 9. Results obtained with one specimen of silver 2×10^{-5} cm. thick. Abscissae, reciprocal of the frequency; ordinates, difference between A.C. and D.C. settings in cm. of bridge wire.

Consider the upper curve in Figure 8. A needle was stuck through the paper at the point A and another at B. The spring was passed under A and over B, and under a third needle C which was moved up and down the axis of ordinates at the same time that slight trial readjustments were made in the positions of A and B until that combination was found at which the spring coincided most closely with the observed points, particularly at the high frequencies. It will be seen that under these conditions the spring crosses the axis in a straight line, which is one of the demands. The spring also passes through A as a straight line, which it theoretically should not do. Any error due to this cannot have a perceptible effect on the extrapolated value. A number of times I made independent extrapolations by the free hand method and by the use of the spring, with essentially the same results.

Additional evidence of the correctness of the extrapolations is the fact that these were all made independently. The extrapolations were first made, and then afterwards the factors were computed by which the extrapolated differences in cm. of bridge wire were reduced to fractional parts of resistance. When the extrapolations were made I had no knowledge of whether the results would conform to the results with other specimens or not.

Successful results were obtained with gold 8×10^{-6} and 1.67×10^{-5} cm. thick, and with silver 2×10^{-5} thick.

The collected results for the eleven samples of 8×10^{-6} gold (that is, extrapolated difference between A.C. and D.C. settings as fractional parts of the initial resistance plotted against current density) are shown in Figure 10. The breadth of these samples varied from 0.066 to 0.226 mm., and both gold leaf and spattered gold are included in the results. If the effect is genuine, there should of course be no relation with the breadth. We have seen that there is a striking dependence of heating effects on the breadth, but it is evident that there is no such correlation here, and that within the limits of error points for all samples lie on the same curve. This, I believe, constitutes rather telling evidence for the genuineness of the effect. With regard to the magnitude of the experimental error, it is true that the points are perhaps more scattered than one could wish, but when we consider that we are dealing with an effect whose existence even has not hitherto been established, although much sought for, I do not believe that the scattering is more than one might expect.

The collected results for gold 1.67×10^{-5} thick are shown in Figure 11. This comprises results on four different samples. The results are much more irregular, and the limits of error are greater for the thicker than the thinner gold. The extrapolation is more uncertain because perceptible curvature continues to higher frequencies than for the thinner gold. It was not possible to reach such high current densi-

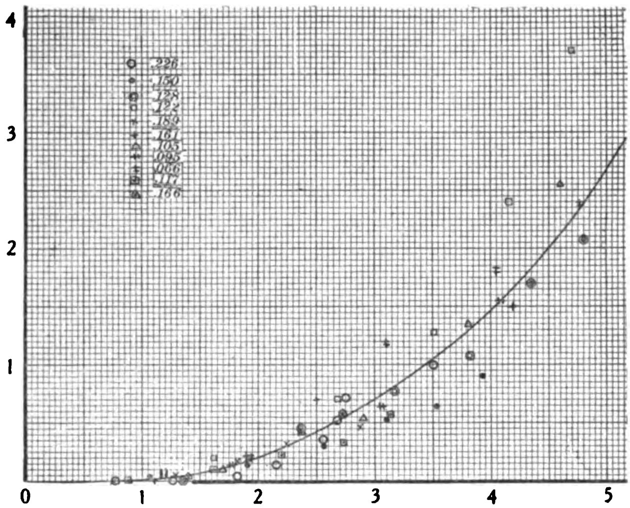

FIGURE 10. Collected results for gold 8×10^{-6} cm. thick. Abscissae, current density in 10^6 amp/cm^2, ordinates, extrapolated difference between A.C. and D.C. resistance in per cent of D.C. resistance. The numbers in the body of the diagram show the breadth of the various samples in mm.

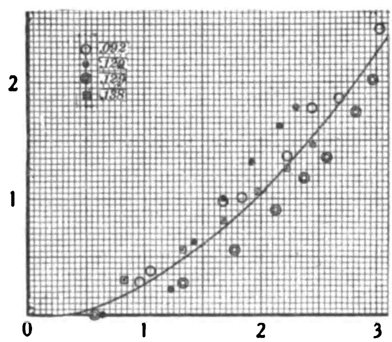

FIGURE 11. Collected results for gold 1.67×10^{-5} cm. thick. Abscissae, current density in 10^6 amp/cm^2; ordinates, extrapolated difference between A.C. and D.C. resistance in per cent of D.C. resistance. The numbers in the body of the diagram show the breadth of the various samples in mm.

ties without the specimen burning out. If the maximum current density without burning out is fixed by the amount of heat that can be carried away by the cooling water, and if this is independent of thickness, as we have seen it is approximately, then the maximum current density for a piece of twice the thickness should be inversely as $\sqrt{2}$. This relation is approximately satisfied. It is to be noticed that the effect is greater for the thicker gold; this would be expected from theoretical reasons.

In addition to these two thicknesses of gold, I tried several samples of the next size gold that I could obtain in the market, namely 5×10^{-5}. Results with this were unsatisfactory. The curvature of the relation between A.C. and D.C. difference and reciprocal frequency continues to much higher frequencies than for the thinner specimens, and in fact is quite marked over the entire frequency range, so that it was not possible to extrapolate. It can be seen at once that the inequalities of temperature in a leaf of five times the thickness are twenty-five times as great for the same current density.

The results for silver 2×10^{-5} cm. thick are shown in Figure 12. Measurements were made on five different samples, of widths varying from 0.072 to 0.165 mm. The results are more scattered than for the thin gold, but again there can be no question of the existence of the effect, and the fact that there is no correlation with the breadth.

In addition to these measurements on gold and silver I attempted to measure the effect in aluminum. The thinnest aluminum leaf that can be obtained in the market is 5×10^{-5} cm. thick, and I found the same trouble with it that I did with the thickest gold, namely that the curvature persists to such high frequencies that the extrapolation cannot be made with any assurance.

Other Possible Explanations of the Effect.

Skin Effect. It is natural to search for explanations of these results other than a departure from Ohm's law. One of the first that suggests itself is the "skin effect." The resistance of a conductor is higher to alternating than direct currents because the alternating current tends to collect in the surface layers, not having time to soak into the interior of the conductor. This effect is very important at wireless frequencies with conductors of the ordinary dimensions. It increases rapidly with increase of frequency, and is greater in large than in small conductors. An upper limit for the conductor used in these experiments may be found by applying

Rayleigh's formula for a conductor of circular section to a conductor of the same area of section as that of the specimens actually used. At the upper limit of frequencies used here the skin effect for such a circular conductor is only one part in 10^{12}, and is hence absolutely not to be considered.

The Electrostriction Effect. The pinch effect is well known in conductors carrying heavy currents, and it might be expected that under the high current densities of these experiments there might be mechanical strains set up in virtue of the magnetic action of the current elements on each other which would alter the resistance. But it is to be noticed that the pinch effect depends on the total current carried by the conductor and not on its density. The currents carried here are only of the order of 1 ampere, and numerical calculation shows that any such effect is beyond the possibility of detection.

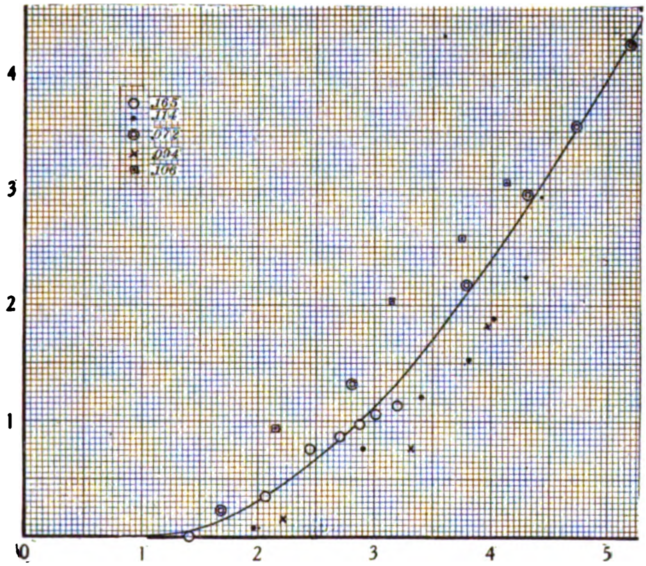

FIGURE 12. Collected results for silver 2×10^{-6} cm. thick. Abscissae, current density in 10^6 amp/cm^2; ordinates, extrapolated difference between A.C. and D.C. resistance in per cent of D.C. resistance. The numbers in the body of the diagram show the breadth of the various samples in mm.

Variation of Resistance with Frequency. The electron theory predicts a change of resistance at high frequencies independent of the skin effect. It is usually considered that this effect is important only at optical frequencies, but it is worth our while to consider it here. The formula may be found on page 432 of Richardson's book on Electron Theory of Matter, and is

$$\sigma_p = \frac{\sigma_0}{1 + p^2 \dfrac{m^2\sigma_0^2}{N^2 e^4}}$$

Here σ_p is the conductivity at the frequency in question, σ_0 the conductivity at zero frequency, p the angular velocity corresponding to the frequency, m the mass of the electron, e the charge on the electron, and N the number of electrons per cm^3. The only quantity in this formula which we do not know definitely is N. Let us assume for the moment that N is equal to the number of atoms. If now we substitute numerical values for silver we find that σ_p differs from σ_0 by 5.3×10^{-18}. (I have used for $p = 2\pi \times 10^4$, which is more than twice the highest experimental value). But now according to my theory of conduction, the number of electrons must be very considerably less than the number of atoms, and this will increase the difference between σ_p and σ_0. If we assume as an extreme value that the number of electrons is 10^{-4} as great as the number of atoms, we find that σ_p still differs from σ_0 by 5.3×10^{-10} at 10,000 cycles. Evidently this effect is not a factor under our conditions.

Computation of the Departure from Ohm's Law.

Referring again to Figure 2, we have defined the departure from Ohm's law as $(\tan \theta - \tan \theta_0)/\tan \theta_0$. What we have actually measured and plotted in Figures 10, 11, and 12 is $(\tan \theta' - \tan \theta_0)/\tan \theta_0$. Given this as a function of current, we require to find the departure from Ohm's law as a function of current. For convenience replace E by y, and I by x. Let the required curve be represented by $y = f(x)$. Experimentally we determined

$$\frac{\dfrac{dy}{dx} - \dfrac{y}{x}}{\left(\dfrac{dy}{dx}\right)_{x=0}} = \varphi(x),$$

where φ is a known function. Without essential restriction we may put $\left(\frac{dy}{dx}\right)_{x=0} = 1$. We now have to solve

$$\frac{dy}{dx} - \frac{y}{x} = \varphi(x).$$

The solution is

$$y = x\left\{\int_0^x \frac{\varphi(x)\,dx}{x} + \text{const.}\right\}.$$

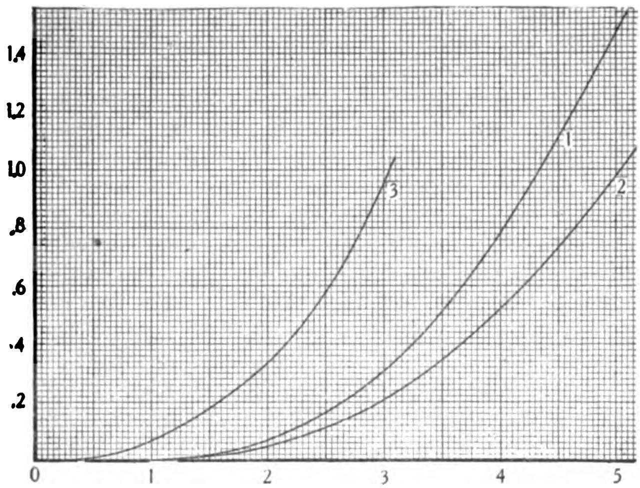

FIGURE 13. Final results, showing the departure from Ohm's law in per cent (ordinates) against current density (abscissae) in 10^6 amp/cm^2. Curve 1 is for silver 2×10^{-6} cm. thick, curve 2 for gold 8×10^{-6} cm. thick, and curve 3 for gold 1.67×10^{-5} cm. thick.

The value of the constant is unity in order to satisfy the condition $\left(\frac{dy}{dx}\right)_{x=0} = 1$. Now the departure from Ohm's law is $\frac{y}{x} - 1$, or

$$\int_0^x \frac{\varphi(x)\,dx}{x}.$$

This may be determined graphically from the experimental curve for φ. The departure from Ohm's law has been computed from the graphs according to this formula for the two thicknesses of gold and for silver, and is reproduced in Figure 13.

Theoretical Discussion.

I have already mentioned that J. J. Thomson has suggested that at extreme current densities the current will increase as the square root of the E.M.F., instead of linearly. As far as I know, no theoretical discussion has ever been given of the magnitude of the effect to be expected at lower current densities, where departures are just beginning to be perceptible. Swann [7] has discussed the resistance of thin films from the standpoint of the classical electron theory of conductivity, and has retained the second order terms, subject to the assumption of the classical theory that after every collision of electron with atom all vestige of the effect of the electric force acting on the electron before the collision is wiped out. Under these assumptions he obtains a departure from Ohm's law in the opposite direction from that found experimentally above, or given by Thomson's theory. Subject to the same assumption I have also carried through an examination from the classical point of view, retaining the second and third order terms in the expressions for the velocity imparted to the free electron by the applied force during its free path, and arrive at a result similar to Swann's, that is, a departure from Ohm's law in the drection of a decrease of resistance with high E.M.F.'s. The first term in the departure involves the square of the E.M.F., as considerations of symmetry show that it must. This simple analysis must be incorrect, however. It is not likely that the trend at very high E.M.F.'s found by Thomson is reversed at lower E.M.F.'s, and it is also exceedingly probable that the assumptions at the basis of the classical theory are not exact. One would certainly expect an effect of higher order on the velocity distribution by the applied force, even if there is no effect of zero order.

I have not been able to make the modifications in the classical analysis which would be necessary to take account of terms of higher order in the velocity distribution. I believe that it must involve the details of atomic and crystal structure.

Not being able to give the exact analysis that we could desire, we may fall back on a dimensional analysis. If the mechanism of conduction is a free path mechanism, as is supposed in the classical theory, and as I think most likely, we see that the departure from Ohm's law is going to depend only on the kinematics of the motion of an electron moving with a certain normal velocity in a free path when acceleration is impressed by an additional force from without. This assumes that the number of free electrons is not changed by the external force. The elements that enter our analysis are the acceleration of the electron (α), its mean free path (l), and its normal velocity in the absence of the external impressed force (v). The departure from Ohm's law is to be expressed in terms of these. As we have defined it, the departure from Ohm's law, D, is a ratio of similar quantities, and is hence dimensionless. The only dimensionless combination of α, l, and v is $\frac{\alpha l}{v^2}$. Hence we have

$$D = f\left(\frac{\alpha l}{v^2}\right).$$

Considerations of symmetry show that the unknown function must be an even function of its argument, since reversing the direction of the acceleration leaves the resistance unaltered. Also the acceleration of the electron is determined by its charge, mass, and the applied force by the equation

$$\alpha = \frac{Ee}{m}.$$

Hence finally we have the result

$$D = \text{even function}\left(\frac{Eel}{mv^2}\right).$$

Maxwell assumed, as was perfectly natural, that the function was algebraic, and hence that the first term was proportional to E^2, it being an experimental fact that the constant term vanishes. In the light of the experiments above, however, it is exceedingly questionable whether this assumption is justified. It seems to me that the curve hugs the axis at the origin much more closely than an algebraic curve, and in fact may have contact of an infinitely high order, like an exponential curve. I have not succeeded in satisfactorily reproducing the course of the curve with two or three terms, although this may be done with a fair degree of approximation. I found this significant thing in trying to fit an algebraic curve to the experimental results. If the first term, that is the square term, corresponds to anything real physically, then its magnitude in a three constant formula should not be very different from its value in a two constant formula, the terms of higher order constituting merely correction terms. This is not the case. The square term in a three constant formula fitting the results for 8×10^{-6} gold was only about one fifth as large as in a two constant formula. It seems to be likely, therefore, both from the appearance of the curve and this evidence from computation that the correct form of the curve is not algebraic, or at any rate that the first term is not a square term.

However, merely in order to afford a basis for comparison with previous results, let us assume for the moment that at comparatively small values of the current the most important term is the square term. My results for 8×10^{-6} gold show that at a current density of 10^6 amp/cm^2 the departure from Ohm's law is certainly not any greater than 10^{-4}. Assuming, in lack of anything better, that the coefficient of the square term is unity, and assuming the square law, this would mean that at a current density of 1 amp/cm^2 the departure from Ohm's law is not greater than 10^{-16}. Maxwell stated the limit as one part in 10^{12} for the same current density. The law is probably actually much closer than 10^{-16} at 1 amp/cm^2.

Another argument against the probability of the square law is the very high values that it would mean for the free path. Let us assume again that the coefficient of the square term is unity, and substitute numerical values. We have to put

$$\left(\frac{Eel}{mv^2}\right)^2 = 10^{-4},$$

where E is the electric field at a current of 10^6 amp/cm^2. In accordance with the usual assumption of classical theory, which I have attempted to show elsewhere is very probably true,[8] we put the energy of the electron equal to the energy of a gas molecule at the same temperature (300° Abs.). We may simplify by writing $mv^2 = 2k\tau$, where k is the gas constant and τ absolute temperature. We have to solve the above equation for l, the only unknown. The specific resistance of gold is 2.4×10^{-6}. To drive 10^6 amp/cm^2 takes 2.4 volts/cm which is 0.08 Abs. E. S. U. Substituting these values gives

$$l = 4.5 \times 10^{-4} \text{ cm.}$$

It would appear on this basis, therefore, that the free path is of the order of 5×10^{-4} cm. long. This is in the direction that I would like to find the path to differ from the previous results of the classical theory, which gave something of the order of 10^{-8} cm., for my theory demands a long path and few electrons,[8] but I believe that the number above goes rather too far in the desired direction. It is, however, perhaps not impossible that the path should be of the order of 10^{-4} cm. In default of a better theory, my measurements do not afford the basis for a better calculation. It would be necessary to refine on the measurements at current densities less than 10^6 in order to pick out the square term (provided that it exists), or to improve on the theory. I do not believe that the latter will be possible without taking into account the detailed structure of the atom. At the same time, I believe that the existence of the effect at current densities so much lower than would be expected on the basis of a free path of the order of 10^{-8} cm. is at least presumptive evidence that this value of the path is too low.

The appearance of the v^2 term in the formula above suggests an essential observation. The measurements, and the results computed from them, do not refer to metal at the same temperature at the different current densities, but the observations at the higher densities are for the metal at higher temperatures, because of the heating effect. This of course was not eliminated by extrapolating to infinite frequency. The change of temperature at the maximum current density was in the neighborhood of 50° for all specimens. If the form of the function were known, it would have been possible to correct for this temperature effect, but in the absence of the knowledge, I thought it better to give the results as obtained without attempting any correction. If conduction is by a free path mechanism, then at higher velocities the departures from Ohm's law are less than at lower ones, (that is, lower temperatures) so that if the corrections had been applied a deviation from Ohm's law even greater than that shown would have been found at the higher current densities.

We have noticed that the deviations are greater for the thick than for the thin gold. Although the results are not as accurate for the thicker as the thinner leaf, there seems to be no possibility that all the difference can be accounted for by errors of observation. If the free path is long, as I have supposed, an effect in precisely this direction would be expected. For the leaf is considerably less in thickness than the length of the normal path, so that increasing the thickness would have the effect of increasing the average path, and so increasing the departure from Ohm's law.

The departure is considerably less in silver than in gold of the same thickness. This would mean a shorter path in silver than in gold. In view of the greater conductivity of silver this may mean that the number of free electrons is greater in silver than in gold. It is, however, perhaps dangerous to drive the comparison too far between the different metals, because the much closer approach to normal of both the specific resistance and the temperature coefficient of resistance of silver suggests that the internal conditions may not be comparable.

Summary.

By the employment of a new method, by which the resistance is measured simultaneously for a heavy direct current and a small superposed alternating current of acoustical frequency, deviations from Ohm's law at high current densities have been detected and measured in leaf gold and silver. The maximum current densities were about 5×10^6 amp/cm^2, and the deviations from Ohm's law were of the order of one per cent. If the mechanism of conduction is a free path mechanism, these results probably mean that the free path is much longer than supposed on the classical electron theory.

It is a pleasure to acknowledge my indebtedness to my assistant Mr. J. C. Slater for making nearly all the readings.

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A TABLE AND METHOD OF COMPUTATION OF ELECTRIC WAVE PROPAGATION, TRANSMISSION LINE PHENOMENA, OPTICAL REFRACTION, AND INVERSE HYPERBOLIC FUNCTIONS OF A COMPLEX VARIABLE.

By George W. Pierce.

A TABLE AND METHOD OF COMPUTATION OF ELECTRIC WAVE PROPAGATION, TRANSMISSION LINE PHENOMENA, OPTICAL REFRACTION, AND INVERSE HYPERBOLIC FUNCTIONS OF A COMPLEX VARIABLE.

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1. **Table I.**— Table I appended contains values of $f(h)$ and $g(h)$ for various numerical values of h, where $f(h)$ and $g(h)$ are defined by the following equations

$$f(h) = +\sqrt{\frac{+\sqrt{1+h^2}+1}{2}}, \qquad (1)$$

$$g(h) = +\sqrt{\frac{+\sqrt{1+h^2}-1}{2}}. \qquad (2)$$

The solution of a number of physical problems involves these functions in a manner to render the table serviceable for computation.

It will be noticed that $f(h)$ and $g(h)$ have the following properties

$$\{f(h)\}^2 - \{g(h)\}^2 = 1, \qquad (3)$$

$$2f(h)\,g(h) = h. \qquad (4)$$

2. **To Extract the Square Root of $1 + jh$, where h is Real and Positive, and $j = \sqrt{-1}$.**—

Let

$$\sqrt{1+jh} = A + jB,$$

then

$$A^2 - B^2 = 1,$$

$$2AB = |h|.$$

These results compared with (3) and (4) show that

$$\sqrt{1+jh} = \pm\{f(h) + jg(h)\}. \qquad (5)$$

Likewise, for h positive,

$$\sqrt{1-jh} = \pm\{f(h) - jg(h)\}. \qquad (6)$$

3. To Extract the Square Root of $P + jU$, where P and U are any Real Quantities.— Factoring out the square root of P, we have by (5) and (6)

$$\sqrt{P + jU} = \pm \sqrt{P}\,\{f(h) + j\,g(h)\}, \text{ if } U/P \text{ is positive,}$$
$$= \pm \sqrt{P}\,\{f(g) - j\,g(h)\}, \text{ if } U/P \text{ is negative,} \quad (7)$$

where

$$h = U/P.$$

4. To Find $\sinh^{-1}(P + jU)$ where P and U are Both Positive and Real.—

Let

$$\sinh^{-1}(P + jU) = A + jB, \quad (8)$$

then

$$P + jU = \sinh A \cos B + j \cosh A \sin B,$$

whence

$$P = \sinh A \cos B, \quad (9)$$
$$U = \cosh A \sin B. \quad (10)$$

Regarding signs, in accordance with the caption, we see that, since $\cosh A$ is always positive, $\sin B$ is positive and $\sinh A$ and $\cos B$ are both positive or both negative.

Taking the sum of the squares of (9) and (10), we have

$$\begin{aligned} P^2 + U^2 &= \sinh^2 A \cos^2 B + \cosh^2 A \sin^2 B \\ &= \sinh^2 A \cos^2 B + (1 + \sinh^2 A)(1 - \cos^2 B) \\ &= 1 - \cos^2 B + \sinh^2 A, \end{aligned}$$

whence by using (9), we have

$$P^2 + U^2 = 1 - \cos^2 B + \frac{P^2}{\cos^2 B}, \text{ and} \quad (11)$$

$$P^2 + U^2 = 1 + \sinh^2 A - \frac{P^2}{\sinh^2 B}. \quad (13)$$

Letting

$$V = \frac{1 - P^2 - U^2}{2}, \quad (14)$$

and solving (11) and (12) as quadratics, we obtain

$$\cos^2 B = V \pm \sqrt{P^2 + V^2},$$
$$\sinh^2 A = -V \pm \sqrt{P^2 + V^2}.$$

Using the proper signs before the radicals to make the expressions positive as demanded by their left hand equivalents, extracting the square roots and employing (1) and (2), we obtain the following results

If $V > 0$, $\sinh A = \pm \sqrt{2V}\, g(h)$, $\cos B = \pm \sqrt{2V}\, f(h)$,

if $V < 0$, $\sinh A = \pm \sqrt{-2V}\, f(h)$, $\cos B = \pm \sqrt{-2V}\, g(h)$,

where $$h = P/V.$$

Having regard to the rule of signs enunciated under (10), we may write

$$\sinh^{-1}(P + jU) = x + j(y + 2\pi n) \text{ and } -x + j(\pi - y + 2\pi n), \quad (15)$$

where

$$\text{if } V > 0,\ x = \sinh^{-1}\{+\sqrt{2V}\, g(h)\},\ y = \cos^{-1}\{+\sqrt{2V}\, f(h)\}, \quad (16)$$

$$\text{if } V < 0,\ x = \sinh^{-1}\{+\sqrt{-2V}\, f(h)\},\ y = \cos^{-1}\{+\sqrt{-2V}\, g(h)\}, \quad (17)$$

with $$h = P/V. \quad (18)$$

Equation (15) gives the value of $\sin^{-1}(P + jU)$, *where P and U are real, positive quantities, in terms of x and y defined by (16) and (17). The value of V is given by (14). The signs in (15) are so chosen that the value of y in the first quadrant is to be employed.*

5. To Find $\sinh^{-1}(P - jU)$, where P and U are Positive, Real Quantities.— This differs from the preceding problem only in the fact that $\sin B$ is negative, whence

$$\sinh^{-1}(P - jU) = x - j(y + 2\pi n) \text{ and } -x + j(y + \pi + 2\pi n), \quad (19)$$

with y in the first quadrant and with x and y defined as in (16), (17) and (18).

6. To Find $\sinh^{-1}(-P - jU)$ and $\sinh^{-1}(-P + jU)$, where P and U are Positive, Real Quantities.— These results may be had directly from Sections 4 and 5 by use of the facts that

$$\sinh^{-1}(-P - jU) = -\sinh^{-1}(P + jU), \quad (20)$$

and

$$\sinh^{-1}(-P + jU) = -\sinh^{-1}(P - jU). \quad (21)$$

7. To Find $\cosh^{-1}(P+jU)$, where P and U are Positive and Real.—

Let

$$\cosh^{-1}(P+jU) = A + jB, \tag{22}$$

then

$$P + jU = \cosh A \cos B + j \sinh A \sin B,$$

whence

$$P = \cosh A \cos B, \tag{23}$$

$$U = \sinh A \sin B. \tag{24}$$

The sum of the squares of these two equations gives

$$P^2 + U^2 = \cosh^2 A \cos^2 B + \sinh^2 A \sin^2 B$$
$$= 1 + \sinh^2 A - \sin^2 B,$$

whence by substitution from (24) and by solution of the resulting quadratic equations we obtain

$$\sinh^2 A = -V \pm \sqrt{U^2 + V^2}, \quad \text{and}$$
$$\sin^2 B = V \pm \sqrt{U^2 + V^2}.$$

These give, with choices of signs to make A and B real and satisfy (24)

if $V > 0$, $\sinh A = \pm\sqrt{2V}\, g(h)$, $\quad \sin B = \pm\sqrt{2V}\, f(h)$, $\quad h = U/V$,

if $V < 0$, $\sinh A = \pm\sqrt{-2V}\, f(h)$, $\sin B = \pm\sqrt{-2V}\, g(h)$, $\quad h = U/V$.

In accordance with (23) and (24), in each line the sinh A and sin B have the same sign before their radicals, and the angle B must be so determined that cos B is positive. Whence

$$\cosh^{-1}(P+jU) = \pm \{a + j(\varphi + 2\pi n), \tag{25}$$

where

if $V > 0$, $a = \sinh^{-1}\{+\sqrt{2V}\, g(h)\}$, $\quad \varphi = \sin^{-1}\{+\sqrt{2V}\, f(h)\}$, (26)

if $V < 0$, $a = \sinh^{-1}\{+\sqrt{-2V}\, f(h)\}$, $\quad \varphi = \sin^{-1}\{+\sqrt{-2V}\, g(h)\}$, (27)

with

$$h = U/V. \tag{28}$$

Equation (25) gives the value of $\cosh^{-1}(P+jU)$, *where P and U are real, positive quantities, in terms of a and* φ *defined by (26), (27) and (28).* φ *is in the first quadrant.* V *is defined by (14).*

8. To Find $\cosh^{-1}(P - jU)$, where P and U are Positive and Real.— This case differs from the preceding only in that, by (24), sinh A and sin B have opposite signs, so that

$$\cosh^{-1}(P - jU) = \pm \{a - j(\varphi + 2\pi n), \qquad (29)$$

Where a and φ have the values given by (26) and (27).

9. To Find $\cosh^{-1}(-P + jU)$, where P and U are Positive and Real.— This case differs from that of Section 7 only in that cos B is negative, so that

$$\cosh^{-1}(-P + jU) = \pm \{a + j(\pi - \varphi + 2\pi n)\}, \qquad (30)$$

where a and φ have the values given in (26) and (27).

10. To Find $\cosh^{-1}(-P - jU)$, where P and U are Positive and Real.— This case differs from that of Section 9 in that sinh A and sin B have opposite signs, whence

$$\cosh^{-1}(-P - jU) = \pm \{a + j(\pi + \varphi + 2\pi n)\}, \qquad (31)$$

where a and φ have the values given in (26) and (27).

11. Attenuation Constant, Retardation Angle, and Surge Impedance of a Smooth Electric Transmission Line Without Leakage.— If

r, c, and l = respectively resistance, capacity, and inductance per loop unit of length of a smooth line,
ω = angular velocity of impressed e.m.f. in radians per second,
α = real attenuation constant of current per unit of length of line,
β = retardation angle per unit of length of line,
z_i = surge impedance of the line,
R_i and X_i = respectively surge resistance and surge reactance of the line,

then [1]

$$\alpha = \omega\sqrt{lc}\, g(h), \qquad (32)$$

$$\beta = \omega\sqrt{lc}\, f(h), \qquad (33)$$

1 These equations are obtained by introducing the g- and f-functions into familiar equations. Compare Pierce: *Electric Oscillations and Electric Waves*, pp. 327 and 329, McGraw-Hill Book Co., 1920.

$$R_i = \sqrt{l/c}\, f(h), \tag{34}$$

$$X_i = -\sqrt{l/c}\, g(h), \tag{35}$$

where

$$h = r/l\omega. \tag{36}$$

12. Constants of a Leaky Line.— If to the list of quantities defined in Section 11, we add

g = leakage conductance per loop unit of length of the line,

then [1]

I. If $lc\omega^2 > rg$,

$$\alpha = \sqrt{lc\omega^2 - rg}\, g(h), \qquad \beta = \sqrt{lc\omega^2 - rg}\, f(h), \tag{37}$$

where

$$h = \frac{rc\omega + gl\omega}{lc\omega^2 - rg}.$$

II. If $lc\omega^2 < rg$,

$$\alpha = \sqrt{rg - lc\omega^2}\, f(h), \qquad \beta = \sqrt{rg - lc\omega^2}\, g(h), \tag{38}$$

where

$$h = \frac{rc\omega + gl\omega}{rg - lc\omega^2}.$$

III. If $gl > rc$,

$$z_i = \sqrt{\frac{rg + lc\omega^2}{g^2 + c^2\omega^2}}\, \{f(h) + j\, g(h)\}, \tag{39}$$

where

$$h = \frac{gl\omega - rc\omega}{rg + lc\omega^2}.$$

IV. If $gl < rc$

$$z_i = \sqrt{\frac{rgc - lc\omega^2}{g^2 + c^2\omega^2}}\, \{f(h) - j\, g(h)\}, \tag{40}$$

where

$$h = \frac{rc\omega - gl\omega}{rg + lc\omega^2}.$$

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[1] Obtained by introducing the f- and g- functions into familiar equations. Compare Kennelly: *Applications of Hyperbolic Functions*, pp. 70 and 125. University of London Press.

13. Constants of an Artificial Electric Line.— If

z_1 = complex impedance of the series elements of an artificial line,
z_2 = complex impedance of the shunt elements,
M = mutual inductance (if any) between adjacent series elements,
φ = retardation angle of current per section of line,
a = real attenuation constant per section of line, and if

we let

$$P + jU = \frac{z_1 + 2z_2}{z_2 - Mj\omega}, \tag{41}$$

and

$$V = \frac{1 - U^2 - P^2}{2}, \tag{42}$$

where P and U are real quantities, then [1]

I. If $V > 0$,

$$a = \sinh^{-1}\sqrt{2V}\, g(h), \quad \varphi = \sinh^{-1}\{\pm\sqrt{2V}\, f(h)\} \tag{43}$$

with $h = U/V$.

II. If $V < 0$,

$$a = \sinh^{-1}\sqrt{-2V}\, f(h), \quad \varphi = \sin^{-1}\{\pm\sqrt{-2V}\, g(h)\} \tag{44}$$

with $h = U/V$.

These equations are to be employed subject to the following rule regarding the quadrant of φ.

Rule Regarding φ.

sign of		quadrant of φ
P	U	
+	+	first
−	+	second
−	−	third
+	−	fourth

[1] Compare Pierce: *Electric Oscillations and Electric Waves*, p. 296, on which the present results are based.

As to surge impedance of the artificial line, its value depends upon the arrangement of elements by which transition is effected from the terminal apparatus to the line. For a line provided at each end with a series element of half the impedance of the internal series elements of the line the complex surge impedance is

$$z_i = \sqrt{\frac{(z_1 + 2z_2)^2}{4} - (z_2 - jM\omega)}. \qquad (45)$$

The real and imaginary parts of this expression may be readily obtained in a numerical case by the method of Section 3.

14. Index of Refraction and Extinction Coefficient for Electric Waves in an Absorbing Medium.— As another example of the utility of the functions here presented attention is called to the following equations from optics and electric wave theory applied to an absorbing medium.

If

γ = specific conductivity in absolute c.g.s. electrostatic units.
ϵ = dielectric constant of the medium,
μ = its permeability,
n = its index of refraction for waves of period T seconds,
κ = extinction coefficient for waves of the period T,

then (see p. 413 *Electric Oscillations and Electric Waves*), using the functions f and g, we may write

$$n = \sqrt{\epsilon\mu}\, f(h), \qquad (46)$$

$$\kappa = \sqrt{\epsilon\mu}\, g(h) \qquad (47)$$

with

$$h = 2\gamma\, T/\epsilon.$$

15. Note on a Relation of $f(h)$ and $g(h)$ to Hyperbolic Functions Giving an Easy Method of Computing the Table of f and g.— Having given some illustrations of the utility of the table, we next call attention to a simple relation employed in computing the table.

If we let

$$h = \sinh x, \text{ then } \sqrt{1 + h^2} = \cosh x,$$

so that it is seen that

$$f(h) = \sqrt{\frac{\cosh x + 1}{2}} = \cosh\left(\frac{1}{2}\sinh^{-1} h\right), \tag{46}$$

and likewise

$$g(h) = \sinh\left(\frac{1}{2}\sinh^{-1} h\right). \tag{47}$$

Equations (46) and (47) were used in computing many of the values of the table by referring to values of hyperbolic functions given in the *Smithsonian Institute Tables.*

16. Extension of Table to Larger Values of *h*.— By expansion in series it is easily shown that

$$f(h) = \sqrt{\frac{h}{2}}\left\{1 + \frac{1}{2h}\right\}, \text{ for } \frac{1}{8h^2} << 1, \tag{48}$$

$$g(h) = \sqrt{\frac{h}{2}}\left\{1 - \frac{1}{2h}\right\}, \text{ for } \frac{1}{8h^2} << 1. \tag{49}$$

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TABLE I.

h = 0 to .286

h	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*
.000	1.000	.0000	.072	1.001	.0360	.144	1.003	.0718	.216	1.006	.1074
.002	1.000	.0010	.074	1.001	.0370	.146	1.003	.0728	.218	1.006	.1084
.004	1.000	.0020	.076	1.001	.0380	.148	1.003	.0738	.220	1.006	.1094
.006	1.000	.0030	.078	1.001	.0390	.150	1.003	.0748	.222	1.006	.1104
.008	1.000	.0040	.080	1.001	.0400	.152	1.003	.0758	.224	1.006	.1114
.010	1.000	.0050	.082	1.001	.0410	.154	1.003	.0768	.226	1.006	.1124
.012	1.000	.0060	.084	1.001	.0420	.156	1.003	.0778	.228	1.006	.1133
.014	1.000	.0070	.086	1.001	.0430	.158	1.003	.0788	.230	1.007	.1143
.016	1.000	.0080	.088	1.001	.0440	.160	1.003	.0798	.232	1.007	.1153
.018	1.000	.0090	.090	1.001	.0450	.162	1.003	.0807	.234	1.007	.1163
.020	1.000	.0100	.092	1.001	.0460	.164	1.003	.0817	.236	1.007	.1172
.022	1.001	.0110	.094	1.001	.0470	.166	1.003	.0827	.238	1.007	.1182
.024	1.001	.0120	.096	1.001	.0480	.168	1.004	.0837	.240	1.007	.1192
.026	1.001	.0130	.098	1.001	.0490	.170	1.004	.0847	.242	1.007	.1202
.028	1.001	.0140	.100	1.001	.0499	.172	1.004	.0857	.244	1.007	.1211
.030	1.001	.0150	.102	1.001	.0509	.174	1.004	.0867	.246	1.007	.1221
.032	1.001	.0160	.104	1.001	.0519	.176	1.004	.0877	.248	1.007	.1230
.034	1.001	.0170	.106	1.001	.0529	.178	1.004	.0887	.250	1.008	.1240
.036	1.001	.0180	.108	1.001	.0539	.180	1.004	.0896	.252	1.008	.1250
.038	1.001	.0190	.110	1.002	.0549	.182	1.004	.0906	.254	1.008	.1260
.040	1.001	.0200	.112	1.002	.0559	.184	1.004	.0916	.256	1.008	.1270
.042	1.001	.0210	.114	1.002	.0569	.186	1.004	.0926	.258	1.008	.1280
.044	1.001	.0220	.116	1.002	.0579	.188	1.004	.0936	.260	1.008	.1289
.046	1.001	.0230	.118	1.002	.0589	.190	1.004	.0946	.262	1.008	.1299
.048	1.001	.0240	.120	1.002	.0599	.192	1.005	.0956	.264	1.008	.1309
.050	1.001	.0250	.122	1.002	.0609	.194	1.005	.0966	.266	1.009	.1319
.052	1.001	.0260	.124	1.002	.0619	.196	1.005	.0975	.268	1.009	.1329
.054	1.001	.0270	.126	1.002	.0629	.198	1.005	.0985	.270	1.009	.1338
.056	1.001	.0280	.128	1.002	.0639	.200	1.005	.0995	.272	1.009	.1348
.058	1.001	.0290	.130	1.002	.0649	.202	1.005	.1005	.274	1.009	.1358
.060	1.001	.0300	.132	1.002	.0659	.204	1.005	.1015	.276	1.009	.1368
.062	1.001	.0310	.134	1.002	.0669	.206	1.005	.1025	.278	1.009	.1377
.064	1.001	.0320	.136	1.002	.0679	.208	1.005	.1035	.280	1.010	.1387
.066	1.001	.0330	.138	1.002	.0689	.210	1.005	.1044	.282	1.010	.1397
.068	1.001	.0340	.140	1.002	.0698	.212	1.006	.1054	.284	1.010	.1406
.070	1.001	.0350	.142	1.003	.0708	.214	1.006	.1064	.286	1.010	.1416

TABLE I.

h = .286 to .572

h	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*
.286	1.010	.1416	.358	1.015	.1762	.430	1.022	.2104	.502	1.029	.2438
.288	1.010	.1426	.360	1.016	.1772	.432	1.022	.2114	.504	1.029	.2447
.290	1.010	.1435	.362	1.016	.1782	.434	1.022	.2123	.506	1.030	.2456
.292	1.010	.1445	.364	1.016	.1792	.436	1.022	.2132	.508	1.030	.2465
.294	1.011	.1455	.366	1.016	.1801	.438	1.023	.2141	.510	1.030	.2475
.296	1.011	.1464	.368	1.016	.1811	.440	1.023	.2150	.512	1.030	.2484
.298	1.011	.1474	.370	1.016	.1820	.442	1.023	.2159	.514	1.031	.2493
.300	1.011	.1484	.372	1.017	.1829	.444	1.023	.2168	.516	1.031	.2502
.302	1.011	.1494	.374	1.017	.1839	.446	1.023	.2177	.518	1.031	.2510
.304	1.011	.1503	.376	1.017	.1848	.448	1.024	.2186	.520	1.031	.2520
.306	1.011	.1513	.378	1.017	.1858	.450	1.024	.2197	.522	1.031	.2529
.308	1.011	.1522	.380	1.017	.1867	.452	1.024	.2207	.524	1.032	.2538
.310	1.012	.1532	.382	1.017	.1877	.454	1.024	.2216	.526	1.032	.2547
.312	1.012	.1541	.384	1.018	.1886	.456	1.024	.2225	.528	1.032	.2556
.314	1.012	.1551	.386	1.018	.1896	.458	1.025	.2234	.530	1.032	.2566
.316	1.012	.1561	.388	1.018	.1905	.460	1.025	.2244	.532	1.033	.2575
.318	1.012	.1570	.390	1.018	.1915	.462	1.025	.2253	.534	1.033	.2584
.320	1.012	.1580	.392	1.018	.1924	.464	1.025	.2262	.536	1.033	.2593
.322	1.013	.1590	.394	1.018	.1934	.466	1.025	.2271	.538	1.033	.2602
.324	1.013	.1600	.396	1.019	.1943	.468	1.026	.2281	.540	1.034	.2612
.326	1.013	.1609	.398	1.019	.1953	.470	1.026	.2290	.542	1.034	.2621
.328	1.013	.1619	.400	1.019	.1962	.472	1.026	.2300	.544	1.034	.2630
.330	1.013	.1629	.402	1.019	.1972	.474	1.026	.2309	.546	1.034	.2639
.332	1.013	.1638	.404	1.019	.1981	.476	1.026	.2318	.548	1.034	.2648
.334	1.013	.1648	.406	1.020	.1991	.478	1.027	.2327	.550	1.035	.2658
.336	1.014	.1658	.408	1.020	.2001	.480	1.027	.2337	.552	1.035	.2667
.338	1.014	.1667	.410	1.020	.2010	.482	1.027	.2346	.554	1.035	.2676
.340	1.014	.1677	.412	1.020	.2020	.484	1.027	.2355	.556	1.035	.2685
.342	1.014	.1686	.414	1.020	.2029	.486	1.028	.2364	.558	1.036	.2694
.344	1.014	.1696	.416	1.020	.2038	.488	1.028	.2372	.560	1.036	.2703
.346	1.014	.1705	.418	1.021	.2048	.490	1.028	.2383	.562	1.036	.2712
.348	1.015	.1715	.420	1.021	.2057	.492	1.028	.2392	.564	1.036	.2721
.350	1.015	.1724	.422	1.021	.2067	.494	1.028	.2401	.566	1.036	.2730
.352	1.015	.1734	.424	1.021	.2076	.496	1.029	.2411	.568	1.037	.2739
.354	1.015	.1743	.426	1.022	.2085	.498	1.029	.2420	.570	1.037	.2748
.356	1.015	.1753	.428	1.022	.2095	.500	1.029	.2429	.572	1.037	.2757

TABLE I.

h = .572 to 3.18

h	$f(h)$	$g(h)$	h	$f(h)$	$g(h)$	h	$f(h)$	$g(h)$	h	$f(h)$	$g(h)$
.572	1.037	.2757	1.04	1.106	.4702	1.76	1.230	.715	2.48	1.356	.915
.574	1.038	.2766	1.06	1.109	.4779	1.78	1.233	.722	2.50	1.359	.920
.576	1.038	.2775	1.08	1.113	.4852	1.80	1.237	.728	2.52	1.362	.925
.578	1.038	.2784	1.10	1.116	.4933	1.82	1.241	.733	2.54	1.366	.930
.580	1.038	.2793	1.12	1.119	.5005	1.84	1.244	.740	2.56	1.369	.935
.582	1.039	.2802	1.14	1.122	.5080	1.86	1.248	.745	2.58	1.373	.939
.584	1.039	.2811	1.16	1.126	.5151	1.88	1.251	.751	2.60	1.376	.945
.586	1.039	.2820	1.18	1.129	.5225	1.90	1.255	.757	2.62	1.379	.950
.588	1.039	.2829	1.20	1.132	.530	1.92	1.258	.763	2.64	1.383	.955
.590	1.040	.2838	1.22	1.135	.538	1.94	1.262	.769	2.66	1.386	.960
.592	1.040	.2847	1.24	1.139	.544	1.96	1.265	.775	2.68	1.390	.964
.594	1.040	.2856	1.26	1.142	.552	1.98	1.269	.780	2.70	1.393	.969
.596	1.040	.2865	1.28	1.146	.559	2.00	1.272	.786	2.72	1.396	.974
.598	1.041	.2874	1.30	1.149	.565	2.02	1.275	.792	2.74	1.400	.979
.60	1.041	.2882	1.32	1.153	.572	2.04	1.279	.798	2.76	1.403	.984
.62	1.044	.2969	1.34	1.156	.580	2.06	1.283	.803	2.78	1.407	.989
.64	1.046	.3059	1.36	1.160	.586	2.08	1.286	.809	2.80	1.410	.993
.66	1.049	.3146	1.38	1.163	.593	2.10	1.290	.814	2.82	1.413	.998
.68	1.051	.3235	1.40	1.167	.600	2.12	1.293	.820	2.84	1.416	1.003
.70	1.054	.3321	1.42	1.170	.607	2.14	1.297	.825	2.86	1.420	1.007
.72	1.057	.3406	1.44	1.174	.613	2.16	1.300	.831	2.88	1.423	1.012
.74	1.060	.3491	1.46	1.178	.620	2.18	1.304	.836	2.90	1.426	1.017
.76	1.062	.3578	1.48	1.181	.627	2.20	1.307	.842	2.92	1.429	1.022
.78	1.065	.3662	1.50	1.184	.634	2.22	1.310	.847	2.94	1.433	1.026
.80	1.068	.3745	1.52	1.188	.640	2.24	1.314	.852	2.96	1.436	1.031
.82	1.071	.3828	1.54	1.191	.647	2.26	1.317	.858	2.98	1.440	1.035
.84	1.074	.3911	1.56	1.195	.653	2.28	1.321	.863	3.00	1.443	1.040
.86	1.077	.3993	1.58	1.198	.659	2.30	1.324	.869	3.02	1.446	1.044
.88	1.080	.4074	1.60	1.202	.666	2.32	1.328	.874	3.04	1.449	1.049
.90	1.083	.4155	1.62	1.205	.672	2.34	1.331	.879	3.06	1.453	1.053
.92	1.086	.4236	1.64	1.209	.678	2.36	1.335	.884	3.08	1.456	1.058
.94	1.089	.4316	1.66	1.212	.685	2.38	1.338	.889	3.10	1.459	1.062
.96	1.093	.4392	1.68	1.216	.691	2.40	1.342	.894	3.12	1.462	1.067
.98	1.096	.4471	1.70	1.219	.697	2.42	1.345	.900	3.14	1.465	1.072
1.0	1.099	.4552	1.72	1.223	.703	2.44	1.349	.904	3.16	1.469	1.076
1.02	1.102	.4628	1.74	1.226	.710	2.46	1.352	.910	3.18	1.472	1.080

TABLE I.

h = 3.18 to 6.04

h	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*
3.18	1.472	1.080	3.90	1.585	1.230	4.62	1.692	1.365	5.34	1.794	1.490
3.20	1.475	1.085	3.92	1.588	1.234	4.64	1.695	1.369	5.36	1.797	1.493
3.22	1.478	1.089	3.94	1.591	1.238	4.66	1.698	1.372	5.38	1.800	1.497
3.24	1.482	1.093	3.96	1.595	1.241	4.68	1.701	1.376	5.40	1.802	1.499
3.26	1.485	1.098	3.98	1.598	1.245	4.70	1.704	1.379	5.42	1.804	1.502
3.28	1.489	1.102	4.00	1.601	1.249	4.72	1.706	1.383	5.44	1.807	1.506
3.30	1.492	1.106	4.02	1.604	1.253	4.74	1.709	1.387	5.46	1.810	1.509
3.32	1.495	1.110	4.04	1.607	1.257	4.76	1.712	1.390	5.48	1.813	1.513
3.34	1.498	1.115	4.06	1.610	1.261	4.78	1.715	1.394	5.50	1.815	1.515
3.36	1.502	1.119	4.08	1.613	1.265	4.80	1.718	1.397	5.52	1.818	1.518
3.38	1.505	1.123	4.10	1.616	1.269	4.82	1.721	1.400	5.54	1.821	1.522
3.40	1.508	1.127	4.12	1.619	1.273	4.84	1.723	1.403	5.56	1.824	1.526
3.42	1.511	1.132	4.14	1.622	1.276	4.86	1.726	1.407	5.58	1.827	1.529
3.44	1.514	1.136	4.16	1.625	1.280	4.88	1.729	1.411	5.60	1.829	1.531
3.46	1.517	1.140	4.18	1.628	1.284	4.90	1.732	1.415	5.62	1.831	1.535
3.48	1.520	1.145	4.20	1.631	1.288	4.92	1.735	1.418	5.64	1.834	1.537
3.50	1.523	1.149	4.22	1.634	1.291	4.94	1.738	1.422	5.66	1.837	1.540
3.52	1.526	1.153	4.24	1.637	1.295	4.96	1.741	1.425	5.68	1.840	1.546
3.54	1.529	1.158	4.26	1.640	1.299	4.98	1.743	1.429	5.70	1.842	1.548
3.56	1.533	1.161	4.28	1.642	1.303	5.00	1.746	1.432	5.72	1.844	1.550
3.58	1.536	1.165	4.30	1.645	1.307	5.02	1.749	1.435	5.74	1.847	1.553
3.60	1.539	1.170	4.32	1.648	1.310	5.04	1.752	1.439	5.76	1.850	1.557
3.62	1.542	1.174	4.34	1.651	1.314	5.06	1.755	1.442	5.78	1.853	1.560
3.64	1.545	1.178	4.36	1.654	1.318	5.08	1.758	1.445	5.80	1.855	1.562
3.66	1.549	1.181	4.38	1.657	1.322	5.10	1.760	1.449	5.82	1.857	1.564
3.68	1.552	1.186	4.40	1.660	1.325	5.12	1.763	1.452	5.84	1.860	1.568
3.70	1.555	1.190	4.42	1.663	1.328	5.14	1.766	1.455	5.86	1.863	1.571
3.72	1.558	1.194	4.44	1.666	1.332	5.16	1.769	1.459	5.88	1.866	1.575
3.74	1.561	1.198	4.46	1.669	1.336	5.18	1.772	1.463	5.90	1.868	1.577
3.76	1.564	1.202	4.48	1.672	1.340	5.20	1.774	1.465	5.92	1.871	1.581
3.78	1.567	1.206	4.50	1.675	1.343	5.22	1.777	1.468	5.94	1.874	1.584
3.80	1.570	1.210	4.52	1.678	1.346	5.24	1.780	1.473	5.96	1.877	1.588
3.82	1.573	1.214	4.54	1.681	1.350	5.26	1.783	1.477	5.98	1.880	1.592
3.84	1.576	1.218	4.56	1.684	1.354	5.28	1.786	1.480	6.00	1.882	1.594
3.86	1.579	1.222	4.58	1.686	1.358	5.30	1.788	1.482	6.02	1.885	1.598
3.88	1.582	1.226	4.60	1.689	1.362	5.32	1.791	1.486	6.04	1.888	1.602

TABLE I.

h = 6.04 to 8.90

h	$f(h)$	$g(h)$	h	$f(h)$	$g(h)$	h	$f(h)$	$g(h)$	h	$f(h)$	$g(h)$
6.04	1.888	1.602	6.76	1.979	1.708	7.48	2.066	1.808	8.20	2.151	1.904
6.06	1.890	1.604	6.78	1.981	1.710	7.50	2.069	1.811	8.22	2.153	1.906
6.08	1.892	1.606	6.80	1.984	1.713	7.52	2.071	1.813	8.24	2.156	1.909
6.10	1.895	1.609	6.82	1.986	1.716	7.54	2.074	1.816	8.26	2.158	1.912
6.12	1.898	1.613	6.84	1.989	1.719	7.56	2.076	1.819	8.28	2.160	1.915
6.14	1.900	1.616	6.86	1.991	1.722	7.58	2.078	1.822	8.30	2.163	1.918
6.16	1.903	1.619	6.88	1.994	1.725	7.60	2.081	1.825	8.32	2.165	1.920
6.18	1.905	1.621	6.90	1.996	1.727	7.62	2.083	1.828	8.34	2.167	1.923
6.20	1.908	1.625	6.92	1.999	1.730	7.64	2.086	1.830	8.36	2.170	1.926
6.22	1.911	1.628	6.94	2.001	1.733	7.66	2.088	1.833	8.38	2.172	1.928
6.24	1.913	1.630	6.96	2.004	1.736	7.68	2.090	1.836	8.40	2.174	1.930
6.26	1.916	1.633	6.98	2.006	1.739	7.70	2.093	1.838	8.42	2.176	1.933
6.28	1.918	1.636	7.00	2.009	1.743	7.72	2.095	1.841	8.44	2.179	1.935
6.30	1.921	1.639	7.02	2.011	1.745	7.74	2.097	1.844	8.46	2.181	1.938
6.32	1.924	1.641	7.04	2.014	1.748	7.76	2.100	1.847	8.48	2.183	1.941
6.34	1.926	1.645	7.06	2.016	1.751	7.78	2.102	1.850	8.50	2.186	1.944
6.36	1.929	1.649	7.08	2.019	1.754	7.80	2.104	1.852	8.52	2.188	1.946
6.38	1.931	1.652	7.10	2.021	1.756	7.82	2.106	1.854	8.54	2.190	1.949
6.40	1.934	1.655	7.12	2.023	1.758	7.84	2.109	1.857	8.56	2.193	1.952
6.42	1.936	1.657	7.14	2.026	1.761	7.86	2.112	1.860	8.58	2.195	1.954
6.44	1.939	1.661	7.16	2.028	1.764	7.88	2.114	1.863	8.60	2.197	1.956
6.46	1.941	1.664	7.18	2.030	1.767	7.90	2.117	1.866	8.62	2.199	1.959
6.48	1.944	1.667	7.20	2.033	1.770	7.92	2.119	1.868	8.64	2.202	1.962
6.50	1.946	1.669	7.22	2.035	1.772	7.94	2.122	1.871	8.66	2.204	1.965
6.52	1.949	1.672	7.24	2.038	1.775	7.96	2.124	1.873	8.68	2.207	1.967
6.54	1.951	1.675	7.26	2.040	1.778	7.98	2.126	1.876	8.70	2.209	1.970
6.56	1.954	1.678	7.28	2.042	1.781	8.00	2.128	1.878	8.72	2.211	1.972
6.58	1.956	1.680	7.30	2.045	1.784	8.02	2.130	1.881	8.74	2.214	1.975
6.60	1.959	1.684	7.32	2.047	1.786	8.04	2.133	1.884	8.76	2.216	1.977
6.62	1.962	1.689	7.34	2.050	1.789	8.06	2.135	1.886	8.78	2.218	1.980
6.64	1.964	1.690	7.36	2.052	1.792	8.08	2.138	1.889	8.80	2.220	1.982
6.66	1.966	1.692	7.38	2.054	1.795	8.10	2.140	1.892	8.82	2.222	1.984
6.68	1.969	1.696	7.40	2.057	1.798	8.12	2.142	1.894	8.84	2.225	1.987
6.70	1.971	1.698	7.42	2.059	1.800	8.14	2.144	1.897	8.86	2.227	1.989
6.72	1.974	1.702	7.44	2.062	1.803	8.16	2.147	1.900	8.88	2.229	1.992
6.74	1.976	1.705	7.46	2.064	1.805	8.18	2.149	1.902	8.90	2.231	1.994

TABLE I.

h = 8.90 to 22.4

h	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*
8.90	2.231	1.994	9.7	2.319	2.092	13.3	2.678	2.484	16.9	2.995	2.823
8.92	2.233	1.997	9.8	2.329	2.104	13.4	2.687	2.494	17.0	3.003	2.832
8.94	2.236	2.000	9.9	2.340	2.116	13.5	2.696	2.503	17.1	3.011	2.841
8.96	2.238	2.002	10.0	2.351	2.128	13.6	2.705	2.513	17.2	3.019	2.850
8.98	2.240	2.005	10.1	2.362	2.140	13.7	2.714	2.523	17.3	3.027	2.859
9.00	2.242	2.007	10.2	2.372	2.151	13.8	2.723	2.533	17.4	3.035	2.868
9.02	2.244	2.009	10.3	2.382	2.162	13.9	2.732	2.543	17.5	3.044	2.876
9.04	2.247	2.012	10.4	2.392	2.173	14.0	2.741	2.553	17.6	3.052	2.885
9.06	2.249	2.014	10.5	2.403	2.185	14.1	2.750	2.563	17.7	3.060	2.894
9.08	2.251	2.016	10.6	2.413	2.196	14.2	2.759	2.573	17.8	3.068	2.903
9.10	2.253	2.019	10.7	2.423	2.207	14.3	2.768	2.582	17.9	3.076	2.910
9.12	2.255	2.021	10.8	2.433	2.218	14.4	2.777	2.592	18.0	3.084	2.919
9.14	2.258	2.024	10.9	2.443	2.221	14.5	2.787	2.602	18.1	3.093	2.928
9.16	2.260	2.026	11.0	2.454	2.242	14.6	2.796	2.612	18.2	3.101	2.937
9.18	2.262	2.029	11.1	2.465	2.253	14.7	2.805	2.621	18.3	3.109	2.945
9.20	2.264	2.031	11.2	2.475	2.264	14.8	2.814	2.631	18.4	3.117	2.952
9.22	2.266	2.034	11.3	2.484	2.274	14.9	2.823	2.640	18.6	3.133	2.969
9.24	2.269	2.036	11.4	2.494	2.285	15.0	2.832	2.649	18.8	3.148	2.986
9.26	2.271	2.039	11.5	2.504	2.296	15.1	2.841	2.659	19.0	3.164	3.002
9.28	2.273	2.042	11.6	2.514	2.307	15.2	2.850	2.669	19.2	3.180	3.019
9.30	2.275	2.044	11.7	2.524	2.318	15.3	2.859	2.678	19.4	3.196	3.035
9.32	2.277	2.046	11.8	2.534	2.329	15.4	2.867	2.688	19.6	3.227	3.052
9.34	2.280	2.049	11.9	2.544	2.340	15.5	2.876	2.697	19.8	3.231	3.068
9.36	2.282	2.051	12.0	2.554	2.350	15.6	2.885	2.706	20.0	3.243	3.085
9.38	2.284	2.053	12.1	2.563	2.360	15.7	2.894	2.715	20.2	3.258	3.101
9.40	2.286	2.056	12.2	2.573	2.370	15.8	2.903	2.724	20.4	3.273	3.117
9.42	2.288	2.058	12.3	2.582	2.381	15.9	2.911	2.733	20.6	3.288	3.132
9.44	2.291	2.061	12.4	2.592	2.392	16.0	2.919	2.742	20.8	3.303	3.148
9.46	2.293	2.063	12.5	2.601	2.401	16.1	2.928	2.751	21.0	3.318	3.164
9.48	2.295	2.065	12.6	2.610	2.411	16.2	2.936	2.760	21.2	3.333	3.179
9.50	2.297	2.067	12.7	2.620	2.421	16.3	2.945	2.769	21.4	3.348	3.195
9.52	2.299	2.069	12.8	2.630	2.432	16.4	2.953	2.778	21.6	3.362	3.210
9.54	2.302	2.072	12.9	2.640	2.443	16.5	2.961	2.787	21.8	3.377	3.226
9.56	2.304	2.075	13.0	2.650	2.454	16.6	2.970	2.796	22.0	3.392	3.241
9.58	2.306	2.077	13.1	2.660	2.465	16.7	2.979	2.805	22.2	3.401	3.256
9.60	2.308	2.080	13.2	2.669	2.475	16.8	2.987	2.814	22.4	3.421	3.272

TABLE I.

h = 22.4 to 107

h	$f(h)$	$g(h)$	h	$f(h)$	$g(h)$	h	$f(h)$	$g(h)$	h	$f(h)$	$g(h)$
22.4	3.421	3.272	29.6	3.912	3.782	36.8	4.348	4.232	72	6.042	5.959
22.6	3.436	3.287	29.8	3.924	3.795	37	4.359	4.243	73	6.084	6.001
22.8	3.450	3.303	30.0	3.937	3.808	38	4.417	4.302	74	6.123	6.041
23.0	3.465	3.318	30.2	3.950	3.821	39	4.473	4.360	75	6.166	6.084
23.2	3.480	3.333	30.4	3.963	3.835	40	4.528	4.416	76	6.205	6.124
23.4	3.494	3.348	30.6	3.976	3.848	41	4.584	4.473	77	6.245	6.168
23.6	3.509	3.363	30.8	3.989	3.862	42	4.638	4.528	78	6.285	6.205
23.8	3.523	3.378	31.0	3.002	3.875	43	4.692	4.584	79	6.326	6.246
24.0	3.538	3.393	31.2	4.014	3.888	44	4.743	4.636	80	6.364	6.284
24.2	3.551	3.407	31.4	4.026	3.900	45	4.797	4.691	81	6.405	6.326
24.4	3.563	3.423	31.6	4.039	3.913	46	4.848	4.744	82	6.443	6.364
24.6	3.577	3.437	31.8	4.051	3.925	47	4.899	4.795	83	6.481	6.403
24.8	3.590	3.452	32.0	4.063	3.938	48	4.952	4.850	84	6.519	6.442
25.0	3.603	3.467	32.2	4.075	3.951	49	5.000	4.898	85	6.558	6.481
25.2	3.618	3.481	32.4	4.087	3.963	50	5.050	4.950	86	6.597	6.521
25.4	3.632	3.495	32.6	4.100	3.976	51	5.099	5.000	87	6.633	6.557
25.6	3.647	3.509	32.8	4.112	3.988	52	5.147	5.049	88	6.669	6.593
25.8	3.661	3.524	33.0	4.124	4.001	53	5.198	5.101	89	6.706	6.637
26.0	3.676	3.538	33.2	4.136	4.014	54	5.244	5.148	90	6.746	6.671
26.2	3.689	3.552	33.4	4.148	4.026	55	5.293	5.198	91	6.782	6.708
26.4	3.702	3.565	33.6	4.161	4.039	56	5.341	5.246	92	6.819	6.744
26.6	3.716	3.579	33.8	4.173	4.051	57	5.387	5.294	93	6.857	6.783
26.8	3.729	3.592	34.0	4.185	4.064	58	5.433	5.340	94	6.894	6.821
27.0	3.742	3.606	34.2	4.197	4.076	59	5.478	5.386	95	6.928	6.859
27.2	3.756	3.620	34.4	4.209	4.088	60	5.524	5.433	96	6.966	6.894
27.4	3.769	3.634	34.6	4.220	4.100	61	5.568	5.477	97	7.001	6.929
27.6	3.783	3.648	34.8	4.232	4.112	62	5.612	5.522	98	7.035	6.964
27.8	3.796	3.662	35.0	4.244	4.124	63	5.656	5.567	99	7.056	6.985
28.0	3.810	3.676	35.2	4.256	4.136	64	5.701	5.613	100	7.105	7.035
28.2	3.823	3.689	35.4	4.267	4.148	65	5.746	5.658	101	7.141	7.070
28.4	3.836	3.703	35.6	4.279	4.160	66	5.789	5.712	102	7.176	7.106
28.6	3.848	3.716	35.8	4.290	4.172	67	5.832	5.745	103	7.212	7.142
28.8	3.861	3.730	36.0	4.302	4.184	68	5.875	5.788	104	7.247	7.178
29.0	3.874	3.743	36.2	4.314	4.196	69	5.916	5.830	105	7.279	7.211
29.2	3.887	3.756	36.4	4.325	4.208	70	5.959	5.875	106	7.321	7.244
29.4	3.899	3.769	36.6	4.336	4.219	71	6.000	5.917	107	7.349	7.280

TABLE I.

h = 107 to 250

h	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*	*h*	*f(h)*	*g(h)*
107	7.349	7.280	143	8.459	8.453	179	9.463	9.458	215	10.371	10.366
108	7.381	7.313	144	8.488	8.482	180	9.489	9.482	216	10.395	10.390
109	7.414	7.347	145	8.518	8.512	181	9.516	9.511	217	10.419	10.414
110	7.440	7.380	146	8.547	8.541	182	9.542	9.537	218	10.443	10.438
111	7.485	7.417	147	8.576	8.580	183	9.568	9.563	219	10.467	10.462
112	7.518	7.451	148	8.605	8.599	184	9.594	9.589	220	10.491	10.486
113	7.552	7.485	149	8.634	8.628	185	9.620	9.615	221	10.514	10.510
114	7.582	7.515	150	8.663	8.657	186	9.646	9.641	222	10.538	10.533
115	7.616	7.550	151	8.692	8.686	187	9.672	9.667	223	10.562	10.557
116	7.650	7.584	152	8.721	8.715	188	9.698	9.693	224	10.585	10.581
117	7.680	7.615	153	8.749	8.744	189	9.724	9.719	225	10.609	10.604
118	7.714	7.649	154	8.778	8.772	190	9.749	9.744	226	10.633	10.628
119	7.745	7.680	155	8.806	8.804	191	9.775	9.770	227	10.656	10.651
120	7.780	7.715	156	8.835	8.829	192	9.800	9.796	228	10.679	10.675
121	7.811	7.746	157	8.863	8.857	193	9.826	9.821	229	10.703	10.698
122	7.842	7.778	158	8.891	8.885	194	9.851	9.846	230	10.726	10.722
123	7.872	7.809	159	8.919	8.914	195	9.877	9.872	231	10.749	10.745
124	7.908	7.844	160	8.947	8.942	196	9.902	9.897	232	10.773	10.768
125	7.939	7.876	161	8.975	8.969	197	9.927	9.922	233	10.796	10.791
126	7.941	7.934	162	9.003	8.997	198	9.952	9.947	234	10.819	10.814
127	7.972	7.966	163	9.031	9.025	199	9.978	9.973	235	10.842	10.838
128	8.003	7.997	164	9.058	9.053	200	10.003	9.993	236	10.865	10.861
129	8.034	8.028	165	9.086	9.080	201	10.028	10.023	237	10.888	10.884
130	8.065	8.059	166	9.113	9.108	202	10.052	10.047	238	10.911	10.906
131	8.096	8.090	167	9.141	9.135	203	10.077	10.072	239	10.934	10.929
132	8.127	8.121	168	9.168	9.163	204	10.102	10.097	240	10.957	10.953
133	8.158	8.152	169	9.195	9.190	205	10.127	10.122	241	10.980	10.975
134	8.189	8.182	170	9.222	9.217	206	10.151	10.146	242	11.002	10.998
135	8.219	8.213	171	9.249	9.244	207	10.176	10.171	243	11.025	11.020
136	8.249	8.243	172	9.276	9.271	208	10.201	10.196	244	11.048	11.043
137	8.280	8.274	173	9.303	9.298	209	10.225	10.220	245	11.070	11.066
138	8.310	8.304	174	9.330	9.325	210	10.249	10.245	246	11.093	11.088
139	8.340	8.334	175	9.357	9.351	211	10.274	10.269	247	11.115	11.111
140	8.370	8.364	176	9.384	9.389	212	10.298	10.293	248	11.136	11.133
141	8.399	8.393	177	9.410	9.405	213	10.322	10.308	249	11.158	11.156
142	8.429	8.423	178	9.437	9.431	214	10.347	10.342	250	11.180	11.178

(Continued from page 3 of cover.)

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ARTIFICIAL ELECTRIC LINES WITH MUTUAL INDUCTANCE BETWEEN ADJACENT SERIES ELEMENTS.

By George W. Pierce.

ARTIFICIAL ELECTRIC LINES WITH MUTUAL INDUCTANCE BETWEEN ADJACENT SERIES ELEMENTS.

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1. General Principles.

1. Introduction.— So far as I have been able to ascertain, the artificial lines with lumped sections heretofore employed have been generally devoid of appreciable mutual inductance between adjacent elements.

In artificial line construction the inductive elements are placed mutually at right angles to each other or are wound in toroidal form so as to reduce to a minimum the mutual inductance caused by magnetic leakage from one inductive element to the next.

In my Text *Electric Oscillations and Electric Waves*,[1] Chapter XVI, I have treated theoretically artificial lines in which mutual inductance exists between adjacent series elements.

It is proposed here to show, with the aid of the general treatment in my text, that such mutual inductance, if properly chosen, has a decidedly beneficial effect in the following two types of apparatus:

A. An Electric Compensator designed to introduce into circuits a time lag substantially independent of the frequency of impressed e.m.f.;

B. An Artificial Line designed to simulate an actual smooth line.[2]

2. General Type of Line.— Let us direct our attention to the general type of artificial line shown in Figure 1.

The shunt elements of complex impedance z_2 may be of any character. The series elements of complex impedance z_1 may likewise be of any character, and have between such of these elements as are adja-

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[1] Pierce: *Electric Oscillations and Electric Waves*, McGraw-Hill Book Co., New York, 1920. (See also corrected reprint in press 1921.)

[2] Since sending this paper to press I have been informed by Mr. K. S. Johnson that application of the present device in Case B has been in use for several years at the Western Electric Company, but has not been described in publications.

cent a mutual inductance M, which may be set equal to zero in case results without M are required. The mutual inductance between non-adjacent elements is supposed to be negligible in all cases.

The line is supposed to terminate at both ends in a series half-element of complex impedance $z_1/2$ so constructed as to have also mutual inductance M with its neighboring series section.

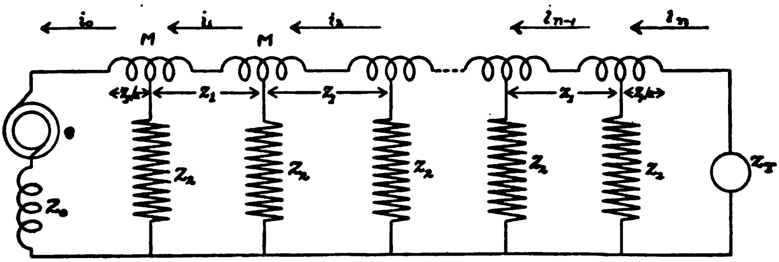

FIGURE 1. General type of artificial line containing mutual inductance between adjacent series elements.

The line is shown connected to an input terminal apparatus of complex e.m.f. e and complex impedance z_0 and connected to an output apparatus of complex impedance z_T.

The artificial line here shown may be regarded as consisting of $n-1$ loops and two terminal series half elements, or it may be considered to consist of n sections, known as T-sections, each of the character shown in Figure 2.

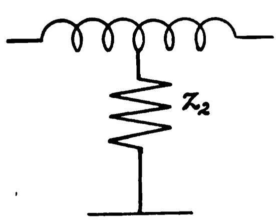

FIGURE 2. A T-section.

It may be noted that the treatment of other general types of lines (for example, lines of π-sections) are deducible from the treatment of the line of Figure 1 by postulating certain changes in the terminal conditions.

3. General Definitions.—

z_1 = *complex impedance* of the series elements.

z_2 = *complex impedance* of the shunt elements.

M = *mutual inductance* between adjacent series elements, estimated positive when the coils are wound in the same sense.

φ = *retardation angle of current* per section of the line = the angle of lag of steady-state current in any section behind the current in the preceding section.

T = *time lag of current* in seconds per section.

a = *real attenuation constant of current* per section of the line = the exponent in the factor ϵ^{-a}, by which factor the steady-state current-amplitude in any series element of an infinite or non-reflective line must be multiplied to give the current-amplitude in the next series element.

z_t = *complex surge impedance* of the line = complex impedance of a line of an infinite number of sections or a line with non-reflective output impedance.

ω = *angular velocity* of impressed sinusoidal e.m.f. in radians per second.

4. Abbreviations and Subsidiary Notation.— The following notation taken from *Electric Oscillations and Electric Waves* with slight augmentation will be employed.

$$P + jU, = \frac{z_1 + 2z_2}{z_2 - Mj\omega} \tag{1}$$

where P and U are real quantities;

$$V = \frac{1 - U^2 - P^2}{2}; \tag{2}$$

$$h = U/V; \tag{3}$$

$$f(h) = +\sqrt{\frac{+\sqrt{1+h^2}+1}{2}}, \quad g(h) = +\sqrt{\frac{+\sqrt{1+h^2}-1}{2}}. \tag{4}$$

Other notation will be introduced below in Section **10**.

5. Equations for a and φ for Steady-State Current.— It is proved in *Electric Oscillations and Electric Waves*, equations (49) and (50), p. 296, that, for the steady-state current under the action of an impressed sinusoidal e.m.f. of constant amplitude and frequency, a and φ are given by expressions which factored may be written

$$\left.\begin{aligned} a &= \sinh^{-1}\left\{+\sqrt{\pm V}\sqrt{\sqrt{1+h^2}\mp 1}\right\}, \\ \varphi &= \sin^{-1}\left\{+\sqrt{\pm V}\sqrt{\sqrt{1+h^2}\mp 1}\right\}, \end{aligned}\right\} \qquad (5)$$

in which V and h have the value given in (2) and (3), and *in which the upper signs or the lower signs are to be used together in each case and are to be chosen so as to make a and φ both real quantities.* For definitions of a and φ see §3.

6. Modified Form of Equations for a and φ.— Using the f- and g-functions given in (4), we may write (5) as follows:

I. If $V > 0$,

$$a = \sinh^{-1}\sqrt{2V}\, g(h), \quad \varphi = \sin^{-1}\{\pm\sqrt{2V}\, f(h)\}$$

with $$h = U/V.$$

II. If $V < 0$,

$$a = \sinh^{-1}\sqrt{-2V}\, f(h), \quad \varphi = \sin^{-1}\{\pm\sqrt{-2V}\, g(h)\}$$

with $$h = U/V. \qquad (6)$$

These equations are to be employed subject to the following rule regarding the quadrant of φ:

Rule Regarding φ.

sign of		quadrant of φ
P	U	
+	+	first
−	+	second
−	−	third
+	−	fourth

Equations (6) are the general expressions for the real attenuation constant a and the real retardation angle φ per section in terms of U, V, $f(h)$, *and $g(h)$, which are defined in equations (1), (2), (3), and (4). These equations apply to the general type of line given in Figure 1.*

In a paper now in press [1] entitled *A Table and Method of Computation of Electric Wave Propagation, Transmission Line Phenomena, Optical Refraction, and Inverse Hyperbolic Functions of a Complex Variable* I have given a table of the functions $f(h)$ and $g(h)$ for various values of h, so as to render very simple the computations of a and φ of equations (6).

7. General Equation for Surge Impedance z_i.— Before passing to a further discussion of a and φ, we shall introduce the general expression for surge impedance z_i, taken from *Electric Oscillations and Electric Waves*, Equation (34), p. 292, as follows:

$$z_i = \pm \sqrt{\frac{(z_1 + 2z_2)^2}{4} - (Mj\omega - z_2)^2}. \tag{7}$$

In Equation (7) the sign before the radical must be chosen to make the real part positive.

It may be noted that this equation also permits of easy computation by the method of the paper referred to in Section **6**.

8. Time Lag per Section.—

Let

T = time lag in seconds per section of the line introduced into the current by the line,

ω = angular velocity in radians per second of the impressed e.m.f.

In the steady state, the current will also have the angular velocity ω and the time lag per section will be given by

$$T = \varphi/\omega. \tag{8}$$

The steady-state time lag in seconds per section is the retardation angle per section in radians divided by the angular velocity in radians per second.

[1] These PROCEEDINGS: Vol. 57, No. 7.

II. The Electric Compensator.

Improvement Introduced by Proper Mutual Induction between Series Sections.

9. **Brief Description.**— In the determination of the direction of submarine sound signals and in giving to submarine sound apparatus directive qualities so as to permit discrimination of certain sounds from other sounds coming from a different direction, Professor Max Mason [1] of Wisconsin University has made use of two or more sound detectors (rubber nipples in the water) communicating with the ear of the observer through paths (air pipes) capable of adjustment as to time of travel (by adjusting pipe lengths), so that, when the detectors are struck, one after the other, by a sound-wave front, the impulses set up in the transmission paths connected to the several receivers may all be brought to the ear together by a suitable adjustment of retardation by the paths (compensation). In this way the setting of the apparatus (a compensator) to give a maximum of sound will, when the apparatus is properly calibrated, give a direct reading of the direction of the incident sound. Sounds coming from other directions (as, for example, noises from the listener's boat) will in general not be compensated to give a maximum of intensity and will be discriminated against by the apparatus.

Professor Mason developed this apparatus in a form known as the *Accoustical System* employing rubber nipples on the ends of tubes as sound detectors, and introducing compensation by varying the lengths of air colums through which the resulting sound waves in the air columns were transmitted to the ear of the observer. The Mason system gives excellent results in practice.

It readily occurred to those who knew of Mason's Device that some advantage might result by using microphones, or other electrical detectors, in the place of the rubber nipples, provided electrical methods could be employed to produce the required retardations of currents set up at the microphones.

An apparatus for this purpose was devised by me at the Naval Experimental Station at New London, and is called an *Electric Compensator.*

An Electric compensator is a device for giving electrically to an electric

[1] See H. C. Hayes, *Detection of Submarines*, Proc. Am. Phil. Soc., Vol. 59, pp. 1–47, 1920: and *U. S. Navy MV-Type of Hydrophone as an Aid and Safeguard to Navigation*, Ibid., Vol. 59, pp. 371–404; Max Mason: *Submarine Detection by Multiple Unit Hydrophones*, Wisconsin Engineer (1921).

current any desired time-retardation substantially independent of the frequency over a significant range of frequencies.

An artificial line consisting of inductances in series and capacities in shunt and provided with suitable switching devices is an electric compensator. To have practical application the attenuation of the compensator must be low and the steps must be made small enough in inductance and capacity values to obviate filtering effects, and the dimensions of the coils must be chosen in a manner to diminish dispersion and other distortions. The principles governing the design of such a compensator are given in my book *Electric Oscillations and Electric Waves*, pp. 285–323.

It is proposed here to show how improvements result from the use of suitable mutual inductance between the neighboring series inductances of the electric compensator.

10. Proof that Introduction between neighboring sections of Mutual Inductance Approximately Equal to One-Tenth of the Self Inductance of the Series Elements Greatly Reduces Dispersion in an Electric Compensator.— It is proposed to prove

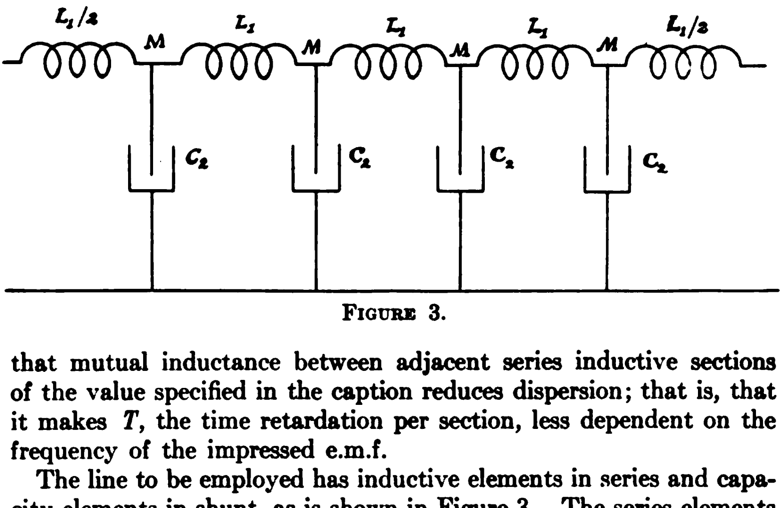

FIGURE 3.

that mutual inductance between adjacent series inductive sections of the value specified in the caption reduces dispersion; that is, that it makes T, the time retardation per section, less dependent on the frequency of the impressed e.m.f.

The line to be employed has inductive elements in series and capacity elements in shunt, as is shown in Figure 3. The series elements have an inductance L_1 and resistance R_1 per loop, and the shunt elements have each a capacity C_1. Let there be mutual inductance M between the inductances of each two adjacent loops, and let there be zero mutual inductance between inductances not adjacent.

We shall then have (compare *El. Osc. and El. Waves*, p. 316)

$$z_1 = R_1 + jL_1\omega, \tag{9}$$

$$z_2 = -j/C_2\omega, \tag{10}$$

These values substituted into (1) give

$$U = \eta Q/2, \tag{11}$$

$$V = \frac{Q}{2}\left\{1 - \frac{Q}{4}(1+\eta^2)\right\}, \tag{12}$$

in which the following abbreviations are employed

$$L = L_1 + 2M, \qquad R = R_1, \tag{13}$$

$$\eta = R/L\omega, \quad \text{and} \quad Q = \frac{LC_2\omega^2}{1 + MC_2\omega^2}. \tag{14}$$

As a further abbreviation, let us write

$$A = 1 - \frac{Q}{4}(1+\eta^2), \tag{15}$$

so that equation (12) becomes

$$V = QA/2. \tag{16}$$

Then by the general equations (6), we have for φ the value

$$\text{if} \qquad V > 0, \quad \varphi = \sin^{-1}\sqrt{\frac{QA}{2}}\sqrt{\sqrt{1+\frac{\eta^2}{A^2}}+1}\cdot \tag{17}$$

This equation is the equivalent of (142), p. 317, of *Electrical Oscillations and Electric Waves.* We now proceed to develop the subject further.

Confining our attention to this case in which V is greater than zero (that is, A positive), and expanding the inner radical in (17), we obtain, after transposition and squaring,

$$\sin^2\varphi = QA\left\{1 + \frac{\eta^2}{4A^2} - \frac{\eta^4}{16A^4} + \cdots\right\} \tag{18}$$

If now we consider the resistance per section to be so small in comparison with the inductive reactance per section that

$$\eta^2 << 1 \text{ and } \eta^2 << 4A^2 \tag{19}$$

we have

$$\sin^2\varphi = QA = Q - Q^2/4. \tag{20}$$

Let us now expand the two sides of (20) in series, obtaining

$$\varphi^2 - \frac{\varphi^4}{3} + \frac{2\varphi^6}{45} - \cdots = LC_2\omega^2 (1 - MC_2\omega^2 + M^2C_2^2\omega^4 - \cdots)$$
$$- \frac{L^2C_2^2\omega^4}{4} (1 - 2MC_2\omega^2 + 3M^2C_2^2\omega^4 - \cdots). \tag{21}$$

By reference to (8) it will be seen that to make T, the time retardation per section, independent of the frequency, we shall require φ^2 to be proportional to ω^2. This requirement will be consistent with (21) provided we can so choose M that

$$\varphi^2 = LC_2\omega^2, \tag{22}$$

and

$$\frac{-\varphi^4}{3} + \frac{2\varphi^6}{45} = -\left(\frac{L^2C_2^2\omega^4}{4} + \frac{LMC_2^2\omega^4}{1}\right) + \left(\frac{L^2MC_2^3\omega^6}{2} + \frac{LM^2C_2^3\omega^6}{1}\right) \tag{23}$$

are both satisfied.

Equations (22) and (23) are equations from which to determine M. Instead of an exact determination of M we shall content ourselves by an approximate determination of M by substituting (22) into (23) and equating separately terms of the same power of ω, obtaining

$$\frac{L^2C_2^2\omega^4}{12} = LMC_2^2\omega^4, \text{ and} \tag{24}$$

$$\frac{2L^3C_2^3\omega^6}{45} = \frac{L^2MC_2^3\omega^6}{2} + LM^2C_2^3\omega^6. \tag{25}$$

Equation (24) gives

$$M = L/12, \tag{26}$$

which substituted into (25) gives

$$L^3/45 = L^3/41. \tag{27}$$

Equation (27) is not true, but since terms in which L^3 enters are of a higher order than terms in L and L^2, we may consider the difference between the two sides of equation (27) as negligible, and employ (26) as the approximate condition required.

By the use of (13), this condition (26) becomes

$$M = L_1/10, \tag{28}$$

and by (8) and (28) equation (22) gives

$$T = \sqrt{LC_2} = \sqrt{1.2L_1C_2}. \tag{29}$$

Equation (28) gives the value of the Mutual Inductance M to be introduced between the adjacent series coils of Self Inductance L_1 (and of negligible resistance) in order to make the time-retardation T per section be as nearly as possible independent of the frequency and to have over a wide range of frequencies approximately the value given in (29).

If the resistance of the coils is not negligible it is possible to modify the relation (28) slightly in a manner that will enhance slightly the constancy of T over a given range of frequencies, but in practice, where a reasonable effort is made to keep resistances small, the relation (28) is sufficiently accurate.

In any case the exact value of T can be calculated by the use of (17) and (8).

11. Computed Table and Curves Showing the Performance of a Line with Mutual Inductance Equal to One Tenth of Self Inductance in Comparison with a Line of Zero Mutual Inductance.— Table I of which four columns were computed from the exact Equation (17) with the use of (8), gives the values of $T/\sqrt{LC_2}$ for different values of $\omega\sqrt{LC_2}$.

TABLE I.

Values of $T/\sqrt{LC_2}$ for Various Values of $\omega\sqrt{LC_2}$ for the Case of $R/R_0=0$ and the Case of $R/R_0=.1$, with $M=0$ and $M=.1L_1$ in each case.

	$R/R_0=0$		$R/R_0=.1$			
	$M=0$ $L=L_1$	$M=.1L_1$ $L=1.2L_1$	$M=0$ $L=L_1$	$M=.1L_1$ $L=1.2L_1$	$M=0$ $L=L_1$	$M=.1L_1$ $L=1.2L_1$
$\omega\sqrt{LC_2}$	$T/\sqrt{LC_2}$	$T/\sqrt{LC_2}$	$T/\sqrt{LC_2}$	$T/\sqrt{LC_2}$	a	a
.00	1.0000	1.0000	∞	∞	.00000	.00000
.01	1.0000	1.0000	2.347	2.347	.02131	.02131
.02	1.0000	1.0000	1.745	1.745	.0287	.0286
.03	1.0000	1.0000	1.497	1.497	.0334	.0334
.05	1.0000	1.0000	1.272	1.270	.0394	.0394
.10	1.0007	1.0000	1.099	1.098	.0456	.0456
.15	1.0017	1.0000	1.050	1.049	.0479	.0478
.20	1.002	1.000	1.030	1.029	.0489	.0487
.25	1.003	1.000	1.022	1.019	.0495	.0494
.30	1.004	1.000	1.017	1.013	.0500	.0498
.35	1.005	1.000	1.016	1.009	.0503	.0499
.40	1.007	1.000	1.015	1.007	.0507	.0503
.50	1.011	1.000	1.016	1.005	.0514	.0508
.60	1.016	1.000	1.018	1.004	.0523	.0514
.70	1.022	1.001	1.023	1.002	.0532	.0521
.80	1.032	1.001	1.031	1.003	.0545	.0528
1.00	1.050	1.002	1.046	1.004	.0577	.0547
1.20	1.075	1.004	1.072	1.007	.0624	.0574
1.40	1.121	1.009	1.109	1.010	.0700	.0605
1.60	1.156	1.019	1.160	1.016	.0834	.0659
1.80	1.239	1.027	1.241	1.026	.1142	.0736
2.00	1.570	1.047		1.044		.0862
2.20		1.082		1.079		.1129
2.40		1.172		1.144		.2122

The last two columns contain corresponding values of the attenuation constant a computed by the use of the exact equation (6), for the case of $R/R_0 = .1$. For $R = 0$ the value of a is zero throughout the range of the table.

In the table the values of R are specified by specifying values of R/R_0, where $R_0 = \sqrt{L/C_2}$. Only two values of R/R_0 (namely, zero and .1) were employed in computing the tables. The case in which the ratio is zero is a case of zero attenuation, while the case with the ratio .1 is a case of much higher attenuation than would arise with coils built with a reasonable effort to keep the ratio of resistance to inductance small, so that the table gives extreme values.

Curves of the data of the table are presented in Figures 4, 5 and 6.

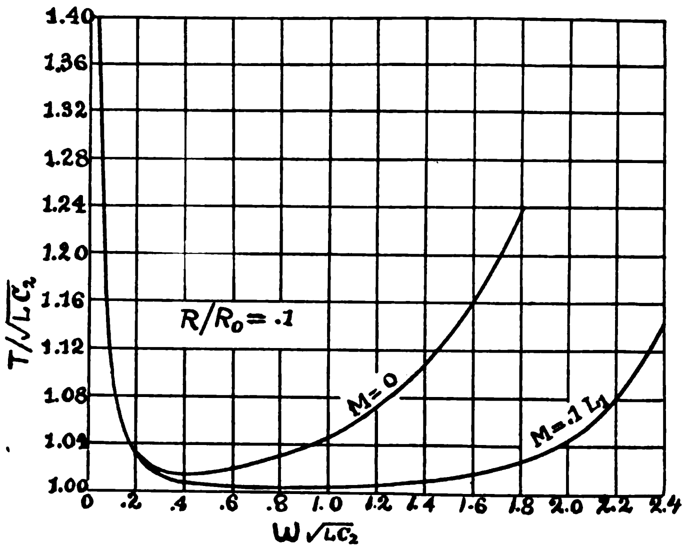

FIGURE 4.

12. Comparison of Line of Mutual Inductance equal to One Tenth of Self Inductance with Line of Zero Mutual.— By reference to the curves of Figures 4 and 5 it is seen that the range of values of ω for which the time retardation T per section is within a given percentage of constant is much larger in the case $M = .1L_1$ than in the case $M = 0$. With $M = .1L_1$ the range of frequencies over which T is within one percent of constant is three times as great as when no

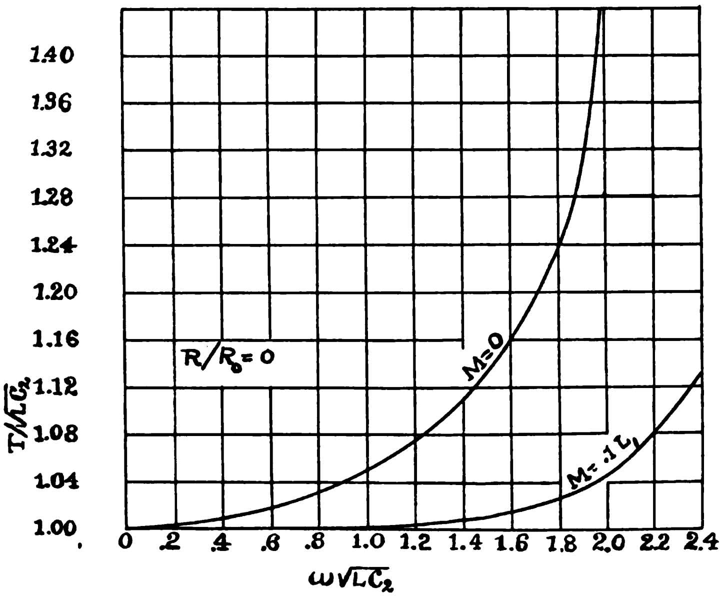

FIGURE 5.

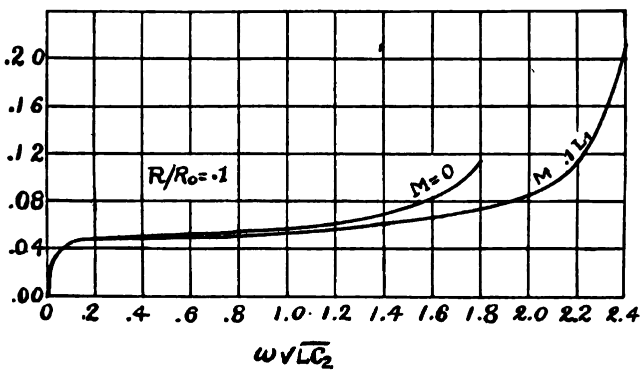

FIGURE 6.

mutual inductance is introduced between the series coils. Reference to Figure 6 shows that the attenuation constant a' is also slightly smaller with the mutual inductance than without it.

If air-core coils are to be used it requires less space and is much easier to supply the required mutual inductance than it is to leave it out, so that by using the mutual inductance between sections we can increase the frequency range for a specified constancy of time retardation to three times the range without the mutual inductance, and this can be done with a better space factor and with a significant diminution of cost.

13. Constancy of Surge Impedance also Somewhat Improved by Introducing Mutual Inductance = $.1 L_2$ between Series Sections of a Low Resistance Line.— Equation (7) is the general expression for surge impedance of the line of Figure 1. When the series impedances are inductances and the shunt impedances are capacities, as in Figure 3, z_1 and z_2 take on the values given in (9) and (10). These substituted into (7) give

$$z_i = \pm \sqrt{\frac{L}{C_2}} \sqrt{1 + \left(\frac{R}{2R_0}\right)^2 - \frac{L - 4M}{4} C_2\omega^2 + j\left\{\frac{R}{2R_0}\sqrt{LC_2}\,\omega\left(1 - \frac{2M}{L}\right) - \frac{R}{R_0}\frac{1}{\sqrt{LC_2}\,\omega}\right\}}\,. \tag{30}$$

Equation (30)[1] *is the general expression for surge impedance of a line of the type shown in Figure 3. In this equation* $L = L_1 + 2M$, and $R_0 = \sqrt{L/C_2}$.

It is seen that, if the resistance of the line is low, the imaginary term in (30) is small, and the real term tends to approach independence of ω as $4M$ approaches L. Making $M = .1L_1 = L/12$, as is required in order to make T less dependent on frequency, has the effect of cutting the real term containing ω^2 by about $\frac{1}{3}$ and hence the introduction of $M = .1L_1$ reduces the dependence of surge impedance on frequency for low resistance lines.

[1] The corresponding equation (142) p. 317 of *El. Osc. and Waves* has in the first printing of the book an error, that has been corrected in the Second Impression of 1921.

III. An Artificial Line to Simulate an Actual Smooth Line.

Improvement as to Attenuation and Surge Impedance brought about by Proper Mutual Induction between Series Sections.

14. Constants for Actual Smooth Line.— In the case of an actual smooth line, let

r, l, c = respectively the resistance, self-inductance, and capacity per loop unit of length;
α = real attenuation constant per unit of length;
β = angle of lag of current per unit of length;
z_i = surge impedance of the line;

then

$$\alpha = \omega \sqrt{\frac{lc}{2}} \sqrt{\sqrt{\frac{r^2}{l^2\omega^2} + 1} - 1} \tag{31}$$

$$\beta = \omega \sqrt{\frac{lc}{2}} \sqrt{\sqrt{\frac{r^2}{l^2\omega^2} + 1} + 1} \tag{32}$$

$$z_i = \pm \sqrt{\frac{l}{c}} \sqrt{1 - j\frac{r}{l\omega}}\,. \tag{33}$$

These equations are the familiar expressions. See, for example *El. Osc. and El. Waves*, pp. 329 and 330.

15. To Design an Artificial Line with Lumped Sections that Will Simulate the Smooth Line as to Surge Impedance.— For this purpose we shall postulate a line of the type shown in Figure 3, and shall determine what value of M brings the surge impedance of this line, as is given in (30), most nearly into the form of (33) in so far as dependence of z_i on ω is concerned. Since the real part of (30) is generally much larger than the imaginary part, it is seen by inspection that a good approximation to this result is made by making

$$M = L/4; \tag{34}$$

which by (13) means

$$M = L_1/2. \tag{35}$$

By making M have the value given in (34) equation (30), in view of the fact that

$$R_0 = \sqrt{L/C_2}, \tag{36}$$

becomes

$$z_i = \pm\sqrt{\frac{L}{C_2}}\sqrt{1 + \frac{R^2C^2}{2L} - j\frac{R}{L\omega}\left\{1 - \frac{LC_2\omega^2}{4}\right\}}. \tag{37}$$

Equation (37) gives the surge impedance of a lumpy artificial line with $M = L_1/2$. Since the imaginary term in the surge impedance of a smooth-line (equation (33)) is generally small over practical ranges of frequency, it is seen that (37) is essentially of the form of (30) in so far as concerns dependence of surge impedance on frequency.[1]

16. To Determine the Mutual Inductance between Series Section in a Lumpy Artificial Line to Bring it into Close Similarity with A Smooth Line as to Attenuation Constant.—Postulating a line of the type of Figure 3, and substituting (9), (10), (13), (14) and (15) into the value of a given in (5) we obtain

If $V > 0$,

$$a = \sinh^{-1}\sqrt{\frac{QA}{2}}\sqrt{\sqrt{1 + \frac{\eta^2}{A^2}} - 1}. \tag{38}$$

Equation (38) is the general equation for attenuation constant for a line of the type of Figure 3, under the condition $V > 0$.

Equation (38) expanded with neglect of higher powers of η^2/A^2 gives

$$\sinh a = \frac{\eta}{2}\sqrt{\frac{Q}{A}}\left\{1 - \frac{\eta^2}{8A^2} + \frac{\eta}{128}\frac{\eta^4}{A^4} + \cdots\right\} \tag{39}$$

A corresponding expansion of (31) gives approximately for the smooth line

$$a = \frac{\eta_0}{2}\sqrt{lc}\,\omega\left\{1 - \frac{\eta_0^2}{8} + \cdots\right\}, \tag{40}$$

[1] This fact was called to my attention by Mr. Phillip Machanik, who based his observation in an examination of the equations in *Electric Oscillations and Electric Waves*.

where

$$\eta_0 = r/l\omega. \tag{41}$$

Since the second terms in (40) and (39) are usually small, and since a is also sufficiently small to make sinh a essentially equal to a, the two equations reduce to nearly the same form in respect to ω if

$$Q/A = LC_2\omega^2. \tag{42}$$

Replacing A in (42) by its value from (14) with neglect of η^2 in (15) we obtain

$$Q = \frac{LC_2\omega^2}{1 + LC_2\omega^2/4},$$

which compared with (14) shows that (42) is approximately satisfied when

$$M = L/4 = L_1/2. \tag{42}$$

A substitution of this value of M into the exact equation (38) gives in a careful approximation

$$\sinh a = \frac{\eta}{2}\sqrt{LC_2}\,\omega\left\{1 - \frac{\eta^2}{8} + \frac{\eta^2}{16}\left(LC_2\omega^2 + \frac{3}{8}L^2C_2^2\omega^4\right)\right\}. \tag{43}$$

If now we expand the hyperbolic sine into

$$\sinh a = a + a^3/6,$$

and replace the a in a^3 by the first term of (43), we obtain

$$a = \frac{\eta}{2}\sqrt{LC_2}\,\omega\left\{1 - \frac{\eta^2}{8} + \frac{\eta^2}{96}\left(LC_2\omega^2 + \frac{9}{4}L^2C_2^2\omega^4\right)\right\}. \tag{44}$$

The approximation of equation (44) shows that even when η is as large as .5 and with $LC_2\omega^2$ as large as 1 the introduction of a mutual inductance of the value given in (42) makes the attenuation constant of the artificial lumpy line of the same form as the attenuation constant a of the smooth line, and that the two attenuation constants can be thus made to agree over a wide range of frequencies.

IV. Conclusions.

The following results, believed to be novel, for artificial line construction have been here derived.

1. To obtain a minimum dependence of time lag per section on frequency of an electric artificial line of low resistance, the line should be constructed with mutual inductance between neighboring series inductive elements equal to one-tenth of the self inductance of each series inductive element.

2. To most closely simulate a real smooth electric line as to attenuation constant and surge impedance by the use of an artificial electric line with lumpy sections, there should be in the artificial line a mutual inductance between adjacent loops equal to one-half of the series self inductance per loop.

3. Details for calculating the performance of lines constructed according to 1. and 2. are given.

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CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION FOR RESEARCH. No. 135.

THE PARASITIC WORMS OF THE ANIMALS OF BERMUDA.

I. TREMATODES.

By Franklin D. Barker.

With Three Plates.

CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION FOR RESEARCH. NO. 135.

THE PARASITIC WORMS OF THE ANIMALS OF BERMUDA.

I. TREMATODES.

Franklin D. Barker.

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Introduction.

Through the courtesy and generous assistance of Professor E. L. Mark, Director of the Bermuda Biological Station, and of the National Academy of Science, it has been my privilege to spend two seasons at the Bermuda Station, collecting and studying the parasitic fauna. The following paper is the first of a series which will embody the results of these investigations.

The two forms here described were found in the stomach of a Hawk's-bill Turtle, *Chelonia imbricata* (Linn.), at the Bermuda Biological Station.

Pachypsolus brachus, n. sp.

(Pls. I and II, Figs. 1–8, 12).

1. Morphology.

General Appearance.— The description of the following species is based on the study of 27 preserved specimens, 11 of which were killed and fixed in 2% formol and 16 in vom Rath's osmio-sublimate mixture. Little difference can be seen as a result of the different killing reagents other than in color. Specimens fixed in formol are grayish-yellow, those fixed in vom Rath's fluid black. A detailed study has been made of specimens in toto, both unmounted and mounted, and of series of frontal and sagittal sections.

The body is oval and plump (Pl. II, Fig. 8), being one half as thick as wide. The length varies from 3 mm. to 3.7 mm., the mode being 3 mm., which is the length of 50 per cent of the individuals. The width varies from 1.5 to 1.9 mm., the mode being 1.7 mm., which is the width attained by 60 per cent. The ends are bluntly rounded, the anterior end slightly more tapering than the posterior. In the median line at the posterior end is a well defined terminal invagination, which marks the position of the excretory pore. The dorsal surface of the body is strongly arched, the ventral surface slightly cupped. The sides are nearly parallel with the exception of a wide, shallow constriction midway between the ends at the level of the acetabulum. Spines or scales were not found anywhere on the body.

In the preserved specimens, the anterior third of the body shows a marked and constant tendency to flex ventrad, which gives rise to a well defined and rather deep ventral cup between the oral and ventral suckers. This cup-like depression persists in compressed specimens and possibly functions as a secondary holdfast (Pl. II, Fig. 8).

The oral sucker is comparatively large, well defined, nearly circular in outline and ventral in position, with its dorso-ventral axis at right angles to the chief axis of the body. In compressed specimens the oral sucker is 0.80 mm. to 0.82 mm. wide by 0.66 mm. to 0.82 mm. long. In frontal sections it measures 0.82 mm. wide by 0.82 mm. long. The ventral sucker, or acetabulum, lies in the median area at the posterior margin of the anterior half of the body and faces obliquely cephalad. It is of approximately the same size and shape as the oral sucker. It measures 0.60 mm. to 0.74 mm. in length by 0.70 mm. to 0.74 mm. in width in compressed specimens and 0.72 mm. by 0.80 mm. in frontal sections.

The genital pore is not salient and lies in the median line, at the anterior margin of the acetabulum, or else slightly to the left of, and just anterior to the acetabulum.

In the middle quarters of the body (Pl. I, Fig. 2), along the sides, and extending well toward the median line on the dorsal surface, can be seen the characteristic dark colored, convoluted tubular and finely granular vitelline glands in moss-like patches. The uterus appears as a dark coiled mass nearly filling the ventral field of the posterior third of the body.

Digestive System.— The transversely oval mouth leads into the triangular lumen of the oral sucker, which is 0.90 mm. deep in sagittal sections with thick muscular walls. A thick walled, large and powerful cup-shape pharynx follows immediately (Pl. II, Fig. 12). The pharynx measures 0.58 mm. long by 0.52 mm. wide by 0.44 mm. deep in sagittal sections. Eight longitudinal muscular ridges or folds project from the inner wall of the anterior two thirds of the pharynx into its lumen. Of the four larger or primary ridges, one is dorsal, one ventral, and two lateral; alternating with these are four smaller or secondary ridges. An œsophagus is not present, the common transverse caecum following immediately behind the pharynx. On each side of the pharynx a single well defined diverticulum extends cephalad from the transverse caecum lateral, or dorso-lateral, to the oral sucker to a height of one half the sucker's depth. These diverticula may or may not bifurcate at their terminations. The lateral caeca are broad and deep and they extend in an undulating course to the posterior end of the body, where they end blindly, giving off in their course numerous small lateral and deep ventral diverticula. The caeca are deeper than wide and lie in a plane mid-way between the dorsal and ventral surfaces in the lateral fields of the body (Pl. II, Fig. 12).

Male genitals.— The two testes (Pl. I, Figs. 1, 5), of medium and nearly equal size, irregular in shape and with undulating to slightly lobed margins, are situated in the same transverse plane midway between the anterior and posterior ends of the body. The testes vary in size from 0.58 mm. to 0.68 mm. by 0.34 mm. to 0.54 mm. The right one is slightly behind the posterior margin of the acetabulum and its left end projects beyond the median plane into the left half of the body. The bulk of the left testis is farther from the median plane than is the left margin of the acetabulum, and its anterior end extends a little farther forward than the posterior margin of the acetabulum. From the antero-dorsal margin of each testis a small vas efferens passes obliquely forward and toward the median plane. These unite just cephalad and dorsad to the ovary to constitute the vas deferens (Pl. I, Fig. 3); this continues in the median area to the base of the cirrus pouch, which it enters. The cirrus pouch, though comparatively short, is an elongated pear shaped organ situated immediately anterior to the acetabulum, with its long axis nearly perpendicular to the frontal plane of the body. It is 0.96 mm. long and from 0.24 mm. to 0.28 mm. in diameter, being a little longer than the acetabulum is deep, so that its base lies slightly dorsad and cephalad to the acetabulum. It is so short that it does not bend around the acetabulum. The vas deferens upon entering the base of the cirrus pouch unites with the enlarged transversely coiled seminal vesicle, which fills the basal third of the pouch. The seminal vesicle connects, in turn, with a comparatively wide prostatic duct, which has an undulating course and tapers toward the distal end of the pouch, where it merges into the ejaculatory duct of the short cirrus.

The cells forming the prostate gland (Pl. I, Fig. 3) occupy the peripheral portion of the pouch and extend from the seminal vesicle nearly to the distal end of the pouch. Fine ducts leading from the prostate cells occupy the medullary portion of the pouch and enter the prostatic duct. The lumen of the cirrus is lined with cuticula, while the lumen of the prostatic duct is covered with high filamentous papillae.

The wall of the cirrus pouch possesses a heavy outer sheet of longitudinal muscle fibers and a thin inner one of circular fibers. The cirrus, which is approximately one-fifth the length of the cirrus pouch, has an outer and an inner muscular component. The outer component comprises an outer sheet of longitudinal muscle fibers and a heavier inner sheet of circular fibers. The inner muscular component immediately surrounds the lumen of the cirrus and the prostatic duct and is likewise composed of an inner sheet of circular muscle fibers and an outer sheet of longitudinal fibers. The cirrus pouch is anchored and possibly controlled by a pair of oblique muscles which are attached respectively to the cephalic and caudal faces of its base.

The cirrus opens into a common genital atrium, which has its outlet in the genital pore lying in, or a little to the left of, the median line and slightly anterior to, or just under, the anterior margin of the acetabulum.

Female genitals.— The ovary (Pl. I, Figs. 1, 5, 6, Pl. II, Fig. 7) lies near the middle of the body, in the median area, dorsal to the posterior portion of the acetabulum and is from one-half to two-thirds the bulk of one of the testes, globular in general form with undulating or slightly lobed outline. In the specimen figured (Fig. 5) it measured 0.38 mm. by 0.38 mm.

The oviduct leaves the ovary from the middle of its anterior margin and at once turns sharply mediad; after making several loops it passes caudad in descending transverse coils, lying a little to the right of the median plane, to the end of the body, where, turning, it winds cephalad, a little to the left of the median plane, in ascending transverse coils; the terminal portion passes between the testes and thence to the left and dorsally over the acetabulum; finally it turns to the right and crosses obliquely the distal third of the cirrus pouch (Pl. I, Fig. 3), where it enters a well defined metraterm or vagina. The base of the metraterm is enlarged and lies across the left side of the terminal portion of the cirrus pouch, but the neck parallels the pouch and terminates anteriorly and to the left of the pouch in the common genital atrium. The wall of the metraterm is thickened and supplied with an inner sheet of circular muscle fibers and an outer sheet of longitudinal fibers. An invagination of the cuticula appears to form the lining of its lumen, the wall of which is transversely ridged.

A diffuse, but well defined, shell-gland (Pl. I, Fig. 6), or gland of Mehlis, lies dorsal, and for the most part anterior, to the ovary. Its posterior portion covers the anterior third of the ovary. The oviduct penetrates the mass of shell-gland cells, which are connected with the oviduct by numerous minute ducts.

A globular compact receptaculum seminis (Pl. I, Figs. 1, 5, 6), one-third the bulk of the ovary, lies dorsal to the shell-gland and the anterior half of the ovary. A small duct leaves its anterior margin and turning mediad joins the oviduct soon after it leaves the ovary.

At the junction of the receptaculum duct with the oviduct a tubular Laurer's canal (Fig. 6) originates and in a slightly undulating course makes its way dorsad and opens on the dorsal surface of the body dorsal to the ovary and the posterior part of the acetabulum and slightly to the right of the median line.

The vitellarium (Pl. I, Figs. 1, 2, 5) is rather striking in appearance and is composed of two masses of convoluted tubules grouped in moss-like patches, which lie in the lateral and latero-dorsal fields in the middle three-fifths of the body. The patches are fairly definite and constant in number, three patches being present on the right and four on the left side. They extend forward of the anterior margin of the acetabulum a distance approximately half of the diameter of that organ, the posterior limit being about mid-way between the testes and the posterior end of the body. The latero-dorsal patches are dorsal to the testes and uterine coils. A fine vitelline duct (Pl. I, Fig. 6) connects with each other the patches or groups of each side and a larger vitelline duct leaves the central group of each side and passes transversely mediad to unite with the one from the opposite side to form a small, but distinct, vitelline reservoir, which lies dorsal to the left margin of the ovary. From the reservoir a small duct leads cephalad and joins the oviduct a short distance beyond the union of the receptaculum duct with the oviduct.

The eggs (Pl. I, Fig. 4) are numerous, spindle shaped, light brown in color, with thick shell. A comparatively large, well-defined and easily separated operculum is present, and a slight opercular rim can be detected. The opercular pole is the more pointed. The eggs measure 0.0375 mm. to 0.0450 mm. in length by 0.015 mm. to 0.020 mm. in width. The older eggs contain a well developed embryo, but many appear empty, which probably indicates a non-fertile condition.

Excretory System.— The excretory system (Pl. II, Fig. 7) is voluminous and consists of an enormous median dorsal reservoir, with a pair of anterior prolongations. The reservoir is one-fourth the width and one-half the depth of the body, and extends from the posterior end of the body to the posterior margin of the ovary, where it bifurcates, one arm passing to the left and one to the right of the ovary; the arms extend cephalad around the oral sucker to the anterior end of the body. The reservoir and arms give off numerous long lateral and deep ventral diverticula, but these do not anastomose. The reservoir terminates behind in a short narrow median canal at the posterior end of the body, which ends in a well-defined excretory pore, terminal in position and nearer the ventral than the dorsal surface. The short excretory canal appears to be lined with cuticula. The entire excretory system is filled with a mass of fine globular, gray and golden, glistening particles among which are numerous larger globular bodies which stain a bright blue with methylen blue.

2. Taxonomy.

Braun (1901, p. 36) described under the name of *Distomum irroratum* R. a trematode found in the intestine of a sea turtle, *Thalassochelys caretta*, from New Guinea, which has a number of characters similar to the trematode described in this paper. Looss (1901, p. 558) described a similar trematode found in the stomach of a sea turtle, *Thalassochelys corticata*, from Triest, which he named *Pachypsolus lunatus*. In a later paper Looss (1902, p. 485), after a careful comparative study of new adult specimens, as well as the forms described by Braun and by himself, reached the conclusion that all were specimens of *Distomum irroratum* Rudolphi, those described by Braun and by himself in his earlier paper being young forms, while those studied by himself later were mature. He accordingly classified all of them as *Pachypsolus irroratus* (R.).

Looss (1902, p. 503) gives the following characters for the genus *Pachypsolus*, "Mittelgrosse Distomen mit sehr kräftigem, dickem, vorn und hinten abgerundetem, auf dem Querschnitte kurz ovalem Körper. Saugnäpfe gross und kräftig, Haut besonders im Vorderkörper mit scheinbaren Bündelen feiner stäbchenartiger Stacheln bewaffnet. Darm mit starkem Pharynx, ganz kurzem Oesophagus und Darmschenkeln, die bis auf einige von ihren Angfangstheilen nach vorn abgehende Blindsäcke einfach sind. Excretionsblase Y förmig, mit bis zum Keimstock reichendem Stamme und bis ins Kopfende sich erstreckenden Schenkeln. Stamm und Schenkel mit mässig zahlreichen, weiten und zum Theil wieder gespaltenen Seitenzweigen, die nach der Bauchseite hinabsteigen mit Ausnahme des vordersten

Paares, welches uber dem Mundsaugnapfe eine einfache Queranastomose der Schenkel bildet. Genitalporus etwas linksseitig von dem Bauchsaugnapfe, Copulationsorgane vorhanden. Cirrusbeutel cylindrisch, von beträchtlicher Länge, in seinem Innern eine mehrfach gewunden, schlanke Samenblase, lange, cylindrische Pars prostatica und dicker Penis, der sich im ausgestülpten Zustande nach seiner Spitze zu merklich verjüngt. Hoden stark seitlich hinter dem Bauchsaugnapfe. Keimstock seitlich vor ihnen; Laurer'scher Canal und Receptaculum seminis vorhanden. Dotterstöcke in den Seiten und unter der Rückenfläche, aus in der Jugend deutlich sternförmigen Follikelgruppen zusammengesetzt, Uterusschlingen hauptsächlich hinter den Hoden die ganze Breite des Körpers ausfüllend und nur die Enden der Darmschenkel freilassend. Eier zahlreich, klein, mit zugespitztem Deckelpol und dickerm Hinterende, zwischen 0,04 und 0,05 mm. lang. Bewohner des Magens von Seeschildkröten. Typus: *P. irroratus* (R.)."

The trematode from *Chelonia imbricata* which I have described has, in general, the characters of the genus *Pachypsolus*, and I do not hesitate to place it in that genus. When compared with the trematodes described by Braun and Looss under the name *irroratus* several essential differences are evident. Externally the following may be noted. The absence of spines, or scales, which may, however, have been lost, the very large and more nearly equal size of the suckers, the ventral cup-like depression and the non-salient genital pore. Internally, the position of the testes and ovary nearer the acetabulum and the less diffuse arrangement of the vitelline masses, which are more nearly like those described by Braun, may be noted. The most striking and essential difference, however, is the size and position of the cirrus pouch, which in *Pachypsolus irroratus* (Pl. II, Fig. 11) bends around the acetabulum, its posterior end extending to the level of, or posterior to, the ovary, while in the form here described (Pl. I, Fig. 3) the cirrus pouch is much shorter, parallel with the dorsoventral axis of the body and entirely anterior to the acetabulum.

Linton (1910, p. 24) has described a new species, *Pachypsolus ovalis*, found in large numbers in the intestine of a Loggerhead Turtle (*Caretta caretta*) from the Tortugas. A third species, *Pachypsolus tertius*, has been described by Pratt (1914, p. 416) from the small intestine of the same host and of the same locality. The species described by Linton and by Pratt differ from *P. irroratus* in minor points and distinctively in the position and extent of the cirrus pouch. Pratt (1914, p. 418) describes the cirrus sac in *P. tertius* (Pl. II, Fig. 9) as "a long cylindrical structure, extending from the genital pore around the dorsal side of the acetabulum to the vicinity of the ovary and the shell-gland, and in some cases to the anterior border of the testes." According to Linton (1910, p. 25) the cirrus pouch in *P. ovalis* (Pl. II, Fig. 10) is "relatively short, reaching barely to the posterior edge of the acetabulum." Both Linton and Pratt consider the differences in the extent of the cirrus pouch, together with other minor differences, to be of specific rank. It is evident that the form which we have described resembles *P. ovalis* Linton more than it does *P. tertius* Pratt or *P. irroratus* (R.) Looss; but it differs from *P. ovalis* Linton in minor characters and distinctively in the position and lesser extent of the cirrus pouch. The difference in the length of the cirrus pouch in *P. ovalis* Linton and in the trematode here described is greater than that between *P. ovalis* and *P. tertius* Pratt and decidedly greater than that between *P. tertius* Pratt and *P. irroratus* (R.) Looss. We agree with Pratt that the "actual position is undoubtedly dependent upon the condition of contraction," but it seems improbable that this constant and marked difference could be due entirely to the contraction of the acetabulum or the body.

We feel warranted in ascribing to this difference in the position and extent of the cirrus pouch, taken together with the minor differences noted, a specific value, and therefore class this trematode as a new species in the genus *Pachypsolus*, designating it as *Pachypsolus brachus*.[1]

In the four species of *Pachypsolus* now recorded we find, in addition to differences of secondary importance, a striking gradation in the position and size of the cirrus pouch, which is the distinctive specific character. The old question, raised by Looss, arises as to specific differences and the specific effects of different hosts on the same species.

From the standpoint of geographical distribution, it is of interest to find in the Hawk's-bill Turtle from the Bermudas a different species of *Pachypsolus* from that found in the Loggerhead Turtles of New Guinea and the Mediterranean and from those found in the Loggerhead Turtles of the Tortugas.

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1 βραχύς, short, having reference to the cirrus pouch.

Synechorchis megas, n. g. et n. sp.

(Pl. III, Figs. 13–22).

1. Morphology.

General Appearance.— Twenty-four specimens of this trematode were studied, twelve of which were fixed in 2% formol and twelve in vom Rath's osmio-sublimate mixture. In general the body is boat- or cradle-shaped, the dorsal surface being strongly convex both longitudinally and transversely and the ventral surface correspondingly strongly concave. The body tapers slightly toward the anterior end making the posterior end the broader and more bluntly rounded. In unmounted specimens the length varies from 4.2 mm. to 9 mm. and the width from 2.2 mm. to 3.2 mm. The thickness of the body is 1.04 mm. with slight variations. No cuticular spines or scales were found.

At the anterior end of the body is a well defined terminal cephalic hood or collar (Pl. III, Figs. 18, 19), 1.50 mm. to 1.65 mm. wide and 1.05 mm. to 1.17 mm. long. The dorsal margin of the hood is unbroken but the ventral margin is indented by a wide shallow notch or hilus (Fig. 17). The whole hood has the general appearance of a cocked hat; its ventral face is slightly concave, with the lappets not prominent, giving the whole somewhat the shape of a kidney. A well defined muscular oral sucker lies in the ventral cupped face of the hood, but an acetabulum is not present (Figs. 17, 18, 22).

At the posterior end of the body, on the dorsal surface, in the median line is a funnel-shaped opening, which marks the termination of the excretory system. The male and female genital pores are separate and salient; they lie on the medial side of the left intestinal caecum (Figs. 17, 22) at the level of the posterior margin of the anterior fourth of the body. The large cirrus was extruded in several of the specimens examined (Fig. 17).

Digestive System.— The oral sucker, having a fairly well developed musculature, opens directly into the œsophagus. It measured 0.66 mm. wide by 0.60 mm. long. A pharynx is not present. The length of the œsophagus varies much; in some specimens it appears to be wanting, in others it may reach a length of 0.50 mm. The wide intestinal caeca occupy a lateral and dorsal position in the body and extend from the oral sucker in an undulating course to near the end of the body, where they end blindly. The caeca throughout their course are folded or pleated, which gives rise to distinct but irregular pockets along their course.

Male genitals.— One of the most characteristic features of this trematode is the testes (Fig. 22), twelve in number, arranged in two groups of six each. They are small, irregular, lobed bodies situated in the posterior third of the worm lying on each side of the body immediately ventral to the terminal portions of the intestinal caeca. Taken together the testes have the shape of a horseshoe, with its open end directed cephalad and extending from the level of the ovary and vitelline glands caudad to the ends of the intestinal caeca. The testes may be separated from, or may overlap, one another. A small duct connects all the testes comprising each group and a larger duct, the vas efferens, passes mediad from the anterior testis of each group. The vas efferens from the left side passes transversely across the body and unites with the short vas efferens from the right side. The vasa efferentia uniting from a short vas deferens, which passes cephalad in the right mediolateral field and joins a long tubular convoluted seminal vesicle, which runs anteriorly, mediad to the right intestinal caecum, and enters the cirrus pouch (Fig. 22). The seminal vesicle is lined with a high columnar epithelium.

The cirrus pouch (Figs. 22, 20) is a very muscular elongated sac lying between the intestinal caeca at the level of the posterior margin of the anterior fourth of the body. It extends obliquely across the body from right-dorsal to left-ventral and enters the male genital pore. It is provided with an outer thick sheet of strong longitudinal muscle fibers and an inner (toward its lumen) thin sheet of circular muscle fibers. Parenchymal tissue fills the space between these muscle sheets. The seminal vesicle enters the base of the cirrus pouch, where it enlarges to form a short tubular pars prostatica, which is surrounded by the prostate cells. The pars prostatica enters a cone-shaped cavity, the ductus ejaculatorius, which is one-fourth of the length of the pouch and is lined with high columnar epithelial cells having the appearance of coarse cilia. The ductus ejaculatoris is followed by a narrow canal which forms the lumen of the cirrus (Fig. 20).

The cirrus is strongly developed and consists of two distinct regions, both of which are protrusile. The basal proximal portion is bulbous and in one specimen measured 0.33 mm. long by 0.25 mm. in diameter; the distal portion is more slender and tapering and in the same specimen measured 0.50 mm. long by 0.125 mm. in diameter. The distal portion can be retracted into the bulbous portion. The entire extruded cirrus may be 0.85 mm. long. The cirrus is covered with a cuticula in which, on the distal portion, are lightly embedded minute spinelets. It is supplied with an outer sheet of circular muscle fibers and an inner thicker sheet of longitudinal muscle fibers. Its lumen is lined with cuticula. The external opening of the cirrus pouch is separate from that of the metraterm, or vagina, and lies mesad to it and to the left intestinal caecum.

Female genitals.— The ovary (Figs. 13, 22) is a little larger than a single testis and is irregular in outline with a lobed margin. It lies at the right of the median plane, posterior to the uterus and at the level of the most anterior testes. A short oviduct leaves the dorso-medial portion of the ovary and, proceeding obliquely dorsad and to the left, is joined by the Laurer's canal, whence it turns posteriad and mediad across the dorsal surface of the shell-gland (gland of Mehlis) and is joined in the central area of the shell-gland, by a duct from the yolk reservoir. The common duct now enters the shell-gland and enlarges, forming the oötype, which receives numerous small ducts from the shell-gland, after which it passes ventrad and posteriad through the shell-gland to its posterior margin, whence, after making several coils, it turns cephalad along the left side of the shell-gland and continues as the uterus. The uterus makes its way cephalad, in the median area, in wide compact transverse folds, which may extend laterally as far as the outer edge of the intestinal caeca and vitelline glands. The uterus terminates in a well defined metraterm, or vagina, which opens to the exterior through a separate female genital pore (Figs. 17, 20, 22) at the left of the male genital pore and ventral to the left intestinal caecum. The metraterm is an elongated slightly convoluted tubular organ, approximately as long as the cirrus pouch, and lies caudad, and almost parallel, to the pouch. Its wall is strikingly thick and muscular, being provided with a thick outer layer of longitudinal muscle fibers and a thick inner (toward the lumen) layer of circular fibers. Its lumen is lined with a thick layer of cuticula, which is raised into longitudinal ridges.

A compact, irregularly shaped, shell gland, or gland of Mehlis (Figs. 13, 22), as large as the ovary, lies in the median field at the left of, and more dorsal than, the ovary. A receptaculum seminis is not present. A short Laurer's canal leaves the oviduct near the ovary and proceeds dorsad and cephalad opening on the dorsal surface at the right of the median line and slightly anterior to the shell-gland.

The vitellarium (Fig. 22) is composed of two groups of vitelline glands lying in the lateral fields of the third fourth of the body, ventral to the intestinal caeca. Each group is made up of from seven to ten compact coarsely granular glands, which are so arranged as to simulate an anterior prolongation of the free ends of the testicular horseshoe; they extend cephalad to approximately the same level on the two sides of the body. A small vitelline duct connects the successive glands of each group; a larger one leaves the posterior gland of either side and, passing caudad and mediad, unites with its mate to form a yolk reservoir (Fig. 13), which lies dorsal to the anterior portion of the shell gland. A small yolk duct leads from this reservoir and joins the oviduct in the central area of the shell gland.

Excretory System.— Two lateral canals, one on each side of the body, arise at the level of the œsophagus. They parallel the sides of the body and lie slightly external and ventral to the intestinal caeca. At the level of the anterior testes their course becomes obliquely caudomediad and they unite just posterior to the shell-gland to form the excretory bladder, which lies in the median plane at about the level of the more posterior testes and ventral to the intestinal caeca. The bladder extends backward in a straight course from the shell gland and terminates in the excretory funnel near the posterior end of the body. The funnel itself runs dorso-caudad and opens through an excretory pore on the dorsal surface in the median line 0.15 to 0.30 mm. from the posterior margin of the body. According to Looss (1902 : 593) this excretory funnel (Figs. 14–17, 22) is characteristic of the *Pronocephalidae;* it is lined by cuticula raised into 7 to 9 longitudinal ridges. Cilia were not observed in the funnel, the inner end of which is surrounded by numerous gland cells.

The uterus is packed with numberless eggs; those from different parts of the uterus were studied. In mass the eggs appear dark brown, but individual eggs are light brown or golden yellow. In shape (Fig. 21) they vary from short or long oval to ovoid, and every egg bears a tuft of filaments at each pole. The body of the egg in glycerine preparations varies from 0.0287 to 0.0387 mm. in length, and from 0.0162 to 0.0187 mm. in width. The shell is thick and has at the more pointed end a definite flattened operculum, but without an opercular rim. Six to ten coarse filaments, which may attain a length of 7 times that of the egg proper, occur at the opercular pole and there are at the opposite end 12 to 20 coarse very long filaments, 15 times as long as the egg proper, with an equal number of short finer filaments. The diameter of the coarser filaments is about one-half the thickness of the egg shell and they appear to be composed of the same material. The intertwining of these filaments causes a characteristic massing of the eggs, and makes it difficult to separate them.

2. Taxonomy.

The presence of the single sucker, the cephalic hood, and the peculiar funnel-shaped depression which is associated with the excretory pore undoubtedly place this trematode in the Family *Pronocephalidae* as characterized by Looss (1902, p. 611). It cannot, however, be placed in the genus *Charaxicephalus* of Looss on account of the difference in the number, arrangement and position of the testes and other, though minor, differences.

It has many of the characters given by Braun (1901, p. 48) for *Monostomum pandum*, which he describes as follows: "Mir liegt nur ein einziges, wohl erhaltenes Exemplar vor, das folgende Verhältnisse aufweist: es ist 11 mm. lang, kahnförmig gekrümmt, verhältnismässig platt, der Rücken gewölbt, die Bauchfläche konkav; weder der Hinterrand noch die Seitenränder sind wie bei *Mon. trigonocephalum* bauchwärts eingebogen: am Hinterrande keine Spur von irgend welchen Anhängen. Das Kopfende trägt ein nierenförmiges, dem Halskragen der Echinostomen ähnliches Schild (2 mm. breit), aus dem sich ein niedriger, an der Spitze die Mundöffnung tragender Kegel erhebt: offenbar entspricht dieses Schild dem Kopfwulst der bisher besprochenen Monostomen aus Seeschildkröten, der demnach auf der Ventralfläche nur schwach gebogen und nicht winklig ausgeschnitten ist wie bei *Mon. trigonocephalum* Rud. Die Breite des Körpers beträgt in der Höhe der Genitalpori 2.7, am Beginn der Dotterstöcke 3.5, und in der Höhe des Keimstockes 4 mm.; sie nimmt also ganz allmählich von vorn nach hinten zu.

Der Saugnapf ist 0.625 mm. lang und 0.729 mm. breit; vom Oesophagus kann ich etwas bestimmtes nicht angeben, da ich ihn nicht sehe, allem Anschein nach ist er kurz, denn die beiden Darmschenkel sind bei genügend starker Vergrösserung sowohl auf der Rücken- wie Bauchfläche dicht hinter dem Kopfschild bereits erkennbar: sie ziehen, die Endteile der Geschlechtsgänge zwischen sich fassend nach hinten und sind zwischen den Dotterstöcken und dem Uterus bis an die Hoden zu verfolgen; ihr weiterer Verlauf ist nicht mit Sicherheit zu erkennen, sie scheinen dorsal über den Hoden und der Mittellinie etwas mehr genähert bis an den Hinterrand der Hoden sich zu erstrecken. Soweit ich sie deutlich erkenne, sind sie nach aussen wie nach innen mit kurzen Blindsäckchen besetzt.

Vom Exkretionsapparat sind nur die beiden weiten Sammel-röhren aussen von den Dotterstöcken erkennbar.

Wie häufig bei *Monostomiden*, findet sich auch hier je eine Ausmündungsstelle für männliche und weibliche Organe; dieselben liegen dicht neben einander, hinter der Gabelstelle des Darms auf der linken Seite, die Uterusmündung seitlich von der Cirrusmündung. Ganz im Hinterende liegen symmetrisch die beiden grossen (bis 3 mm. langen) vielfach gelappten Hoden; sie berühren sich hinten mit ihren medianen Flächen, vorn weichen sie auseinander. Vom Leitungsweg bemerkt man rechts die gewundene Vesicula seminalis, die durch einen graden Kanal in den langgestreckten und dickwandigen Cirrusbeutel mündet; seine Lange beträgt über 2 mm.

In dem von den vorderen Enden der Hoden freigelassenen Raume liegt rechts der vierstrahlige Keimstock, neben und etwas hinter diesem in der Mittellinie die Schalendrüse. Hier beginnt der Uterus, auch fliessen an dieser Stelle die queren Dottergänge zusammen. Die Dotterstöcke liegen wie gewöhnlich seitlich im Korper und erstrecken sich vom Vorderende der Hoden bis vor die Körpermitte; sie bestehen aus zahlreichen, eine traubige Gruppierung aufweisenden Follikeln.

Die Uterusschlingen breiten sich, quere Richtung einhaltend, in dem Raum zwischen den Dotterstöcken und vor den Geschlechtsdrüsen aus; das neben dem Cirrusbeutel liegende Metraterm ist kurz vor seiner Ausmündung von einer kompakten Drüsenmasse umgeben. Die Eier scheinen Polfäden nicht zu besitzen; sie liegen allerdings so dicht im Uterus, dass sich Filamente den Blicken leicht entziehen könnten, andererseits würde aber, wenn Filamente vorhanden wären, kaum eine sehr dichte Lagerung der Eier möglich sein; Messungen an jungen, sicher der Anhänge entbehrenden Eieren aus dem Anfangsteil des Uterus ergaben 0.035 mm. Länge und 0.01 mm. Breite."

It is evident that the trematode which I have described differs from *M. pandum* not only in minor details but more especially in the larger number of testes.

Pratt (1914, p. 411) has described a monostome trematode, *Wilderia elliptica*, found in the Loggerhead Turtle from the Tortugas, which has many characters in common with both *M. pandum* and the form here described, but differs from both of them in the absence of a cephalic collar or hood. On the ground of the absence of a collar and the presence of several testes Pratt has created the new genus and new species *Wilderia elliptica*. The trematode described in this paper cannot be classed as *M. pandum*, on account of the several testes, nor as *Wilderia elliptica*, on account of the presence of a definite cephalic hood or collar.

If, as Pratt (1914, p. 416) suggests, Braun's description of the testes in *M. pandum* is incorrect and "they are as a matter of fact made up of successive pairs of distinct organs," then the trematode which I have described may be identical with *Monostomum pandum*. Also, if Pratt is in error regarding "the slightest indication of the collar-like cephalic ridge at the forward end of the body" being absent in *Wilderia elliptica*, his species is probably identical with the trematode which I have described and with *Monostomum pandum*. Until these points are determined, it seems advisable to create a new genus and a new species for this trematode, which I accordingly designate as *Synechorchis megas*, making it the type species of a new genus, *Synechorchis*,[1] designed to include those monostome trematodes which have a continuous cephalic collar, and numerous testes placed laterally in the posterior part of the body.

The material, on which these descriptions are based, was collected and sent to me by Prof. E. L. Mark, Director, and Dr. W. J. Crozier, Resident Naturalist, of the Bermuda Biological Station. To both of them I wish to acknowledge my appreciation and indebtedness.

To Mr. Hiram O. Studley, one of my students, I desire to express my appreciation for his assistance in making a preliminary study and drawings of the second form described in this paper.

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[1] Συνεχής, continuous line, and ὄρχις, testicle.

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EXPLANATION OF PLATES.

All drawings were made with the aid of a camera lucida except Figures 10, 11 and 12.

ABBREVIATIONS

act.	Acetabulum	*o'typ.*	Oötype
cae. in.	Intestinal caeca	*phx.*	Pharynx
can. exc.	Excretory canal	*po. exc.*	Excretory pore
can. L.	Laurer's canal	*po. gen.*	Genital pore
cir.	Cirrus	*po. gen.'*	Male genital pore
coll.	Collar	*po. gen.''*	Female genital pore
cstr. vt.	Yolk reservoir	*poc. v.*	Ventral cup
dt. vt.	Yolk duct	*rcp. sem.*	Recaptaculum seminis
fil. pol.	Polar filament	*sac. cir.*	Cirrus pouch
gl. cnch.	Shell gland	*suc. or.*	Oral sucker
gl. prost.	Prostate gland	*te.*	Testis
gl. vt.	Vitelline glands	*ut.*	Uterus
oa.	Ovary	*va. df.*	Vas deferens
oes.	Oesophagus	*vg.*	Vagina
o'dt.	Oviduct	*vs. exc.*	Excretory vesicle
op.	Operculum	*vsl. sem.*	Vesicula seminalis

PLATE I.

Pachypsolus brachus.

FIGURE 1. Ventral view. × 40. Scale line, at the margin, represents 1 mm.
FIGURE 2. Dorsal view of unmounted specimen. × 40. Scale line 1 mm.
FIGURE 3. Cirrus pouch, vagina, acetabulum, etc. Composite view obtained by superposing several successive para-sagittal sections. × 80.
FIGURE 4. Eggs with and without lid. × 110. Scale line 0.1 mm.
FIGURE 5. Dorsal view. × 40. Scale line 1 mm.
FIGURE 6. Reconstruction of female genitals, based on frontal and sagittal sections. × 60.

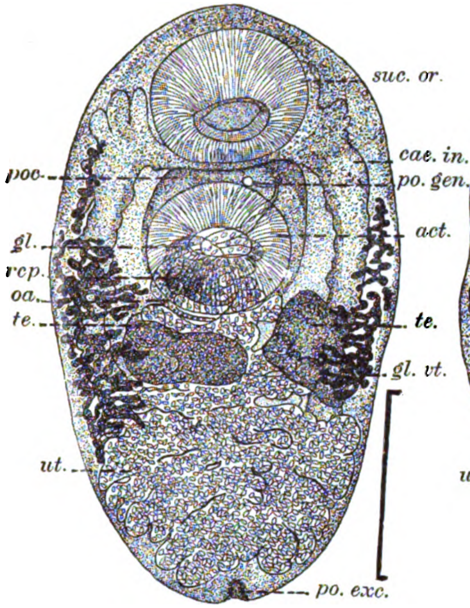


Fig. 1

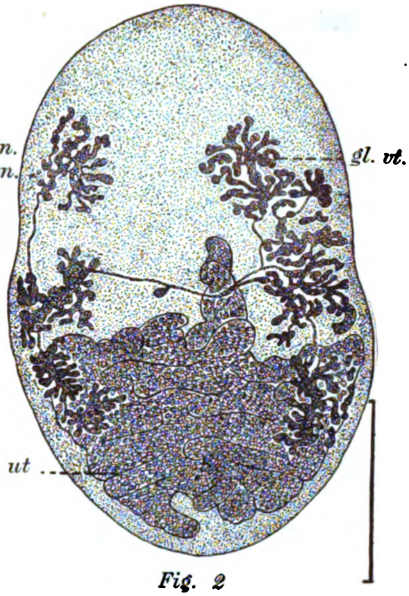


Fig. 2

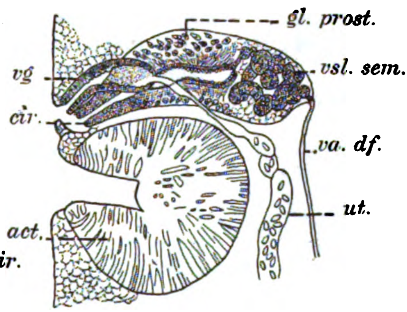
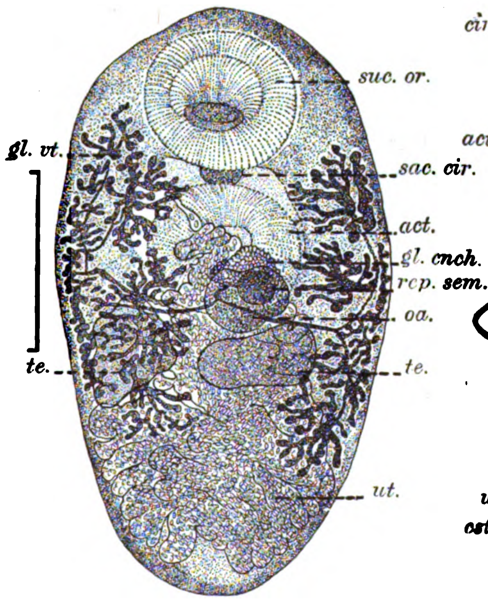


Fig. 3

Fig. 4



J.D.B. del. Fig. 5

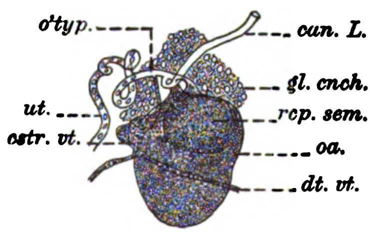


Fig. 6

PLATE II.

FIGURE 7. Reconstruction of excretory system of *Pachypsolus brachus* based on frontal sections. × 50.

FIGURE 8. Ventral view of unmounted specimen of *Pachypsolus brachus*. × 30.

FIGURE 9. Cirrus pouch etc. of *Pachypsolus tertius*, after Pratt 1914, Figure 2. × 64.

FIGURE 10. Cirrus Pouch etc. of *Pachypsolus ovalis*, after Linton 1910, Figure 7. × 60.

FIGURE 11. Cirrus pouch etc. of *Pachypsolus irroratus*, after Looss 1902, Figure 169. × 38.

FIGURE 12. Reconstruction of digestive tract of *P. brachus* based on frontal and sagittal sections. × 50.

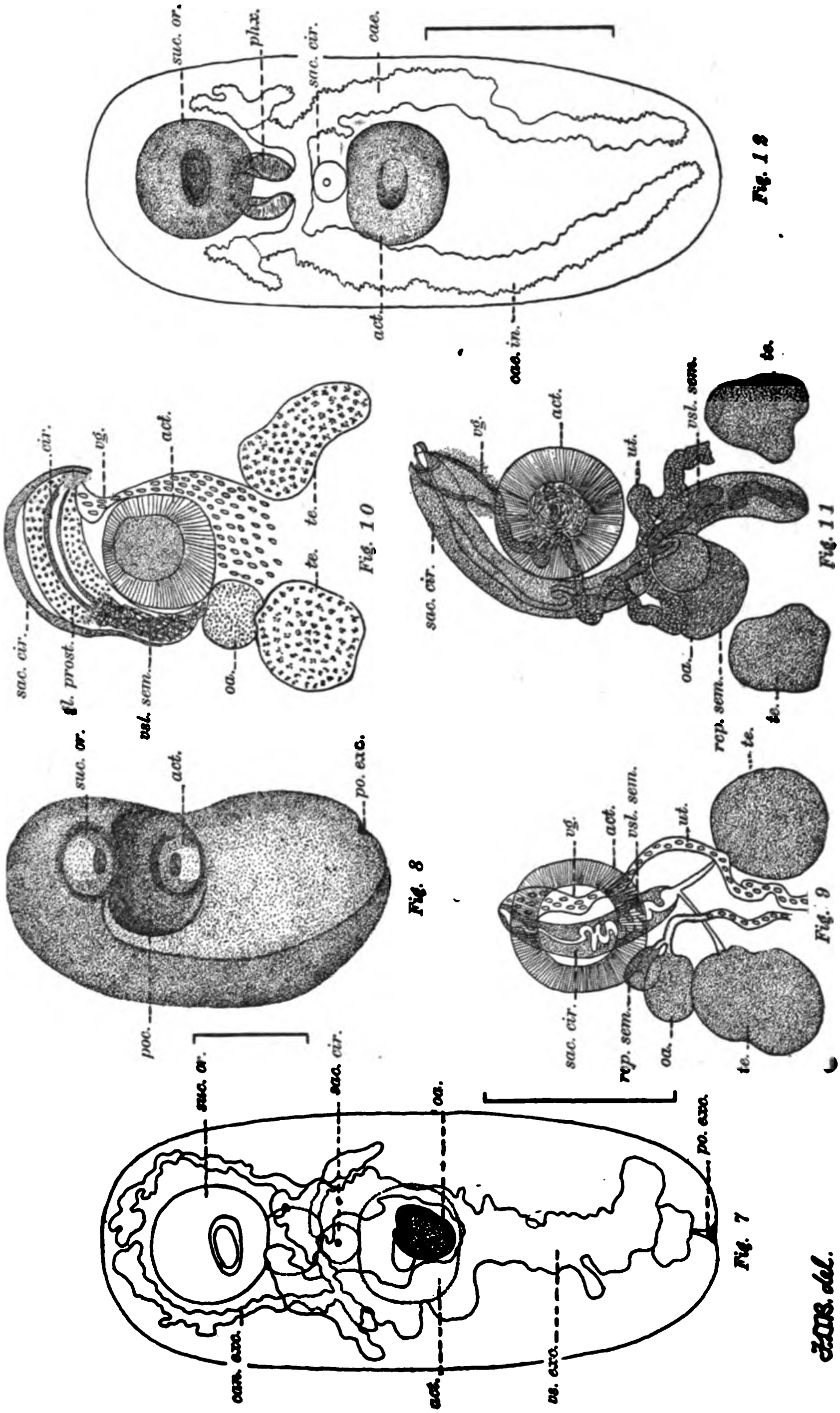

suc. or.
plx.
sac. cir.
cae.
act.
cae. in.
Fig. 12
cir.
vg.
act.
sac. cir.
gl. prost.
vsl. sem.
oa.
te.
Fig. 10
ut.
vsl. sem.
rcp. sem.
Fig. 11
po. exc.
poc.
Fig. 8
Fig. 9
sac. cir.
oa.
can. exc.
vs. exc.
Fig. 7

PLATE III.

Synechorchis megas.

FIGURE 13. Reconstruction of female genitals, dorsal aspect, based on frontal and sagittal sections. × 220. Scale line, below the figure, represents 0.1 mm.

FIGURE 14. Sagittal section of excretory funnel. × 250.

FIGURE 15. Cross section of excretory funnel near surface level. × 250.

FIGURE 16. Cross section of excretory funnel at deeper level. × 250.

FIGURE 17. Ventral view of unmounted specimen. × 23.

FIGURE 18. Ventral view of cephalic region of unmounted specimen. × 68.

FIGURE 19. Sagittal section of cephalic hood. × 75.

FIGURE 20. Details of male and female copulatory organs. × 90. Scale line 0.5 mm.

FIGURE 21. Egg. × 500.

FIGURE 22. Ventral view of mounted compressed specimen. × 33. Scale line 1 mm.

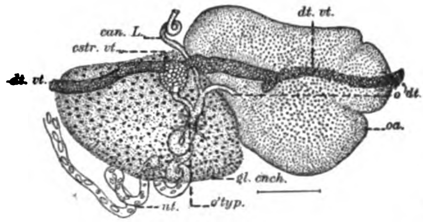


Fig. 13

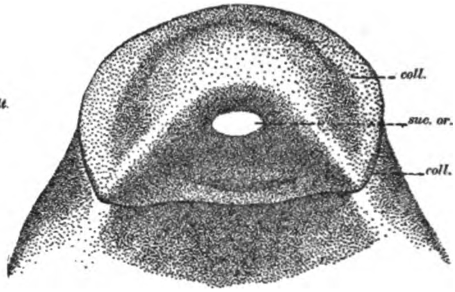


Fig. 18

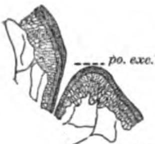


Fig. 14

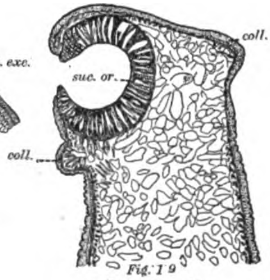


Fig. 19

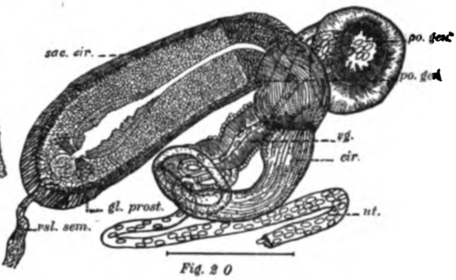


Fig. 20

Fig. 15

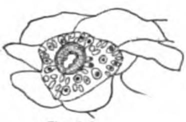


Fig. 16

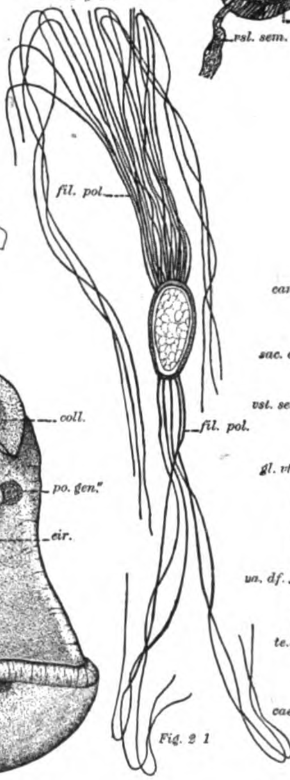


Fig. 21

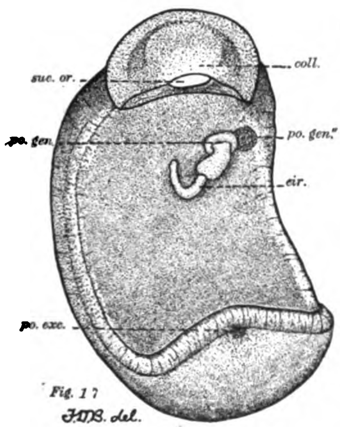


Fig. 17

J.D.B. del.

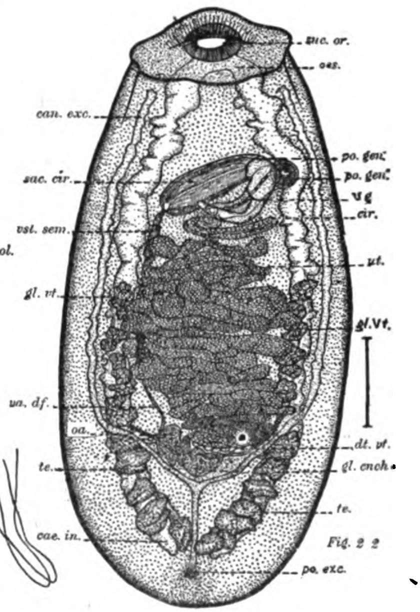


Fig. 22

(Continued from page 3 of cover.)

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CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION FOR RESEARCH. No. 136.

ADDITIONS TO THE HYDROID FAUNA OF THE BERMUDAS.

By Rudolf Bennitt.

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ADDITIONS TO THE HYDROID FAUNA OF THE BERMUDAS.

By Rudolf Bennitt.

Received February 15, 1922. Presented by E. L. Mark.

The hydroids which have been studied in the preparation of this paper are from two sources, namely, the collection made by the writer during the summer of 1921, and those made at various times since 1903 by Dr. E. L. Mark and others in attendance at the Bermuda Biological Station. I am greatly indebted to Dr. Mark, both for having made my stay at Bermuda possible and for having given me the opportunity of examining for hydroid material his miscellaneous collections.

The only papers hitherto written on the Bermuda hydroids are by Congdon (1907) and Ritchie (1909). Congdon described 19 species, of which five (*Eudendrium hargitti*, *Clytia fragilis*, *Sertularella speciosa*, *Sertularia humilis*, and *Thyroscyphus intermedius*) are new. Several others, described by him as new, have been shown by later writers, notably Nutting and Fraser, to belong to already established species. Ritchie discusses the synonymy of one of the Bermuda campanularians, and extends the range of two "Challenger" hydroids from the West Indies to the Bermudas.

Fraser's paper (1912) on the hydroids of Beaufort, N. C., is also a valuable aid in the study of the Bermuda hydroids, since 21 species of the latter, or over half of the Bermuda forms, occur also in the Beaufort region. The strong affiliation of the hydroid fauna of the Bermudas with that of the West Indian region, already suggested by Congdon, is still more strikingly demonstrated by the species now reported from Bermuda; in all, 29 species are common to the two regions.

The distribution of the individual species found in Bermuda is shown in the following table:—

	Bermuda	Beaufort, N.C.	West Indies, Florida, etc.	South America	Eastern North America, north of Beaufort	Western North America	Arctic region	Western Europe	Mediterranean Sea	West Africa	South and East Africa	Indian Ocean	Australia, N. Zealand, Oceania	Antarctic region	Bathymetrical Record (in fathoms)
Bimeria humilis........	×		×		×			×							0–10
Eudendrium hargitti....	×														0–10
Eudendrium ramosum...	×	×	×	×	×	×	×	×	×	×	×		×	×	0–542
Pennaria tiarella.......	×	×	×		×										0–10
Halecium bermudense...	×	×	×												0–10
Halecium nanum........	×	×	×						×	×					0–10
*Halecium tenellum......	×	×	×	×	×	×	×	×	×	×		×	×	×	0–235
Campanularia marginata	×		×												0–440
*Campanularia raridentata	×	×			×	×	×	×	×	×		×	×		0–250
*Clytia bicophora.......	×				×										0–10
*Clytia cylindrica........	×				×	×									0–25
Clytia fragilis..........	×		×												0–14
*Clytia johnstoni........	×	×	×		×	×	×	×	×	×		×	×		0–100
Clytia noliformis........	×	×	×		×			×				×	×		0–10
*Obelia geniculata.......	×	×	×	×	×	×	×	×			×	×	×	×	0–42
Obelia hyalina..........	×	×	×	×				×		×		×			0–68
Lafoea venusta.........	×		×									×			30–324
Hebella calcarata.......	×	×	×					×		×	×	×	×		0–122
Sertularella speciosa.....	×														0–1
*Sertularella tenella......	×		×	×	×	×	×	×			×	×	×	×	0–103
Sertularia brevicyathus..	×	×						×		×			×		0–15
*Sertularia cornicina.....	×		×		×	×	×	×					×		0–8
Sertularia aestuaria.....	×														0–10
*Sertularia stookeyi......	×	×	×												0–10
Sertularia versluysi.....	×	×	×		×					×	×		×		0–30
Thyroscyphus intermedius	×														0–10
Aglaophenia cylindrata..	×		×												21–30
*Aglaophenia lophocarpa..	×		×			×									24–1181
Aglaophenia minuta.....	×	×	×												0–10
*Antennularia pinnata....	×		×												0–100
*Lytocarpus clarkei......	×		×												13–201
Lytocarpus philippinus..	×	×	×	×					×		×	×	×		0–8
*Monotheca margaretta..	×	×	×												0–10
Plumularia diaphana....	×	×	×		×			×	×	×	×		×		0–576
*Plumularia corrugata....	×		×	×		×							×		0–130
*Plumularia inermis.....	×	×	×												0–10
*Plumularia setacea......	×		×	×		×	×	×	×	×	×	×	×		0–106

* The first record of these species from Bermuda is contained in the present paper.

This paper records 37 species, including all the hydroids reported from Bermuda up to the present. Where both trophosome and gonosome have been adequately described elsewhere in readily available papers, as is nearly always the case, I have attempted no taxonomic discussion. References are given to the text and plates of the original description, and also to the standard works dealing with the hydroids of the American shores of the Atlantic, in many of which a more complete bibliography may be found. Similar references are given for the species which I have not seen, but which have been reported from Bermuda by Congdon and others.

Family BOUGAINVILLIDAE.

Genus Bimeria.

Bimeria humilis Allman.

Allman, 1877, p. 8, pl. 5, figs. 3–4.
Congdon, 1907, p. 467, fig. 6.

Family PENNARIDAE.

Genus Pennaria.

Pennaria tiarella McCrady.

McCrady, 1857, p. 51.
Hargitt, 1900, p. 387, 4 pls.
Hargitt, 1901, p. 311, figs. 8, 9.
Nutting, 1901, p. 337, fig. 14.
Congdon, 1907, p. 464.
Fraser, 1912, p. 355, fig. 12.

Pennaria tiarella is common on the buoys and reefs about Hamilton Harbor and Great Sound, and on the flats outside. Specimens examined showed as many as 17 filiform tentacles, confirming Congdon's belief that *P. symmetrica* (Clarke, 1879) of Cuba, which has 18 filiform tentacles, is identical with *P. tiarella*.

Stoloniferous reproduction was here observed for the first time in the family, and, so far as I am aware, for the first time in the whole group of Gymnoblastea. Well-marked stolons extend from the distal ends of the stem and branches (Fig. 1); they are considerably larger than the parts from which they arise, anastomose freely, and have the same appearance, even to the exact color, as the normal hydrorhiza of *Pennaria*. The free ends of the stolons are somewhat knobbed, and along their course appear broken stumps, precisely like the base of the original stem. This colony was growing in a horizontal position on the under side of a floating buoy, and the stolons had grown along the bottom of the buoy, there to give rise to new colonies.

FIGURE 1. *Pennaria tiarella*. Colony showing stolon-formation. × 1½.

Family EUDENDRIDAE.

Genus EUDENDRIUM.

Eudendrium hargitti Congdon.

Congdon, 1907, p. 465, figs. 1–5.

Besides being extremely abundant in Hungry Bay, the shallow inlet on the south shore where it was found by Congdon, *E. hargitti* is generally distributed on buoys, timbers, ledges, and eel-grass all over Hamilton Harbor and Great Sound, just below low-tide mark. Congdon's specimens were 20–50 mm. high, and had 35–45 tentacles. Specimens in my collection from Fairyland Creek reached a height of 80 mm., and the number of tentacles varied from 35 to 60. The distal hydranths are usually larger than the proximal.

Eudendrium ramosum (Linnaeus).

Tubularia ramosa, Linnaeus, 1767, p. 1302.
Eudendrium ramosum, Hargitt, 1901, p. 309, figs. 5, 6.
Eudendrium ramosum, Nutting, 1901, p. 332, fig. 7.
Eudendrium ramosum, Congdon, 1907, p. 464.
Eudendrium ramosum, Fraser, 1912, p. 349, fig. 8.

A few colonies, 50–75 mm. high, were found on a floating buoy in Hamilton Harbor.

Family HALECIDAE.

Genus Halecium.

Halecium bermudense Congdon.

Congdon, 1907, p. 472, figs. 16–20.
Fraser, 1912, p. 367, fig. 28.
Stechow, 1914, p. 134.
Stechow, 1919, p. 33.

This is one of the most abundant species in Bermuda, growing on a great variety of structures in almost every locality where hydroids are to be found. My specimens attained a height of 75 mm., Congdon's 25–35 mm.

Halecium nanum Alder.

Halecium nanum, Alder, 1859, p. 355.
Halecium marki, Congdon, 1907, p. 474, figs. 21–23.
Halecium nanum, Fraser, 1912, p. 367, fig. 29.
Halecium nanum, Stechow, 1914, p. 135.
Halecium nanum, Stechow, 1919, p. 36.

This minute species was often found on floating Sargassum; a few colonies were also found with *H. bermudense* on Pennaria from Cow-Ground Flat. The colonies reached a maximum height of 8 mm.; Congdon's specimens were 1½–3 mm. high.

Halecium nanum appears to have two modes of growth; the resulting forms are shown to belong to the same species by the presence of the characteristic female gonosome on both. One form is short and scrubby, the other longer and with a few irregular branches coming off just below the hydrophores. The trophosome of the latter variety agrees so well with what Fraser (1912, p. 368, fig. 30) doubtfully called *H. repens* Jäderholm, that I believe the two are identical, and that he observed this straggling variety of *H. nanum*.

Halecium tenellum Hincks.

Hincks, 1861, p. 252, pl. 6, figs. 1–4.
Hincks, 1868, p. 226, pl. 45, fig. 1.
Nutting, 1901, p. 357, fig. 52.
Fraser, 1912, p. 369, fig. 31.
Stechow, 1919, p. 41.

A few colonies were found, in all stages of growth, on Sargassum at Somerset Bridge, the hydrorhiza forming an extensive network over an alga. The gonosome, essential for a satisfactory determination of the species, which Fraser failed to find in his Beaufort specimens, was present in the Bermuda material.

Family CAMPANULARIDAE.

Genus Campanularia.

Campanularia marginata (Allman).

Obelia marginata, Allman, 1877, p. 9, pl. 6, figs. 1, 2.
Campanularia insignis, Congdon, 1907, p. 469, figs. 10, 12.
Leptoscyphus insignis, Ritchie, 1909, p. 3.
Campanularia marginata, Nutting, 1915, p. 44, pl. 6, figs. 5–7.

Campanularia raridentata Alder.

Alder, 1862, p. 315, pl. 14, fig. 5.
Fraser, 1912, p. 357, fig. 14.
Nutting, 1915, p. 39, pl. 4, fig. 1.

A single small colony of two or three individuals was found on floating Sargassum. Identification is somewhat doubtful, owing to the absence of the gonosome, but the trophosome agrees in every way with Nutting's description. The ten pointed teeth, the 3–5 annulations at the ends of the pedicel, the tubular hydrotheca, and the considerable variation in the height of the pedicel, together seem sufficient to place the Bermuda specimen in this species.

Genus Clytia.

Clytia bicophora Agassiz.

Agassiz, L., 1862, p. 304, pl. 29, figs. 6–9.
Nutting, 1901, p. 343, fig. 21.
Nutting, 1915, p. 56, pl. 12, figs. 1–3.

Fraser and many other writers consider *Clytia bicophora* identical with *C. johnstoni* (Alder). Nutting, with some hesitation, regards it as a separate species, on the basis of the following points: 1) the tenuity of the hydrothecal wall; 2) the smaller size of the hydrotheca; 3) the presence of a simple instead of a complex diaphragm. My specimens of *C. bicophora*, found growing on *Pennaria* from Cow-Ground Flat, have hydrothecae which are distinctly smaller than those of *C. johnstoni*, and show many cases of the collapsed hydrothecal wall. They also have only 12 marginal teeth, and there are annulations in the middle of the pedicels, which are sometimes annulated throughout. None of my specimens of *C. johnstoni* show these features, and I have found no stages intermediate between the two; this seems sufficient to establish *C. bicophora* as a separate species.

Clytia cylindrica Agassiz.

Agassiz, L., 1862, p. 306, pl. 27, figs. 8, 9.
Nutting, 1901, p. 342.
Fraser, 1912, p. 358, fig. 16.
Nutting, 1915, p. 58, pl. 12, figs. 6, 7.

A few colonies were found on Sargassum at Agar's Island, and on floating Sargassum off the north shore.

Clytia fragilis Congdon.

Congdon, 1907, p. 471, fig. 13.
Nutting, 1915, p. 62, pl. 15, fig. 1.

A number of colonies, 10–12 mm. high, were found on Sargassum at Somerset Bridge. The gonosome was absent, but the trophosome is quite characteristic in this species.

Clytia johnstoni (Alder).

Campanularia johnstoni, Alder, 1857, p. 36.
Clytia johnstoni, Hincks, 1868, p. 143, pl. 24, fig. 1.
Clytia grayi, Nutting, 1901, p. 344, fig. 23.
Clytia johnstoni, Fraser, 1912, p. 358, fig. 17.
Clytia grayi, Stechow, 1914, p. 128, fig. 5.
Clytia johnstoni, Nutting, 1915, p. 54, pl. 11, figs. 1–6.
Clytia johnstoni, Stechow, 1919, p. 43.

This was one of the commonest species on floating Sargassum. The characteristic annulated gonangia were extremely numerous in the specimens collected. In one colony a stolon twice as long as the pedicel extended out from the middle of the pedicel, establishing connection with the substratum. This is the first case of stolon-formation that I have seen in the genus *Clytia*. There were never less than 14 marginal teeth, and I found no cases of the collapsed hydrothecal wall; these points, with the greater size of the colonies, made them readily distinguishable from *C. bicophora*.

Clytia noliformis (McCrady).

Campanularia noliformis, McCrady, 1858, p. 92.
Clytia noliformis, Nutting, 1901, p. 343, fig. 22.
Clytia simplex, Congdon, 1907, p. 472, figs. 14, 15.
Clytia noliformis, Fraser, 1912, p. 359, fig. 19.
Clytia noliformis, Nutting, 1915, p. 57, pl. 11, figs. 7–10.

Colonies of *Clytia noliformis* are very numerous on floating Sargassum. My specimens showed many intergradations between *C. simplex* as described by Congdon and *C. noliformis* as described by Nutting.

Genus Obelia.

Obelia geniculata (Linnaeus).

Sertularia geniculata, Linnaeus, 1758, p. 812.
Obelia geniculata, Nutting, 1901, p. 351, fig. 38.
Obelia geniculata, Fraser, 1912, p. 362, fig. 23.
Obelia geniculata, Nutting, 1915, p. 73, pl. 18, figs. 1–5.

A few colonies were found on floating Sargassum.

Obelia hyalina Clarke.

Obelia hyalina, Clarke, 1879, p. 241, pl. 4, fig. 21.
Obelia hyalina, Congdon, 1907, p. 468, figs. 7–9.
Obelia congdoni, Hargitt, 1909, p. 375.
Obelia hyalina, Fraser, 1912, p. 363, fig. 24.
Obelia hyalina, Nutting, 1915, p. 76, pl. 18, figs. 6, 7.

This is one of the hydroids found most often on the floating Sargassum, and a number of colonies 2–3 cm. high were found on a fish-car at Agar's Island. There were many cases of stolon-formation from the ends of the branches, and in one case these stolons were thickly intertwined with similar stolons of *Aglaophenia minuta*. *Obelia hyalina* often grows far out on colonies of *Sertularia stookeyi* no larger than itself.

Family LAFOEIDAE.

Genus Lafoea.

Lafoea venusta Allman.

Allman, 1877, p. 11, pl. 6, figs. 3–4.
Ritchie, 1909, p. 260.

Specimens of *Lafoea venusta* were dredged by the "Challenger" "off the Bermudas, 30 fathoms."

Family HEBELLIDAE.

Genus Hebella.

Hebella calcarata (A. Agassiz).

Lafoea calcarata, A. Agassiz, 1865, p. 122.
Lafoea calcarata, Hargitt, 1901, p. 387, fig. 24.
Hebella calcarata, Nutting, 1901, p. 353, fig. 56.
Lafoea calcarata, Congdon, 1907, p. 467.
Hebella calcarata, Fraser, 1912, p. 371, fig. 34.

Family SERTULARIDAE.

Genus Sertularella.

Sertularella speciosa Congdon.

Congdon, 1907, p. 476, figs. 24–28.

Sertularella tenella Alder.

Alder, 1857, p. 23.
Hartlaub, 1901, p. 63, Taf. 5, figs. 21–24, Taf. 6, figs. 2, 4, 7, 9, 10.
Nutting, 1904, p. 83, pl. 18, figs. 1, 2.

A large number of colonies, about 9 mm. high (Nutting's specimens were 12.5 mm. high), were found among branching Bryozoa in a collection made in 1903 by Dr. A. W. Weysse. The trophosome agrees in every way with that described by Nutting and by Hartlaub; occasional branches are given off at right angles to the stem, and the hydrothecal walls may be nearly smooth or may have six or seven well-marked annulations. The gonangia are one and a half to two times the length of the hydrothecae.

Genus Sertularia.

Sertularia brevicyathus (Versluys).

Desmoscyphus brevicyathus, Versluys, 1899, p. 40, figs. 9, 10.
Sertularia brevicyathus, Nutting, 1904, p. 60, pl. 6, figs. 1, 2.
Sertularia brevicyathus, Congdon, 1907, p. 481.

Numerous colonies of this little *Sertularia* were found on a gorgonian stem and on algae dredged in 1903 at four stations on the Challenger Bank, about 15 miles southwest of Bermuda, in 31-70 fathoms. Others were collected on Sargassum near Agar's Island.

Sertularia cornicina (McCrady).

Dynamena cornicina, McCrady, 1858, p. 102.
Sertularia complexa, Nutting, 1901, p. 360, fig. 57.
Sertularia cornicina, Nutting, 1901, p. 359, fig. 56.
Sertularia cornicina, Nutting, 1904, p. 58, pl. 4, figs. 1–5.
Sertularia cornicina, Fraser, 1912, p. 374, fig. 38.

Sertularia cornicina was found on the ledges and on Sargassum at both Agar's Island and Somerset Bridge, also on a gorgonian dredged in 32 fathoms on Challenger Bank. The latter specimens showed the formation of unusually long stolons from the tip of the colony back to the hydrorhiza. No sign of the often epizoic *Hebella calcarata* was seen.

Sertularia aestuaria Stechow.

Sertularia humilis, Congdon, 1907, p. 479, figs. 29–32.
Sertularia aestuaria, Stechow, 1919, p. 157.

This very common sertularian frequently formed thick mats over the ledges at about the low-tide mark in practically all the localities visited. The specific name *humilis* was used in 1879 by Armstrong (Jour. As. Soc. Bengal, vol. 48, p. 101, tab. 9) for *Desmoscyphus humilis* of the Indian Ocean, and Stechow has suggested for Congdon's *S. humilis* the name *S. aestuaria*, descriptive of its habitat at tide-level.

Sertularia stookeyi Nutting.

Nutting, 1904, p. 59, pl. 5, figs. 6, 7.
Fraser, 1912, p. 375, fig. 39.

A large number of colonies, with gonangia, were taken on floating Sargassum both off the north shore and in Hamilton Harbor. In many cases there was profuse growth of stolons from the extremities.

Sertularia versluysi Nutting.

Desmoscyphus gracilis, Allman, 1888, p. 71.
Desmoscyphus inflatus, Versluys, 1899, p. 42.
Sertularia versluysi, Nutting, 1904, p. 53, pl. 1, figs. 4–9.
Sertularia versluysi, Congdon, 1907, p. 481.
Sertularia versluysi, Fraser, 1912, p. 375, fig. 40.

Genus Thyroscyphus.

Thyroscyphus intermedius Congdon.

Congdon, 1907, p. 482, figs. 33–36.

Family PLUMULARIDAE.

Genus Aglaophenia.

Aglaophenia cylindrata Versluys.

Versluys, 1899, p. 49, figs. 19–21.
Ritchie, 1909, p. 261.

Specimens of *Aglaophenia cylindrata* were dredged by the "Challenger" "off the Bermudas, 30 fathoms." The species is very similar to *A. rhyncocarpa* Allman, being separated from it by differences in the corbulae.

Aglaophenia lophocarpa Allman.

Allman, 1877, p. 41, pl. 24, figs. 1–4.
Nutting, 1900, p. 92, pl. 18, figs. 6–8.

Several immature colonies, about 25 mm. high, were found on the stem of a large colony of *Lytocarpus clarkei*, dredged in 32 fathoms on Challenger Bank. The gonosome was absent, but the complex trophosome is sufficient for identification.

Aglaophenia minuta Fewkes.

Fewkes, 1881, p. 132.
Nutting, 1900, p. 96, pl. 31, figs. 1–3.
Congdon, 1907, p. 483.
Fraser, 1912, p. 378, fig. 43.

There is a dense growth of this little plumularian on many pieces of floating Sargassum; specimens have also been found at Agar's Island and among material dredged in 32 fathoms on Challenger Bank. The two nematophores noted by Congdon in the axil of each hydrocladium are mentioned in Nutting's description, though Congdon must in some way have overlooked this statement. No gonosome was found.

Genus Antennularia.

Antennularia pinnata Nutting.

Nutting, 1900, p. 71, pl. 5, figs. 5, 6.

Growing among encrusting Bryozoa on a floating buoy in Hamilton Harbor and reaching a height of 37 mm., were a large number of colonies of this hydroid, whose canaliculated coenosarc and unprotected gonangia place it in the genus *Antennularia*. The trophosome agrees with that described by Nutting for *A. pinnata*, except that I was able to find only one nematophore, instead of two, in the axil of each hydrocladium, and none at all on the stem, although Nutting states that they are "scattered over the stem."[1] There is also considerable disparity in size between my specimens and his, but this is not conclusive evidence of specific difference. Some of the colonies are sparsely branched, and the arrangement of the hydrocladia is invariably alternate or subalternate.

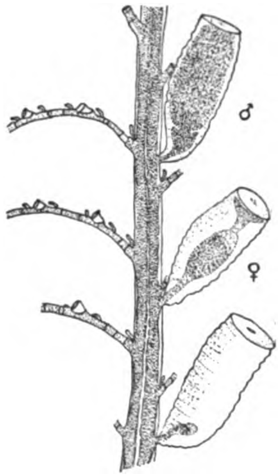

FIGURE 2. *Antennularia pinnata*. Portion of colony bearing male and female gonangia. × 12.

The gonangia (Fig. 2) are unprotected, oblong-ovate, coarsely and irregularly annulated, about 20 times as long as the hydrothecae, with strictly terminal apertures, and are borne singly on short processes from the stem opposite the hydrocladia. Both male and female gonangia are found in the same colony. The female blastostyle bears usually a single gonophore, which is situated on one side. The male blastostyle is entirely surrounded by the mass of male reproductive cells. The position of the gonangia, their annulated walls, their

1 Professor Nutting has kindly corroborated my identification of this species and of *Plumularia inermis*.

terminal apertures, and their comparatively large size, make this gonosome, previously undescribed, distinct from that of any other American species of *Antennularia*.

Genus Lytocarpus.

Lytocarpus clarkei Nutting.

Nutting, 1900, p. 124, pl. 32, figs. 5–7.

Large colonies of *Lytocarpus clarkei*, measuring from 100 to 300 mm. in length, were dredged at five stations on Challenger Bank, in 31–70 fathoms. The gonosome is absent, but the trophosome agrees completely with that described by Nutting. The color of the perisarc in the preserved specimens varied from light brown to deep chocolate-brown.

Lytocarpus philippinus (Kirchenpauer).

Aglaophenia philippina, Kirchenpauer, 1872, Pt. 1, p. 45, Taf. 1, 2, Taf. 7, fig. 26.
Lytocarpus philippinus, Nutting, 1900, p. 122, pl. 31, figs. 4–7.
Lytocarpus philippinus, Congdon, 1907, p. 484, fig. 37.
Lytocarpus philippinus, Fraser, 1912, p. 379, fig. 45.
Lytocarpus philippinus, Stechow, 1919, p. 132.

An immature colony, about 25 mm. high, was taken on Sargassum at Somerset Bridge. The gonosome was absent.

Genus Monotheca.

Monotheca margaretta Nutting.

Nutting, 1900, p. 72, pl. 11, figs. 1–3.
Fraser, 1912, p. 380, fig. 47.

Several colonies in good condition, 6–12 mm. high, were found on floating Sargassum. The gonosome is unknown; the trophosome agrees in detail with that described by Nutting.

Genus Plumularia.

Plumularia diaphana (Heller).

Anisocalyx diaphanus, Heller, 1868, p. 42, tab. 2, fig. 5.
Plumularia alternata, Nutting, 1900, p. 62, pl. 4, figs. 1, 2.

Schizotricha tenella, Nutting, 1900, p. 80, pl. 4, figs. 4, 5.
Schizotricha tenella, Nutting, 1901, p: 365, fig. 70.
Plumularia alternata, Congdon, 1907, p. 484.
Plumularia alternata, Fraser, 1912, p. 381, fig. 48.
Schizotricha tenella, Fraser, 1912, p. 383, fig. 52.
Plumularia diaphana, Bedot, 1914, p. 89, tab. 5, figs. 14–16.
Plumularia diaphana, Stechow, 1919, p. 114.

Plumularia diaphana is rather common on floating Sargassum. Branches were observed in a few cases, though the colonies are nearly always unbranched. Stechow noticed that in many colonies the proximal three or four hydrocladia were paired instead of alternate; I find this to be almost universally the case in Bermuda specimens. The gonosome is unknown.

Plumularia corrugata Nutting.

Nutting, 1900, p. 64, pl. 6, figs. 1–3.

A few colonies, 10–12 mm. high, were found on floating Sargassum. The gonosome was absent. The colonies were unbranched, and the stem showed a pair of internal ridges at both the proximal and distal end of each internode.

Plumularia inermis Nutting.

Nutting, 1900, p. 62, pl. 5, figs. 1, 2, 2a.
Fraser, 1912, p. 382, fig. 50.

This delicate hydroid covered thickly a large area of eel-grass in the shallow water of Fairyland Creek; the colonies attained a height of 18 mm. The trophosome agrees with Nutting's description, except that the intermediate internodes are much more numerous than one would infer from his reference to their "occasional appearance," and there are often one or two short intermediate internodes between the proximal hydrotheca and the stem. The hydrocladia rarely bear more than three hydrothecae, and are often prolonged into stolons.

The gonosome, heretofore unknown, was found in abundance. The gonangia (Figs. 3, 4) are 20–30 times as long as the hydrothecae, unprotected, oblong-ovate, decidedly annulated throughout, and differing from those of all other American species of *Plumularia* in springing directly from the hydrorhiza. The colonies are dioecious; the female blastostyle (Fig. 3) shows the thick, rounded "Deckenplatte" of ectodermal cells about the terminal orifice, and bears usually a single gonophore on one side; the male blastostyle (Fig. 4) is surrounded by a solid mass of sperm-producing cells.

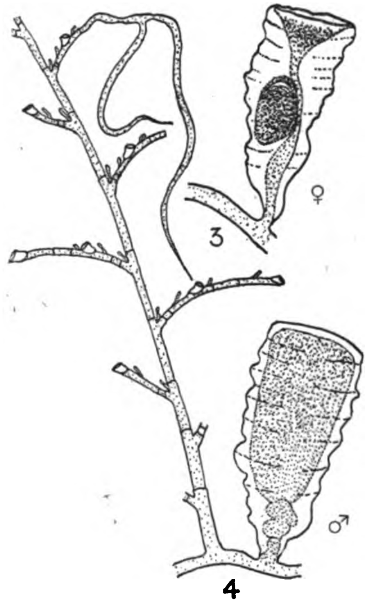

FIGURE 3. *Plumularia inermis.* Female gonangium. × 11.
FIGURE 4. *Plumularia inermis.* Male gonangium and portion of colony, showing stolon-formation. × 11.

Plumularia setacea (Ellis).

Corallina setacea, Ellis, 1755, p. 19, pl. 11.
Plumularia setacea, Nutting, 1900, p. 56, pl. 1, figs. 1–4.

Several colonies of *Plumularia setacea* were found on Sargassum; in one group of colonies stolon-formation was extensive.

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(Continued from page 3 of cover.)

VOLUME 57.

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SOME HYMENOPTEROUS PARASITES OF LIGNICOLOUS ITONIDIDÆ.

By Charles T. Brues.

CONTRIBUTION FROM THE CRYPTOGAMIC LABORATORIES OF HARVARD UNIVERSITY.

LXXXIX. A REVISION OF THE ENDOGONEAE.

By Roland Thaxter.

Received March 17, 1922. **Presented April 12, 1922.**

In preparing the present Contribution concerning the Endogoneae it has not been my intention to consider the subject in all its aspects, phylogenetic, cytological and other; and this revision has been undertaken chiefly with a view to the improvement of the systematic status of the family. For although relatively small, it has not escaped the taxonomic confusions and uncertainties which so frequently beset the path of the systematic mycologist, and it has seemed worth while to make at least an attempt to clear up some of the moot points relating to it, and at the same time to add such new information as I have been able to accumulate from personal observation or otherwise. I have therefore endeavored to obtain authentic information in regard to as many of the known forms as possible, and personally to examine as complete a representation of the type-material as could be assembled. Such value as this account possesses is therefore largely due to the courtesy of correspondents who have been so kind as to assist me in accomplishing these objects; and in this connection I desire to express my great obligation to Professor Abrams, of Leland Stanford, who has allowed me to examine all the Harkness types of Endogone in the University Herbarium: to the Abbé Bresadola, who has sent me a specimen of his *E. reniformis* collected by Rick in Brazil: to Dr. C. W. Dodge for Californian material collected by himself and by Mr. H. E. Parks: to Professor E. C. Jeffrey for a very interesting collection from Little Metis, Quebec, given to me many years ago: to Professor G. Lindau for the privilege of examining portions of all the types of Hennings and Bresadola in the Berlin Museum; to Mr. C. G. Lloyd for a portion of his *Endogone tuberculosa* and other interesting forms; to Professor O. Mattirolo who has sent me for examination specimens of all his material of Endogone, including the types of *E. Pampaloniana* and *E. Tozziana;* to M. N. Patouillard for confirming my determination of his *E. lignicola* and for portions of the types of *Ackermannia*

Dussii and *A. coccogena;* to Professor Carlos Spegazzini for communicating the types of *Endogone fuegiana* and *E. argentina;* and to Miss E. A. Wakefield for opportunity to see Berkeley's types of *Endogone australis* and *Glaziella vesiculosa.*

INDEX.

The fungi which are grouped in this assemblage of somewhat diverse forms are, in general, rather infrequently met with; owing in part to their apparent rarity, and partly to the fact that certain of the species, at least, are truly hypogaeous, and may develop at a depth of several inches below the ground, or beneath thick mats of Sp hagna or other mosses. While types of this sort are thus usually encountered by accident, or through the acquisition of what may be called a "hypogaeous instinct" which may enable one, after experience, to judge by various indications what situations are the most promising for the collection of these and other fungi hypogaei, the recognition of others is surrounded by no such difficulty, since their fructifications may be developed free to the air, on mosses, rotten wood, leaves, dung or other substances above the leaf cover, or emerging from it.

It seems not improbable that the vegetative hyphae of all the Endogoneae are at first continuous. In a majority of cases, however, the hyphae of sporulating conditions show at least occasional septa, which, in highly developed sporocarps like that of Glaziella, become very abundant. The spores developed from these hyphae are either zygospores, thick-walled acrogenous chlamydospores or thin walled spores formed endogenously in sporangia.

The true relationships of the group to other families of fungi have long been a matter of conjecture, as is evident from the terms — asci, sporangia, cysts, vesicles etc.— which have been applied by various authors to the chlamydospores alone. But although the admirable researches of Bucholtz, who first (1912) published an account of its sexual reproduction, have thrown much needed light on its affinities, the group as a whole has been assumed to include forms of considerable diversity.

The inclusion in a single genus of the zygosporic and chlamydosporic types, has hitherto been based entirely on a general resemblance in habit and habitat, and a similarity in the appearance of the two types of spore, and there has been no evidence which would indicate that the two were ever produced simultaneously in the same sporocarp, or were closely associated in their natural habitats. This assumption proves, however, to have been justified; since in a single instance, among the northern forms collected by Professor Jeffrey and herewith described, zygospores and chlamydospores are so intimately associated in the same spore mass, that there can hardly be any question as to their specific identity.

Although the zygosporic or chlamydosporic nature of these spores is usually manifest, except in very old material, it is not always easy in cases where they are surrounded by densely compacted hyphal tissue, to determine whether their origin is sexual or not, the elements involved being so compressed and distorted that conjugating processes, unless very conspicuously differentiated, might well escape notice. This is true for example in *Endogone incrassata*, Figures 17–19, or *E. tuberculosa*. There seems, however, at least in the genus Endogone, to be a rather fundamental difference between the two types. In the zygospores, which are usually surrounded by a more or less definite hyphal envelope, an outer wall is present, within which a continuous endospore is laid down, so that the contents is completely separated from the cavity of the origins. The contents is also more fatty and dense, often composed of distinct elements which may be very regular in size and shape (Fig. 9) and might even be mistaken for endospores.

The chlamydospores, on the other hand, although they may closely resemble the zygospores, do not appear, as far as I have seen, to produce a continuous endospore; unless the otherwise anomalous Glaziella in which such an endospore is clearly distinguished (Fig. 91) proves to be an exception. For this reason, in a majority of species, the protoplasm of the chlamydospore and that of the sporophore are continuous, being connected by a protoplasmic isthmus which may remain unbroken even in mature spores (Fig. 46), or may be finally pinched off in the middle by the gradual thickening of the lateral walls. In a smaller number of instances, the separation between the cavities of the spore and sporophore is accomplished at an early stage through the formation of an independent septum, Figures 52–59 and Figure 85. Spores of the latter type have, for the most part, much thinner walls than those of the former, and were regarded by Bucholtz as perhaps young sporangia. An examination of various species and copious material, however, has convinced me that they are homologous with chlamydospores of the first mentioned type.

The so-called sporangia which have been above alluded to, Figures 60–78, which have been associated only with the genus Endogone, are quite unlike the other types of reproduction; and although I have followed previous writers by including them in this genus, there is no evidence beyond a certain resemblance between the sporangiocarp in the one case and the sporocarp in the other, which would tend to confirm the correctness of this reference. These sporangia are terminal vesicles, formed in a solid mass at the extremities of branching sparingly septate filaments which radiate more or less definitely from a cushion-like base. The spores which they contain are variable in size, form and number, thin-walled, with dense or fatty contents, and result from a total cleavage of the sporangial protoplasm. They are so characteristic that it would be quite impossible to mistake them for the spore-like masses above mentioned which may occur in zygospores or sometimes even in chlamydospores.

Baccarini (1903) was of the opinion that these sporangial types should be removed from the Endogoneae and placed in the Mortierelleae; the sporangium in both cases being separated from the sporangiophore by a simple septum. As Bucholtz remarks, this disposition appears to be somewhat premature. It must be confessed, however, that if these sporangial forms are rightly included in the Endogoneae, it seems very probable that the two families should be regarded as very closely related, at least; since they are similar in two other important characters; namely, through the production of specialized zygosporic envelopes, and the presence of highly specialized acrogenous chlamydospores.

As far as I am aware, there has as yet been no successful attempt to germinate the spores of any of these fungi, or to grow them under artificial conditions; and in my own experience I have been unable, after repeated attempts, to induce the zygospores of *Endogone pisiformis* Lk. (*sphagnophila* Atk.) to germinate; or to procure any characteristic growth when uncontaminated spore-masses have been transferred to agar nutrients. The spore-masses of this species, when wintered over out of doors, have also failed to develop further. When placed on fresh sphagnum in a moist chamber for a protracted period during the summer, they usually become covered by a thin white coating of nondescript hyphae: but although various peculiar Zygomycetes, to which reference has been made in a former paper, (Thaxter (1897) p. 12) have at times been observed in such cultures, there is no reason to believe, even though, as in some instances, they seemed to grow from the masses themselves, that the association was other than an accidental one. It seems very probable that the thick-walled spores of the Endogoneae, as in various other instances, germinate as a rule only after special preparation, or under special conditions, and that in Nature they are eaten by various animals; continuing their development after being voided. This is suggested by the fact that I have myself observed uninjured spores of species of Endogone in the stomach-contents of shrews and of myriopods. Until successful cultures have been made, and the development of the three spore types has been successfully followed, or at least until more careful and extended field observations have given some evidence of their actual connection it cannot be assumed that they should all three be included within the limits of a single genus.

The literature of the Endogoneae, since the type species of the genus Endogone was described by Link in 1809, has been scattered and not very voluminous. With the exception of the paper by Bucholtz, above mentioned, and the enumerations in the Kryptogamenflora of Rabenhorst and of Cohn and the Pflanzenfamilien, there has, I think, been no general summary even of this genus. Von Höhnel in his Fragmenta, Nos. VI, X and XV, discusses the synonymy and relationships of the genera Endogone, Endogonella, Sclerocystis, Xenomyces, Ackermannia and Sphaerocreas; and numerous other references to the genus Endogone, or descriptions of new species, are to be found here and there in various other publications. In the appended list of literature, however, only such titles are included as are in some measure essential, and those which have reference merely to records of occurrence have been omitted.

Turning first to the genus Endogone, and following the conception of the genus which has been adopted by Bucholtz and all recent writers, one is forced to include in it all the three categories of spore-forms above enumerated, namely, zygospores, sporangia and chlamydospores. In order to avoid new names and combinations I have adopted this procedure as a provisional solution. It may be well to repeat, however, that although the sporangial forms arise in general from a similar vegetative body, and are associated in somewhat similar aggregations in similar habitats, their connection with the other types has not been definitely indicated, even by close association in nature, and their inclusion in the same genus is based on a pure assumption. Whether it may prove desirable to retain the name Endogone for the sexual and chlamydosporic forms and to apply a different name to those which form sporangia is not as yet clear. It may further be pointed out that the presence of isogamy and of heterogamy, of specialized spore envelopes or their absence, as well as of simple and multiple aggregations of the zygospore masses, may similarly lead to a subdivision of the sexual forms themselves, under more than one designation. The desirability or the reverse of either of these procedures will, however, doubtless become more clear as the lacunae in our knowledge of the group are gradually filled.

The reasons which have determined the selection of the sexual forms as the true representatives of the genus, as originally founded, are based on an examination of the original figures and description given by Link (1808), of the type-species, *Endogone pisiformis*, on p. 33 of the apparently rare publication in which his paper is contained. The exact wording of this description is as follows:

"38. Endogone. Sporangium subglobosum, extus floccosum, intus grumosum sporangiola minuta, globosa, membranacea, sporidiis repleta.

"Praecedenti generi affine, (Tuber), supra terram in muscis crescit hypothallo radiciformi. Membrana externa sporangii tenuis floccosa. Contextus caeterum vesiculosus, microscopio simplici inspectus grumosus, at compositi ope conspiciuntur sporangiola, ut in praecedenti genere, dispersas inter vesiculos multo minores. Sporidia minuta, globosa, sporangiolis inclusa. Unica species.

"E. pisiformis, irregulariter globosum, lutescens membrana floccosa inductum. Magnitudine pisi. Fibrillis paucis muscis adnascitur in silvis abietinis. Membrana floccosa inducta tenuissima, sporangium intus colore lutescente Tuberis, at non venosum, sed grumoso granulosum. Segmenti transversalis particulam, V. fig. 52a, sporangiola cum sporidiis ibid. lit. b."

The figure "a" referred to, shows a portion of the spore-mass covered by a radiating sterile tomentum (membrana floccosa tenuissima) of tapering filaments, evidently more or less diagrammatically represented. The spores, which are shown embedded in the general mass (sporangium), are not subspherical, but more nearly elliptical, with the exception of those which may be assumed to be viewed end on. Figure "b" shows several of these spores (sporangiola) which have been forcibly and irregularly broken, as is evident from the rent through which the contents is represented as emerging. This contents is made up of granules indicated by single black dots, the "sporidia minuta" of the description, which bear no resemblance to resting spores and could not by any stretch of the imagination be regarded as intended to represent the large thin-walled spores of the sporangial type. This description is sufficiently clear, although, like most descriptions, incomplete, and taken in connection with the figures, which are not bad for the period, afford a reasonably satisfactory basis for determination.

Since *E. pisiformis* is the generic type, it is a matter of much importance to determine with some approach to accuracy, to which of the European forms now recognized it may be assumed to correspond.

Bucholtz, who may have seen transcriptions, only, of the original paper, and may have been misled by the confusing use of the terms sporangia sporangiola and sporidia, has assumed that the classic specimen collected near Naples by Vittadini and distributed in the Fungi Europaei No. 2516 under the name *Endogone microcarpa*, was to be regarded as the true pisiformis. It seems quite impossible, however, to reconcile the characters of the Vittadini form, which is the *Endogone malleola* of Harkness, with the account given by Link whose figures alone are sufficient to preclude the possibility of such a conclusion.

The more important points brought out by Link's account indicate that he was dealing with the type of sporocarp usually found in Endogone, consisting of yellow ellipsoid thick walled spores with coarsely granular contents, associated with smaller vesicular structures, and irregularly disposed in a solid compact rounded mass surrounded by a rather conspicuous "thin floccose membrane," and developed above ground on mosses.

If one compares with this account the characters of the other known European types, none seem to correspond so closely as *E. Ludwigii* Bucholtz (*E. sphagnophila* Atk.). No other species is found, as far as I am aware, growing on mosses above the surface of the ground, while its yellow ellipsoid spores with uniform coarse granular contents, and its conspicuous thin white superficial tomentum further distinguish it. The vesicular swellings of its hyphae, which are sometimes conspicuous among the larger spores, may further correspond to the "vesiculae multo minores" of Link.

Since for the reasons above indicated the reference by Bucholtz of *E. malleola* Hark. to *E. pisiformis* Link cannot be regarded as a possible solution of the difficulty, and since it is quite necessary to form some reasonably plausible opinion as to what constitutes the Type of the genus, I have felt it desirable to follow Krieger (1902) and the earlier opinion of Bucholtz, in referring to *E. pisiformis* Link the species more recently named by the latter (1912) *Endogone Ludwigii.*

ENDOGONE Lk.

Link (1809), p. 33.

Glomus Tulasne (1845), p. 63.

Hypogaeous or epigaeous: producing thick-walled isogamous or heterogamous zygospores with or without specialized envelopes: thick walled acrogenous non sexual chlamydospores: or thin-walled sporangia. The three types, as a rule, produced separately in compact groups, which may be single or associated in a common mass, naked or surrounded by a variably developed pseudoperidium or tomentum, and may form either a definite sporocarp or an indefinite loosely coherent spore-mass.

Type Species.

Endogone pisiformis Link.

(Figs. 1–7.)

Link (1809), p. 33, Taf. II, fig. 52, a & b. Bucholtz (1902), p. 81, Tab. II, fig. 13 and V, fig. 4.

Krieger (1902), Fungi Saxonici, No. 1651.

Endogone Ludwigii Bucholtz (1911), p. 194, Taf. IX, figs. 77–87.
E. sphagnophila Atkinson (1918), p. 16.
E. xylogena Schroeter (1887), p. 260, *nec.* Saccardo (1877), p. 14, sub *Protomyces*. Thaxter (1897), p. 12.

Spore-masses waxy when fresh, horny when dry, pale to golden yellow, becoming somewhat orange yellow, subspherical to reniform, or lobed, less often convolute, flattened, umbilicate below: covered by a thin tomentum, clear white when dry, formed by characteristic, thick-walled hyphae 4–6 μ in diameter with numerous free, projecting, distally attenuated branches. The substance of the spore mass consisting of an irregular plexus of stout branching non-septate filaments, showing numerous irregular vesicular enlargements, becoming more or less obliterated as the irregularly crowded, broadly ellipsoid to ovoid, thick-walled, pale orange yellow zygospores mature. Spore-masses (dry) 2–7 × 1–2 mm. thick. Zygospores, 35–60 × 30–45 μ, the wall subhyaline 3–5.5 μ thick. Peridial hyphae × 3–8 μ.

Usually above, rarely below the leaf cover; on mosses, especially near the tip of Sphagnum; on leaves, twigs, dung, rotten logs, etc., in moist situations, especially in coniferous woods. Temperate Europe and North America.

This species is without doubt very generally distributed in temperate America; since it is already known to occur in Maine, New Hampshire, Connecticut and eastern Tennessee (Thaxter); West Virginia (Sturgis); New York and Maryland (Atkinson), and in Michigan (Kauffman). In my own experience it has proved not at all uncommon, and was first met with at Kittery Point, Maine, in 1886, when young conditions, showing the early stages of conjugation were obtained. Although it is found most frequently at or near the tips of Sphagnum, especially in moist coniferous woods, and is conspicuous in this position from its bright color, it bears no definite relation to this substratum as a host; since it occurs also, as above indicated, on various other substances. Its waxy consistency, when fresh is, as noted by Schroeter, characteristic; as is the hard almost horny character of the dry spore mass, which loses its bright color, becoming dirty yellowish; the variably developed superficial tomentum assuming a more noticeable clear white appearance, owing probably to the refractive character of the thick walled filaments which compose it. The size and form of the spore-mass varies considerably from nearly round to flattened and somewhat convolute. The largest individual seen measures 7 mm. in width when dry.

The early conditions of development are much more difficult to detect, from their small size and much paler color. The process of conjugation is not progressive in the developing mass; but occurs almost simultaneously throughout it, the rather rapid enlargement of the whole being due to the simultaneous increase in size of the individual zygospores. The gametes are subequal, and do not differ from one another more than is frequently the case in other isogamous types. They are subcylindrical and lie parallel to one another, distinguished by a clean cut septum at some distance below their adherent tips, Figure 1. The developing zygospore rises from this point of contact, above and between the extremities of the gametes, Figures 2–6. The successive stages in this process are not unlike those figured by Van Tieghem (1873), Pl. III, figs. 88–93, in *Syncephalis cornu.*

Before full maturity, the hyphal elements of the mass are conspicuous, and rather characteristic from their large size, their branching and the development of vesicular swellings which I have assumed to be the "vesiculi multo minores" mentioned by Link, and which are referred to by Bucholtz as "stellenweise verbreiterungen." As the zygospores mature, these elements become compressed between them, and may be hardly recognizable, their flattened remnants forming, in many cases, an irregular envelope about the individual spores.

The branching terminations of the filaments which form the superficial tomentum are well figured by Bucholtz (1911), fig. 77, and possess great individuality, Figure 7, but are not always conspicuous in older individuals. The prominence of this tomentum varies greatly in different individual masses, and under different conditions. It seldom seems to be so copiously developed as is represented in the figure of Link, which is evidently somewhat diagrammatic, and in older specimens may appear to form a rather even covering of apparently nearly uniform elements.

The description given by Schroeter of *Endogone xylogena* corresponds so closely to this species, that I have included it as a synonym. It seems quite improbable that the plant which he examined could have been the *Protomyces xylogenus* of Saccardo; since the latter is without hyphae, and corresponds in all respects to the sclerotium-condition, "Phylloedia," of some myxomycete: its habitat, buried in soft rotten wood and exposed only by the weathering of the latter; its yellow color, and the general appearance of its spores, being the same. The figures given by Saccardo (1877) in the Fungi Italici, fig. 104, show the somewhat irregular outline and the characteristically thickened, but ill defined, walls of this well-known condition of the myxomycete plasmodium.

With reference to the occurrence of this species in Europe, it may be mentioned that the single specimen collected by Bucholtz in Livonia was found "in einem nadelwald unterirdisch," and was associated with insect-remains, which suggests that it may have grown on the dung of some small animal, a habitat which I have myself observed. The apparently copious material collected in Thuringia by Ludwig, which forms the basis of the account given by Bucholtz, was found on the dung of Liparis caterpillars. The specimens distributed by Krieger were found "Auf Moos, faulenden Blättern, Aestchen, unter Strauchern von Vaccinium myrtilus auf dem Fichtelberge in Erzgebirge."

With regard to mutual identities in connection with this species, it should perhaps be clearly stated that while the use of the name *E. pisiformis* and the inclusion of *E. xylogena* as a synonym represent merely my personal conclusions, Professors Atkinson and Bucholtz have both examined the material on which the present account is based, and have pronounced it identical with *E. sphagnophila* in the one case, and *E. Ludwigii* in the other. It may further be mentioned that one of the specimens distributed by Krieger, has been examined by me personally, and is also identical; although a second specimen in the same copy of this set, the gross appearance of which is very similar, proves to be *Sphaerocreas pubescens*. As it is stated that the fungus was found "sehr selten," it may be assumed that the distribution is a miscellaneous one, accumulated from more than one gathering. The possible relation between *Sphaerocreas pubescens* and *Endogone pisiformis* will be further alluded to under the former species.

For convenient comparison, the description of *E. xylogena* given by Schroeter (l. c.) may be here appended.

"Endogone xylogena (Saccardo (1877): *Protomyces x.*). Fruiting bodies irregularly rounded, flattened, 3–4 mm. broad, 1–2 mm. thick, waxy when fresh, horny when dry, reddish yellow. Peridium thin, formed from 3–5 μ thick, strongly refractive hyphae, smooth. Gleba homogeneous, consisting of closely woven hyphae between which the spores are disposed. Spores spherical to elliptical or ovoid, 35–50 × 26–40 μ, the wall 6 μ thick, nearly hyaline, contents clear orange yellow.

Endogone multiplex nov. sp.

(Figs. 8–10.)

Fruiting body about 15 × 12 mm., dirty whitish, turning yellowish brown in alcohol; somewhat lobed, the surface rough from the projecting contours of the very numerous small, more or less firmly coherent, rounded or somewhat irregular spore-aggregates, of which the mass as a whole is composed, and throughout which a large amount of finely divided humus material is incorporated. Individual spore-groups more or less rounded, or somewhat irregular, mutually coherent, or readily separable, 350–700 μ in diameter, and including from ten to fifty spores each, more or less; each group surrounded by an envelope of hyphae among which a considerable amount of humus material is incorporated; the hyphae variable in diameter, 4–18 μ, thick-walled, rather brittle, freely branched, three or sometimes four branches often radiating from subtriangular or angular enlargements, especially in the larger ones, which are rather conspicuously distinguished, though scanty. Zygospores yellow, spherical, oblong to ovoid or piriform, often irregularly subangular from pressure, 80–90 $\times$ 60–84 μ; the endospore clearly defined, slightly yellowish, about 5 μ; the exospore hyaline and, when freed, swelling to 8–10 μ; the contents rather bright yellow, composed of nearly spherical fatty bodies 4–8 μ in diameter which completely fill the cavity. The attachments of the suspensors clearly defined, sometimes approximated, more often distant: the spore surrounded by a clearly defined, relatively thick, separable envelope, 8–12 μ thick, of closely felted hyphae.

Growing beneath the leaf cover beside a path in mixed deciduous woods (oak and hickory) on Cutts Island, Kittery Point, Maine: September 15, 1902.

This species is most nearly related to *E. tuberculosa*, but differs in various essential points. The individual spore-masses are, as a rule, very readily separable, so that a small fragment of the fruiting body, when teased or rubbed under the cover glass, separates to a mass of rather uniform coarse granules, which represent the individual spore-groups, Figure 10: the envelopes of which are composed largely of humus particles which often wholly conceal the spores within.

The material is unfortunately fully matured, and it is thus impossible to determine the exact nature of the process of conjugation, and even the suspensors are for the most part disorganized to such an extent that their form and limits can no longer be made out. The relation and attachment of the latter to the spore are very characteristic. They are always quite distinct, Figures 8–9, sometimes close together, but usually separated by a considerable interval; in this respect recalling the similar relation so often seen in the zygospores of Choanephora. On treatment with potash, the separable exospore and the surrounding filaments become considerably swollen and gelatinous, so that their limits are determined with difficulty.

The peculiar characters of this species illustrate the culmination of the tendency toward a definite grouping of the spores within the gleba, which is present to a less marked degree in *E. tuberculosa* and *E. fuegiana*. The sexual nature of the spore-origin is unquestionable from the two distinct origins are present in all spores. The alternative that they may be intercalary and represent a lateral bulging, so to speak, in the continuity of the hypha, is an explanation which is rendered quite improbable by our knowledge of spore-formation in all the chlamydosporic types. The conjugation is evidently somewhat peculiar, as is evidenced by the often remote origins, and it is to be regretted that, owing to the fact that the whole spore-mass is hardly distinguishable from a slightly coherent mass of earth, the younger stages are not likely to be found, unless by accident.

Endogone tuberculosa Lloyd.

(Figs. 11–16.)

Lloyd (1918), p. 799; fig. 1239.

This species has been described and its gross appearance well illustrated by Lloyd, to whom the writer is indebted for a small portion of the type material on which the following notes are based. It was collected in New South Wales by Mr. J. B. Cleland, who states that it was found just at the surface of the ground, apparently partly buried in it, if one may judge by the coating of earth which completely envelopes it. Its gross characters are peculiar from the fact that the gleba is not a continuous and undifferentiated spore-mass, but is in a sense compound.

The sporogenous area, which is only visible in sections, Figure 11, is very irregular in outline, pushing indeterminate lobes or extensions outward into the surrounding covering of earth, which thus varies greatly in thickness, and appears to be held together by a scanty penetrating mycelium. It is possible, after slightly moistening the cut surface, to determine that the golden yellow spores are arranged in rounded masses of variable size and shape, or are associated in larger somewhat less definite areas. In either case they are often, though not always, separated by intruding layers of the earthy matrix, the presence of which is indicated by its darker color, and which may be even more intimately incorporated in the general mass, although none appears to occur within the individual spore-groups.

In these spore-groups, or areas, the more clearly defined of which may be from 350–1000 μ in diameter, more or less, the bright yellow spores are closely packed and coherent, each surrounded by a thin, but as a rule clearly defined, envelope of closely matted finer hyphae. Penetrating the larger groups or areas, or separating the smaller ones, vein-like wefts of coarser filaments, forming an irregular pseudoparenchyma, may be present, Figure 12, so that the general appearance of the cut surface is not unlike that of one of the Tuberaceae.

The individual spores, Figures 12–16, are often irregular from pressure, and very variable in size and outline; subspherical or more often longer than broad, elliptical, subpiriform or often elongate, 50 × 42–150 × 90 μ, the average about 90 × 65 μ; the exospores about 5–6 μ, becoming very thick, even 15 μ; the endospore comparatively thin, about 1–2 μ. The yellow contents consists of not always dense, granular fatty protoplasm, usually associated with larger fatty masses or globules; but in certain fully mature individuals, it appears to have lost its color, becoming hyaline; while the exospore is greatly thickened, Figure 14, intruding irregularly, somewhat as in *E. incrassata*, and throwing the endospore into irregular folds.

Although, owing to the mature condition of the specimen, the spore-origins are for the most part shriveled or destroyed when freed from the tenaciously adherent spore-envelopes, a sufficient number have been isolated to satisfy me that two hyphal elements are involved in spore-production, which are associated and differentiated much as in *E. lactiflua;* although relatively smaller and less conspicuously different, one from the other, than in this species. In one instance, only, Figure 13, has it been possible to determine with some exactness the more normal appearance and relation of the two conjugating elements, although many have been observed in which the remains of corresponding structures were clearly traceable.

In the type figured by Lloyd, the surface of the specimen is considerably and irregularly roughened, pitted or lobed, the roughness having apparently suggested the specific name. This tuberculate habit does not, however, appear to be related to the presence of the characteristic spore-groups, and is merely a modification of the earthy covering.

The species is more like *E. pisiformis* in the form and color of its spores, but resembles *E. lactiflua* in its type of conjugation. In the grouping of its spores and its yellow color it recalls *E. multiplex,* which is nevertheless readily distinguished by the two discrete suspensor-insertions which characterize this species. The grouping of the spores is similar to that found in *E. fuegiana,* which, however, forms a compact continuous spore-mass, without incorporated foreign material, and in which the origin of the spores and spore-groups is quite different and apparently non-sexual.

Endogone incrassata nov. sp.

(Figs. 17–19.)

Fruiting body even or somewhat lobed, yellowish, with a whitish scaly or reticulate crust variably developed, about 2–5 mm. in diameter when dry. Gleba firm and compact, yellowish; the hyphae thin-walled and vesicular, or running in strands or bundles between the spores; the thin peridial region of more slender thick-walled filaments. Spores scattered thickly, without definite arrangement, throughout the mass of the gleba, which contains no foreign matter; more nearly isodiametric, somewhat irregular in outline, subspherical to broadly oblong, at first filled with rather uniform yellow subspherical fatty granules, about 3–5 μ, the continuous endospore clearly defined, thinner than the exospore; the two about 8 μ thick; the exospore becoming much thickened, 16–20 μ, intruded toward the center and pushing the endospore into folds, the contents losing its color and granular character. The spores 66 × 64–75 × 85 μ.

Under spruce, about two inches below the surface of the cover; with a distinct alliaceous odor. Gerrish Island, Kittery Point, Maine; August, 1896.

Three specimens of this species were found associated, and close by a single individual of *E. radiata*, of which it may possibly prove the sexual form. The gleba is so dense, and its elements surrounding the spores so vesicular, that it has been impossible to make out with certainty the character of the gametes which are evidently small, not clearly distinguished and almost obliterated by the enlargement of the spores and the consequent pressure. In a few instances, appearances have been seen such as are represented in Figures 18–19; but, in the dense pseudotissue about the spore, it is quite possible that the apparent conjugating spore-origin may be in reality due to an accidental juxtaposition of gleba elements, bearing a superficial resemblance to conjugating structures.

The spores when fully matured, Figure 19, resemble those of *E. tuberculosa*, Figure 14, although the wall of the exospore becomes relatively thicker and the endospore is thrown into deeper and more complicated folds by its intrusion. In this condition it is quite hyaline and impenetrable by stains, the contents losing its granular character entirely. The spore-envelope is thin and not clearly differentiated. The scaly or flecked appearance of the surface of the sporocarp is due to patches of loose hyphae which project from the peridium, and in section appear as flat tufts.

Endogone lactiflua Berkeley (1846).

(Fig. 20.)

Berkeley (1846), p. 81. Tulasne (1862), p. 183. Bucholtz (1912), p. 155, figs. 1–61.
Endogone lanata Harkness (1899), p. 280.

This species has become for the first time thoroughly well known through the researches of Bucholtz, who was not only the first to see and to describe the sexual origin of its spores, but to figure clearly the remarkable envelope which surrounds them at maturity, formed from labyrinthine filaments which eventually become thickened and modified to form what he has called a "flammenkrone," which is firmly adherent to the exospore. Both the envelope and the flammenkrone, however, vary, as is mentioned by Bucholtz, (1912), p. 165, in different individuals, apparently according to the age of the spore-mass, and in some of the Hesse specimens in the Farlow Herbarium neither are striking or easily recognized; while in others they are apparent at a glance. The same is true of material which the writer has collected at various times and in various localities in New England; at South Billerica, Mass.; at Kittery Point, Maine, where seven different gatherings were made; and at Intervale, New Hampshire. In all these gatherings, which were mostly of single specimens, the gross size is smaller and the spores themselves larger than in the Hesse specimens; and while in some the labyrinthine envelope-filaments (Bucholtz, fig. 50), though finer, are quite as distinct and the flammenkrone clearly distinguished, in a majority of cases these structures are not clearly visible, except that a well developed hyphal sheath is always present. Entirely similar conditions are, however, seen in some of the Hesse specimens, so that it seems probable that their distinctness may be a matter of age or some of the circumstances associated with their growth. Although in the Hesse material the spores are usually only 100 μ in diameter, while in the American they are 120–125 μ, specimens received from Hesse by Ed. Fischer are reported to be 115–125 × 70–90 μ, and in the large number of cases reported by Bucholtz, the range of variation is 68–160 × 60–104 μ. The discrepancy is thus not so great as it might at first appear; al though further examination may indicate that more than one specific form is represented in this series.

Although the occurrence of this species in America has not been hitherto recorded, it appears to have been collected several times by Harkness in California. Through the courtesy of Professor Abrams of the Leland Stanford Herbarium, I have had an opportunity to examine all the material of Endogone referred to by Harkness, (1899), in his paper on Californian Hypogaeous Fungi, including "*E. lanata*" sp. nov., "*E. microcarpa*" Tul. and "*E. macrocarpa*" Tul. The portions of these specimens communicated are similar in color and appearance, and it would be impossible to distinguish either of them by their microscopic characters from the eastern material above referred to. In all, the conjugating processes are clearly defined, and the spore-envelope well developed. In the specimen marked "*E. macrocarpa*" this is especially true, the flammenkrone, though not as striking as in the best developed Hesse specimens, being clearly present. The size of the spores in these Californian specimens is also similar, the longer axis varying from 125 μ or less to 160 μ: a range similar to that reported for the European types.

In a single specimen found at Kittery under beech trees, the gleba is dark blackish brown, the color being apparently due to the fact that a large amount of finely divided humus material is incorporated throughout its substance, a condition seen elsewhere in *E. multiplex* and a few other species. The zygospores differ somewhat in possessing a somewhat roughened, smoky brown exospore, distinctly unlike the yellowish wall of the ordinary type. It has not seemed desirable to separate this form specifically, however, on the basis of a single specimen.

For further details in regard to *E. lactiflua*, the admirable and very complete account of Bucholtz should be consulted. The possibility should be borne in mind that the very variable series of forms now included under this name may prove to represent more then one species, when they become more thoroughly known, and their life-histories have been traced. In the present state of our knowledge, however, the use of a single name to designate them seems in every way desirable.

Endogone fasciculata nov. sp.

(Figs. 21–28.)

Spore-masses spongy, loosely coherent, rather thin and irregularly lobed, somewhat amorphous, 10–14 × 4–5 mm., but very variable, incorporating more or less of the substratum (Sphagnum) and other foreign matter. Chlamydospores in rounded or somewhat elongate or irregular coherent groups, associated with less definitely distinguished masses of readily separable zygospores; pale yellowish or faintly brownish, mostly spherical or somewhat longer than broad, 60 × 60–85 × 70 μ, the wall becoming relatively very thick, 6–10 μ. Zygospores immature, irregularly spherical, colorless, about 50 μ, arising from the larger of two unequal gametes.

In Sphagnum. Little Metis, P.Q. E. C. Jeffrey.

This species is in some respects the most interesting member of the genus, since it is not only peculiar from the grouping of its spores, but presents the only instance in which zygospores and chlamydospores have been found intimately associated in the same spore-mass. It thus furnishes the first indubitable evidence that the zygosporic and chlamydosporic types have been rightly included in a single genus.

None of the zygospores examined are mature, but there is no indication that any special envelope is developed about them, as in *E. lactiflua* and some other sexual forms; although the process of formation, Figures 23–26, is very similar to that which occurs in the last mentioned species. The hyphae with which they are associated are thin-walled, scanty and evanescent; so that even in the youngest stages of conjugation, the exact origin and relation of the progametes is not clearly evident. Although this cannot be regarded as determined beyond question, examination of young stages under an immersion seems to show that the type of conjugation is homothallic, and that the progametes arise in proximity to one another from the same filament. The gametes are distinguished much as in *E. lactiflua*, one being larger than the other, and bearing the zygospore, which bulges upward; both remaining attached, with slightly thickened walls and septa. The groups of zygospores are more irregular and undifferentiated than those of the chlamydospores, among which they are irregularly distributed in continuous masses.

The chlamydospores arise from a plexus of clearly defined, thick-walled, variously bent and interlaced branching hyphae, which form a core from which short irregular sporiferous branches grow radially outward. The chlamydospores are thus at first rather firmly associated in grape-like clusters, which may be of definite rounded outline, Figure 21, or longer or more irregular. This definite relation seems to be more or less obscured in older specimens in which the hyphae tend to break up, as in other species of the genus. It should be mentioned that zygospores do not seem to be invariably associated with the chlamydosporic form. The chlamydospores themselves are rather uniform, commonly more or less spherical or but slightly longer than broad, and when fully mature possess a relatively very thick wall, surrounding a coarsely fatty contents.

The species is most nearly related to *E. vesiculifera*, which seems very clearly distinguished by the peculiar clavate empty vesicles which are associated with the chlamydospores. In the grouping of its spores it also bears some resemblance to *E. fuegiana*, which is at once distinguished by its hard continuous gleba.

Endogone vesiculifera nov. sp.

(Figs. 29–32.)

Spore-mass loose in texture and without definite form, about 5–8 × 4 mm., incorporating more or less of the substratum (Sphagnum) and some other foreign matter. Chlamydospores arising in groups, rounded or more elongate, often nor clearly defined; pale yellowish, spherical or slightly longer than broad, rather uniform, about 65 × 65 μ, the larger 80 × 70 μ: arising from fascicles of intricately woven, branching, thick walled hyphae, and borne terminally on short radiating branches; associated with broadly clavate vesicular cells, 100–125 × 50–64 μ, which extend outward beyond them.

In Sphagnum, Little Metis, P.Q. E. C. Jeffrey.

The material of this form is somewhat scanty, although sections of three different individuals are preserved. It resembles *E. fasciculata* very closely, the chlamydospores being very similar in size and shape and similarly grouped about a core of thick-walled hyphae. It is readily distinguished, however, by the presence of numerous pear-shaped or broadly clavate, nearly empty, thin-walled, sterile vesicular structures which arise in company with the chlamydospores from slender short branchlets. These bodies are very characteristic, and although their origin is the same, are by no means ordinary chlamydospores which have failed to develop. They are no doubt the homologues of spores, but cannot be directly compared with the numerous empty abortive vesicle-like spores which are conspicuous, for example in *E. canadensis*. In many cases their broad projecting terminations form a continuous margin about the spore-groups. Those of the latter which are peripheral, may be further enveloped externally by a closely woven layer of fine, thin-walled, hyphae, which may penetrate inward to some extent, between the vesicles and spore-groups, entering the spores themselves and filling them more or less completely, This parasite seems similar to that which attacks *E. lactiflua*, *E. fuegiana* and other species.

ENDOGONE FUEGIANA Spegazzini.

(Figs. 33–34.)

Spegazzini (1887a), p. 6, No. 5; (1887b), p. 120.

Through the courtesy of Professor Spegazzini I have had an opportunity to examine the type of this species collected on Staten Island, Straits of Magellan. In its present condition the type does not show all the characters mentioned in the original description which, since the publication in which it appeared is rare, should perhaps be quoted in extenso.

"Globoso vel elliptico repanda, extus alba, levis vel vix sub lente valida flocculosa, parvula (2–5 mm. diam.), inferne saepius umbilicata vel depresso-rugulosa centroque nodulosa vel subcicatricosa, uda compactiuscula tenacella; sicca dura, fere cornea: cutis carne arcte adnata persistens; caro sordide alba sub sectione fulvo-maculata, ob punctulos rufos dense congestos: puncti 7–8 cellulares, globoso subpolygoni (180 μ diam.), carne innati, nunquam confluentes: cellulae punctulorum sphaeroideae e mutuo pressione saepius ovoideae (80 × 65 μ) laeves, crasse tunicatae ad verticem precipue, inferne subapiculatae ac nodulo majusculo obscuriore donatae, fulvae vel subtestaceae. Inodora, insipida."

Found under moss on Staten and Clarence Islands, Straits of Magellan.

There has been some question as to the true position of this species, owing to the characteristic arrangement of its spores, the "cellulae" of the above description, which are more or less definitely and compactly associated in small groups of six or usually more, Figure 33, separated by variably distinct strands of compact parallel hyphae, an arrangement which gives an irregular and rather faintly areolate appearance to sections of the gleba. This has led to the suggestion that the plant might be an immature condition of some tuberaceous form. The species is, however, a well defined Endogone. The spore-groups are smaller and more clearly defined than those of *E. tuberculosa*. Spegazzini remarks that the spore-groups are never confluent; but a section from the dried material shows that they are not always distinguished with great clearness, and are at least often in close contact.

The spores, unlike those of *E. tuberculosa*, are reddish brown, considerably smaller and more nearly spherical, though usually irregular from mutual pressure. Their greatest diameter seldom exceeds 80 μ, while that of *E. tuberculosa* is often as much as 125 μ. The gleba is a dirty brownish yellow with a reddish tinge, horny when dry, the strand which separates the spore-groups, which are not always clearly marked, having a darker brownish color. The gleba, unlike that of *E. tuberculosa*, is continuous in the sense that, as far as I have seen, it contains no incorporated foreign matter.

The origin of the spore-groups is quite remarkable, and I have had some difficulty in making it out, owing to the scantiness of the material which it was essential to injure as little as possible. Their origin seems unassociated with any sexual process, and careful examination of a section shows that the spores, which are practically sessile, originate by budding in all directions from an enlarged hyphal termination. In the fully mature condition which characterizes the type, this termination is very thick-walled and irregular in outline. At points where a spore-group has been cut nearly through the middle, one may see sections of these thick-walled terminations with one or more definitely related spores in situ, as indicated in Figure 34. Each termination appears to produce as many spores as can be crowded around it, and when the group is viewed from without, it is quite impossible to see any indication of their mode of origin. Although a multiple origin of zygospores from a single conjugation is not necessarily excluded as a possibility in this instance, and might find a certain analogy among the Entomophthorales where two distinct zygospores may be produced in this manner, it may be assumed that the process in this instance is purely asexual and that it is merely a more specialized manifestation of that which occurs in *E. fasciculata*, in which, owing to the loose texture of the general mass, the spores, although arising in crowded groups, are produced in a more nearly normal fashion. This conclusion is further supported by the structure of the individual spores which lack a continuous endospore. A majority of the spores are attacked by a sterile parasite similar to that mentioned in the preceding species and shown in the spore at the right in Figure 34.

ENDOGONE MACROCARPA Tul.

Tulasne (1851), p. 182, Pl. XX, fig. 1. Bucholtz (1912), p. 184, figs. 62–74.
Nec Harkness (1889), p. 279.
Glomus macrocarpus Tul. (1845), p. 63.
Endogone australis Berk. (1860), p. 270.

Bucholtz (1912) gives an extended summary of the occurrence and spore-variation in this species, which indicates that it is perhaps the most frequently observed and variable member of the genus. The only records of its occurrence in America are that of Lloyd (1908), who reports it somewhat doubtfully from the Bahamas; and that of Harkness (1899) who speaks of finding it under Libocedrus at Towles, in the Sierra Nevada Mountains, California. Mr. Lloyd informs me that the Bahama specimen, which was doubtful, and may have been *E. fulva*, has been lost; so that this record must remain very dubious. The California form, which I have examined, proves, as above stated, to be *E. lactiflua* and is identical with what I have called by this name from the East. The spores are clearly zygospores, and the hyphal envelope is well developed, although the "flammenkrone" are not so strikingly differentiated as in some of the Hesse specimens, in the Farlow Herbarium.

In New England I first encountered what I have regarded as this species, growing on earth in greenhouse pots at the Botanic Garden in Cambridge, in company with *Hymenogaster Klotschii* and *Hydnangium carneum*, a habitat and association which has also been noticed in Europe. Of this material, one gathering made in the winter of 1891–92, has spores seldom exceeding 100 μ in greatest diameter, while a second gathering made two years later from the same pots, has spore-masses in which the larger spores measure from 170–200 μ in greatest diameter. In neither of these was any definite peridium developed, possibly owing to the fact that both grew on the surface and were subjected to constant watering.

In addition to these two gatherings, seven others have been made at Kittery Point, Maine. In these instances the fungus was found in moist coniferous and deciduous woods, usually just below the leaf cover rarely on the surface; the spore-masses usually solitary, or but two or three together. This material also shows a considerable range of variation in the size of the spores; although a majority correspond in this respect to the first gathering above mentioned. The larger spores are in general 80–100 μ in greatest diameter. This average maximum is considerably below that given by Bucholtz in his summary of the spore measurements of twenty-seven European gatherings; which includes no case in which the maximum is below 100 μ. When one considers, however, that he gives a variation of the maximum diameter in this summary between 112 μ and 230 μ, the smaller maximum of the American material does not appear significant.

The structure and character of the gleba is also subject to variation which bears no evident relation to the size of the spores. The hyphal matrix is thus quite loose in some individuals, and the spore origins correspondingly conspicuous; while in others it is as densely compacted as in *E. lactiflua*, so that clearly recognizable spore-origins, though readily made out, have to be sought for. Although Baccarini (1903) has made this difference a basis for the separation of his *E. Pampalomana* (vide infra), it hardly seems a sufficient specific distinction.

Through the kindness of Dr. Dodge, I have had an opportunity to examine three gatherings made by Mr. H. E. Parks in California: No. 348 at Saratoga, No. 312 at Aldercroft Creek, and the third at Guadalupe. All of these are unusually well developed. The largest measures 15 mm. dry: the peridium is unusually thick, yellowish white, with adherent humus material. The gleba is firm and dull yellowish in the dry material, although dark brown in the alcoholic specimens. The nearly spherical spores often reach the maximum of 230 μ mentioned by Bucholtz, and the wall, which may reach a thickness of 18 μ, is traversed by radial canals (?) which, although they are much less strikingly developed in a few other specimens examined in which the walls are unusually thick, are here very numerous and conspicuous, and appear to be associated directly with flattened masses of oily material which adhere to the inner surface, and from the middle point of which they seem to spring. In the absence of intermediate conditions, this California form would be specifically separated from the Eastern ones without question. It seems preferable, however, as in the case of *E. lactiflua*, of which they may prove to be the chlamydosporic condition, to include them under one name until we know more about them. It must be acknowledged, nevertheless, that the variations above enumerated may prove too great to justify this procedure, and it is possible that, as in the case of *E. lactiflua*, in the light of further information, more than one species may emerge from this rather too comprehensive assortment.

I am indebted to Miss Wakefield of the Kew Herbarium for an opportunity to examine a portion of the type of *E. australis* Berkeley, from Tasmania. The spores are like those of *E. macrocarpa*, the maximum diameter observed being 170 μ. In all its characteristics it comes well within the variations of the present species, and there seems to be no reason for maintaining it as a distinct form.

Endogone pampaloniana Baccarini (1903), p. 90, has been examined, through the courtesy of Professor Mattirolo, who has kindly communicated a slide of microtome sections from the type of this species. Like most sections of this nature, they are of little use for the purposes of specific determination, and it is difficult to decide from them what the distinctive characters, if such exist, really are. Baccarini based the species on the fact that the hyphae between the spores are more copiously developed and compactly woven than in the usual types of *E. macrocarpa,* in which he conceives the spores, "ampolla," to be simply gregarious, while in *E. pampaloniana* they form a "cumulo," which he regards as a transitional condition between the loose heap formed in *E. macrocarpa,* and the more definite sporocarp of *E. lactiflua.* The different origin of the spores in *E. lactiflua* would, however, destroy any significance in such a series. The spores correspond in size to those of *E. macrocarpa,* 120–140 μ, but have much thinner walls, owing perhaps to the immature condition of the specimen. As has been mentioned above, similar conditions have been found in New England, although the compact "gleba" is characterized by the usual thick-walled spores, and the same is true of Californian material. Until we have much more information concerning the variations of *E. macrocarpa* it seems desirable to regard *E. pampaloniana* as at best no more than a variety of this species.

Endogone tenebrosa nov. sp.

(Fig. 46.)

Spore-mass spongy, easily disintegrating, blackish. Hyphae loose and friable, 8–40 μ in diameter. Chlamydospores spherical or subspherical, 200–270 μ, the largest 260 × 275 μ, brownish yellow, becoming quite opaque at maturity, the reddish brown wall becoming 15–20 μ thick and finally invisible; surrounded by a thin hyaline exospore.

In Sphagnum. Little Metis, P. Q. E. C. Jeffrey.

The material of this species is so broken up in the fluid in which it is preserved that it is difficult to determine what was the original form of the irregular spongy masses. The huge spores are readily visible with the naked eye, and become absolutely opaque from the darkening of the contents, and finally of the thick endospore, which, at maturity, is invisible even with bright illumination, and is surrounded by a very thin hyaline exospore. Though sometimes slightly irregular, or slightly longer than broad, they are as a rule rather uniformly and evenly spherical. In structure and development they correspond to those of *E. macrocarpa:* but are even more closely comparable with those of the species referred to below, which was found in the stomach of a shrew.

ENDOGONE MICROCARPA Tul.

(Figs. 35–37.)

Tulasne (1851), p. 182, Plate XX, fig. 2. Bucholtz (1912), p. 192, figs. 75–76. nec Rabh. Fungi Europaei No. 2516.

Glomus microcarpus Tulasne (1845), p. 63.

This species has been recorded from America only on the authority of Harkness (1899), who collected what he regarded as this form in the forest at Mill Valley, California, No. 237. The description which he gives does not make at all clear what he had before him; but the corresponding number from the Harkness Collection, which has been kindly sent me for examination by the Stanford University Herbarium, proves to correspond to some of the forms of *E. lactiflua*, the spores being clearly zygospores.

A form, however, identical in all respects with the figures and description of Tulasne, has been kindly communicated to me by Dr. C. W. Dodge; who collected it in June, at Aldercroft Creek, Los Gatos, California. The spore-masses are well formed, though rather small, firm and similar to those of *E. macrocarpa* in form and color. The spores are nearly spherical, 40–48 μ, and very thick-walled.

Although there have been various records of this species in Europe, it does not appear, from published accounts, that it has been recognized with certainty since the original records of Tulasne, by whom it was found in Italy and France; and it seems to have been confused with smaller types of *E. macrocarpa*. Some of the latter from America serve in a measure to bridge the gap between the two species, but *E. microcarpa*, with a rather constant maximum spore diameter of 48 μ, seems clearly distinguished. The accounts of Tulasne and of Bucholtz, who reëxamined the original types, should be consulted for further information in regard to this species.

Endogone radiata nov. sp.

(Figs. 47–51.)

Fruiting body variously lobed, whitish, becoming yellowish brown in alcohol, about 10 × 5 mm., the dried specimen about 5 mm. Gleba tough, dense, nearly homogeneous, the closely coherent rather slender elements hardly distinguishable, yellowish with a fibrous appearance; the peridial layer rather thin, darker brownish, the superficial hyphae usually producing terminal and intercalary vesicular enlargements with distinguishing septa. Spores scattered, sometimes rather distant, sometimes with a slight tendency to grouping, rarely spherical, usually with the longitudinal axis considerably greater than the transverse, oblong, elliptical or subpiriform, often irregular from pressure, the long axis more or less coincident with the radius of the fruiting body, 68 × 38–85 × 50 μ, borne terminally on often clearly recognizable simple hyphae, somewhat stouter than those which compose the substance of the gleba. The spore-wall shows no visible distinction between exospore and endospore and is from 4–5 μ thick: the contents rather finely granular, pale brownish yellow.

Under the leaf cover in spruce woods; Gerrish Island, Kittery Point, Maine; Intervale, N. H.; August, 1896 and 1901: in Sphagnum, Little Metis, P. Q. E. C. Jeffrey.

This species was first taken for *E. microcarpa:* but is certainly distinct. Its spores are rarely spherical although they appear to be so when cut transversely; the wall is comparatively thin, and is not visibly double. The radiate arrangement of the spores, which are firmly embedded in a dense fibrous matrix, seems to be characteristic; but is lost as soon as the section deviates from the radial direction. In the specimens from Kittery and Little Metis, the surface of the peridium shows numerous short projecting filaments with swollen terminations, and intercalary vesicular cells of no great size. At Kittery Point this species was found in company with *E. incrassata* which was supposed, at the time, to be the same. It is thus not now possible to say whether it had the same alliaceous odor. None was noticed in the Intervale material. Among the rather numerous individuals collected by Professor Jeffrey, there are no individuals of *E. incrassata*, as far as has been ascertained. Any connection between the two is thus problematical.

Endogone arenacea nov. sp.

(Figs. 38–40.)

Spores associated in an indefinite mass through which the material of the substratum (sand) is uniformly and copiously distributed, the whole bound together in an irregular crust-like aggregation, by a loose white mycelium of occasionally septate hyphae. Spores, chlamydospores, rather uniformly spherical, thick-walled, brownish yellow, about 70 μ in diameter (65–75 μ): the walls 5.5–6.5 μ; with KOH, 8 μ.

Near margin of brook, Maraval Valley, Port of Spain, Trinidad, B. W. I., in sand under trash.

This species was found at no great distance from the gathering of *E. fulva*, hereafter mentioned, from the same locality. The spore-mass has the appearance of a bit of caked sand, about 16 × 15 mm. and about 4 mm. thick when dry. The rather scanty mycelium is visible with a lens over the surface, but it would be unlikely to attract attention, and was preserved and examined almost by accident. The mass is less characteristic and more amorphous than that of any other species, unless it be *E. multiplex*. The spores, although they show occasional variations in outline and slight differences in size, are exceptionally uniform in these respects as compared with other chlamydosporic types, and are usually quite spherical. The very thick endospore is not continuous, and no septum is present: the thin, often hardly distinguishable, exospore is usually externally roughened by adherent more or less granular disorganized material. The hyphae are much bent and tangled between the spores and sand grains, and the spores often arise from a very short branch. Their non-sexual origin is, however, unquestionable. When treated with potash a rather characteristic smoky stain appears about their insertion, Figure 40. The fatty contents is apt to develop acicular fat crystals, Figures 38, 40. The hyphae show the usual irregularities seen in other species of the genus, and are very rarely septate.

Endogone canadensis nov. sp.

(Figs. 52–55.)

Sporocarp subspherical or irregularly lobed; soft, but rather firmly coherent, with a rather well defined whitish (?) peridial layer: gleba dark brown. Spores distinguished by a septum, ovoid to ellipsoid, or somewhat asymmetrical, 70–80 × 54–58 μ very rarely 100 × 65 μ; the wall hyaline or pale yellowish, 4 μ thick. Hyphae 8–14 μ, of the usual type, with occasional clearly defined septa; the sporophores characteristically slender, 5–6 μ.

In Sphagnum, Little Metis, P. Q. E. C. Jeffrey.

The spore-mass in the material examined, which is all alcoholic, is similar to that of *E. radiata*. The gleba, however, does not consist of a firm dense matrix in which the spores are firmly held, but is formed of a loose mesh of friable mycelium, of the usual type, in which the spores are free, and are associated with numerous vesicular mostly spherical abortive spores of variable size which eventually shrivel and turn brownish.

The species is most nearly related to *E. fulva*, but is distinguished by its decidedly smaller and more regularly ovoid spores, which are borne on characteristically slender sporophores, and separated by a septum. The nearly hyaline wall of the spore is relatively distinctly thicker; the exospore thin, but rather clearly defined. The fatty coarsely granular contents is at first hyaline, becoming brownish.

Endogone borealis nov. sp.

(Figs. 44–45.)

Spore-mass irregular, coherent, spongy, dark, almost chocolate brown, about seven to eight mm. in greatest diameter. Gleba of loosely woven hyphae, 10–25 μ in diameter, among which much foreign matter and many abortive spores are incorporated. Spores reddish brown, broadly and rather symmetrically elliptical, about 125 × 100 μ, the larger 145 × 110 μ: the thick red-brown walls about 8μ: borne on rather slender hyphae and frequently subtended by a septum.

In Sphagnum, Little Metis, P. Q. E. C. Jeffrey.

This species seems clearly distinguished by the form and color of its thick-walled spores, the contents of which, in the alcoholic material examined, forms a rather finely granular more or less fibrous protoplasmic network. It does not seem nearly related to other known species unless it be *E. canadensis*, from which it is distinguished by the peculiar color and broadly and symmetrically elliptical outline of its large thick-walled spores. The endospore is not continuous when examined under brilliant illumination although the isthmus is a very narrow one and a small septum appears to be present.

ENDOGONE PULVINATA Henn.

(Figs. 41–43.)

Hennings (1897), p. 212: nec Lloyd (1918), p. 800, fig. 1240.

Dr. Lindau has very kindly allowed me to see a fragment of the type of this species, collected by Gollmer, and found growing on the ground at Caracas, Venezuela. The specimen, which is not in the best condition, resembles *E. fulva* in general appearance and color. The spores, however, although they have thin walls like *E. fulva* and are similarly separated from the hypha by a septum, are distinctly different in general appearance, being more nearly spherical, often asymmetrical, and seldom showing the considerable difference between the two diameters that is so characteristic in the last mentioned species. The larger spores are 85 × 85 μ or 75 × 85 μ, according as the axes tend to vary slightly: the average being about 75 × 75 μ or 75 × 70 μ, with considerable variation below these dimensions, and no little variation in outline. The walls are 2–4 μ thick, as in *E. fulva*, and the hyphae which, in the specimen seen, are for the most part disorganized, appear to be entirely similar and loosely woven.

ENDOGONE FULVA (Berk.) Pat.

(Figs. 56–59.)

Paurocotylis fulva Berkeley (1873), p. 137.
Endogone Moelleri Hennings (1897), p. 211.
Endogone lignicola Patouillard (1902), p. 183. Bucholtz (1912), p. 199, figs. 97–99.
Endogone fulva Patouillard (1903), p. 341; Bucholtz (1912), p. 200, figs. 97–99.
Endogone pulvinata Lloyd (1918), p. 800, fig. 1240, nec *E. pulvinata* Henn., (1897), p. 212.

Patouillard first called attention to the fact that *Paurocotylis fulva* belonged to the genus Endogone and that it was unrelated to *P. pila* Berk. which is the type of the genus. From the data and figures given by Bucholtz, who has examined the original material in both instances, the identity which he suggests between *E. fulva* and *E. lignicola* seems almost certain. The fact that they occur in widely separated regions, the one in Ceylon, the other in the West Indies, is shown to be of little significance; since other species, like *E. malleola* may have, as will be seen, an equally wide distribution.

I have collected this species in abundance in the Maraval Valley near Port of Spain, Trinidad, growing subgregariously along the Maraval brook in moist bamboo trash, fruiting within this material and running out to produce its fructifications on the surrounding sand and pebbles. A single specimen was also found under the leaf cover in the forest about the Grand Etang, Grenada; and I obtained several typical specimens growing exposed on rotten logs in Boggs' Hammock, a short distance south of Cocoanut Grove, Florida.

Dr. Lindau has been so kind as to send me a fragment of the type of *E. Moelleri*, described by Hennings from Brazil. This material is, as above indicated, identical with the Trinidad form, which has been submitted to M. Patouillard and is pronounced by him in all respects the same as his *E. lignicola*. The spores of the Brazilian form have the darker color which seems to be more characteristic of individuals which have developed in humus, without exposure to the light and air, and are, as in the Grenada gathering, sometimes almost opaque when first mounted.

Mr. Lloyd has also been so kind as to send me a portion of the Jamaica material figured by him (l. c.) as *E. pulvinata* Hennings, as well as a second specimen collected by Mr. Brace in the Bahamas. These gatherings also correspond in all respects to the Trinidad form, and must be regarded as typical *E. fulva*. I have further received from Professor Mattirolo for examination, a specimen collected by Rick in Brazil, which also has all the essential characters of the present species, although the spores are not turgescent: and from Professor Spegazzini a gathering from La Boca, Buenos Aires, doubtfully determined as *A. argentina*, which seems quite typical of this species, although not in very good condition.

The spore-masses of *E. fulva* vary from 1½ cm. to a few mm. in diameter when dry, and are usually umbilicate below, subspherical to flattened and irregularly lobed; and even in the same gathering there may be great variation in color. The peridium, which is usually well developed, although in some specimens it may be absent to a greater or less extent, exposing the naked spore-mass, is at first pure white and floccose in young fresh individuals, turning brownish with age, or when handled, the color deepening from ochraceous tawny to chestnut brown.

The hyphae are of the usual type, rather stout, 8–12 μ in diameter, more or less, often nodulose or irregular, showing occasional septa, which are more frequent than in most other species, and are in some cases quite loosely interwoven.

The spores vary considerably in color, even in the same individual; and although sometimes nearly opaque, "atro olivaceis vel atris," may, when produced free to the light and air, have a decidedly pale, yellowish color. Their outline is characteristically oblong, elliptical to oval or even subpiriform, rarely nearly circular in outline, except when viewed end on. They may be more than twice as long as broad, e.g. 125 × 55 μ, and ordinarily show a decided difference between the long and short diameter; the average variation being from 50–125 × 45–70 μ. The wall, although thin as compared with some forms of *E. macrocarpa*, for example, is thick, 2–4 μ, in contrast to the walls of the sporangial types. Bucholtz makes a separate category, a fourth subdivision of the genus, to include this somewhat thinner walled type of spore, and speaks of them as possible sporangia. Having examined a large series of specimens in all stages of development, and from widely separated localities, it seems evident that they are certainly nothing more than chlamydospores, having somewhat thinner walls than those of the more familiar species, and being distinguished by a septum. The attachment of the spore is often sublateral, as is indicated in figure 97 of Bucholtz, and the sporogenous hypha is often, though by no means invariably, somewhat narrower just below the point of attachment.

The contents of the spores may be rather dense and uniformly granular, or is often somewhat stringy in appearance apparently from the presence of fatty crystalline structures. The species is most nearly related to *E. pulvinata* and the other forms in which the spore is distinguished by a basal septum.

ENDOGONE RENIFORMIS Bres.

(Figs. 60–71.)

Bresadola (1896), p. 297.

Endogone ? argentina **Spegazzini (1899), p. 300.**

Through the kindness of Professors Lindau and Spegazzini I have been able to examine the type material of *E. reniformis* Bres. collected by Möller in Brazil and of *E. argentina* collected at Santa Catalina, Llavallol, Argentina. The Abbé Bresadola has also sent me a third specimen collected by Rick in Brazil, and I myself found apparently the same form in the antarctic forest at Punta Arenas, Magellanes, Chile.

A comparison of these four gatherings indicates that, although the spores of the Magellan specimen are distinctly larger, the other three are not separable, and correspond in all essentials. Bresadola, in his description, speaks of monosporic asci in which the spore is clearly distinguished, but was probably misled by the appearance of young sporangia in which the contents was still continuous, not having yet divided into spores.

The sporangiocarps of this species which occur on or just under the leaf cover, are subspherical to reniform, umbilicate, yellowish when dry, nearly white when fresh, 4–10 mm. in diameter, sometimes 20 mm. according to Spegazzini, and arise from a ropy mycelium which may form a more or less distinct stalk as in *E. malleola.* In the specimens examined there is no peridial layer distinguishable, the surface being composed of sporangia and slightly projecting scanty hyphal elements. The fertile hyphae are sparingly septate and branched, bearing the sporangia terminally and diverging from a cushion-like basal region associated with the umbilicus. The sporangia are more commonly spherical, but, as in *E. malleola* may show variations in length and breadth and may be asymmetrical in outline (Figs. 61–62). At maturity the sporangium wall collapses about the spores and follows their irregular contour. The average diameter is about 35–40 μ, but may reach 60 μ or over. The spores, which are evidently formed by cleavage of the whole contents in these sporangia, vary in number from four to a dozen or even more, although Spegazzini mentions eight, only, and are rather variable in size and irregular in shape from mutual pressure. In the Brazilian and Argentine material, Figures 64–71, they are 12–30 $\times$ 12–25 μ the average about 18 $\times$ 20 μ, but in the Magellan material, Figures 60–63, they are for the most part distinctly larger, 20–38 $\times$ 14–34 μ. The number present in a single sporangium varies from four to a dozen or more; although, as stated by Spegazzini, there are often not more than eight. This number is, however, by no means constant or even characteristic. On the rupture of the sporangium wall they are readily set free, although when fully mature, Figures 67–68, they appear to be held by the collapsed sporangial wall and rather firmly coherent. They are quite hyaline and contain, as a rule, one or more large oil globules or coarse dense granules.

A second Argentine collection from La Plata sent me by Spegazzini doubtfully determined as this species, proves to be *E. fulva*, as already mentioned.

Endogone malleola Harkn.

(Figs. 72–78.)

Harkness (1899), p. 280, Plate XLIV, figs. 22 a & b.

Endogone microcarpa Fischer pro parte (1897), p. 121, figs. 4–5. Rahenhorst Fungi Europei, No. 2516, nec Tulasne (1851).
Endogone pisiformis Bucholtz (1912), p. 196, figs. 88–96; nec Link (1809).
E. Torrendii Bresadola. In Torrend (1913), p. 101: (1920), p. 55. Torrend (1913), Fungi Selecti Exsiccati, No. 159.

This species seems to have been responsible for much of the confusion with which the genus has been afflicted, since, although it is fundamentally unlike the majority of the other types which have been included in Endogone, it bears certain resemblances to them which have led to a misconstruction of appearances that are frequently found in the spores of the other two sections of the genus. This misconception has led to the opinion that the chlamydospores, for example, were to be regarded as sporangia, or at least that they might become directly transformed into sporangia. This conclusion, however, seems to have no better basis than the fact that, in many cases, the contents of these spores is so modified, that they become filled with large granules or fatty bodies, often so uniform in size and form that their spore-like character has been assumed. Thus Bucholtz in his Beitrage, influenced probably by the use of the terms sporangium and sporangiolum in Link's description, has assumed that the present form may be regarded as the true *E. pisiformis*, and is thus the type of the genus. The reasons for believing that this reference can hardly be correct, have already been mentioned. In *E. malleola*, however, the large spherical or somewhat irregular bodies which form the fructifying mass are filled with numerous relatively large, separable, walled spores; quite different in appearance from any differentiation such as has been above referred to.

The references to this species which occur in the literature, are for the most part based on the material collected by Vittadini in the vicinity of Naples and distributed in the Fungi Europaei under *E. microcarpa*. Fischer (1896) assuming that the determination was correct, and that the material showed a condition of this species in which the chlamydospores had become transformed into sporangia, regarded it as a demonstration of the sporangial or hemiascoid nature of the spores of Endogone in general.

The significance of this condition has been variously discussed, and the terms ascus and sporangium variously applied to it. Its resemblance to the sporangium of the Mortierelleae was first pointed out by Baccarini (1903), who believed that it should be excluded from the Endogoneae for this reason. The researches of Bucholtz who demonstrated the sexual origin of the spores in certain species, and the necessity of their inclusion among the Mucorales, gave further support to this suggestion of Baccarini, and, assuming that the three sections herewith distinguished actually represent conditions of a single generic type, the view that the members of the family are close relatives, at least, of the Mortierelleae, is, as has been already pointed out, strongly supported by the fact that in this family alone among the Mucorales, does one find zygospores having specialized envelopes, associated with highly developed acrogenous chlamydospores; and sporangia separated from the sporangiophore by a simple septum. It should be remembered, however, that although the two may be provisionally thus associated, the apparent parallelism is not necessarily more than a coincidence.

The second record of this species is that of Harkness (1899) who first described it under the name *E. malleola* from material, collected on Mt. Tamalpais in California, which I have had the privilege of examining, and which differs in no essential from the Naples material, although the maximum diameters of the latter are often greater (Figs. 72–74).

The form was not again reported till specimens collected in Portugal were described as *E. Torrendii* Bresadola, Figures 75–76, in an enumeration by Torrend (1913) of the second century of his Fungi Selecti Exsiccati, published in Brotéria. Quite recently this description has been republished by Bresadola (1920) among his Selecta Mycologica, where, however, the fact of its distribution by Torrend is not mentioned.

Its range has been further extended by its discovery in New Zealand where material, having dimensions somewhat greater than those of the Naples gathering, has been collected by Mr. James Mitchell, and very kindly communicated to me by Mr. Lloyd (Figs. 77–78).

If one compares these different gatherings, although there is a general agreement in the form, structure and color of the fruiting masses, which are very similar to those of *E. argentina*, the average size of the sporangia and the number of spores which they contain is subject to considerable variation. Treatment with potash, slight pressure of the coverglass, and degrees of maturity, have to be considered in such a comparison; but quite apart from these, there is a marked difference observable even between individuals of the same gathering. Thus of two individuals from the Torrend distribution, one shows sporangia with an average diameter of 55–60 μ, while those of the other average from 70–75 μ or slightly over, the latter dimensions corresponding to the Californian and Naples gatherings. Although Bucholtz reports a maximum diameter of 116 μ for the latter, I have not seen any above 100 μ in the specimen examined. The New Zealand form, on the other hand, is distinctly larger, the maximum diameter being 120 μ, diameters of 100 μ being common and the average being 80–85 μ.

The form of the sporangia is normally subspherical, but may be irregular, longer than broad, or even broader than long, or subangular from mutual pressure. The wall usually appears thin, and tends to follow the contour of the contained spores; but, especially when treated with potash, may form a clear gelatinous envelope around the spores, 4–5 μ thick. The spores are somewhat variable in size, subangular from pressure, but often become spherical when free, and possess a distinct thin wall. None have been seen, even in the Torrend material, which closely approach the measurements given by Bresadola, 15–28 × 15–17 μ. Measured in the sporangium they rarely seem to exceed 14–15 μ, and usually average from 8–12 μ: although when set free and treated with potash they may reach 20 μ occasionally. They form a rather viscous mass, and when the sporangium is violently broken, are apt to escape in more or less coherent groups. The filaments, on which the sporangia are borne terminally, are branched and usually rather copiously septate, even submoniliform; the contents above the upper septum, which is often a short distance below the sporangium, being often divided into several superposed spores.

From its general characters this form could probably be cultivated with ease by anyone who was fortunate enough to find it in a fresh condition, and a thorough examination of its development in pure cultures is very much to be desired.

Doubtful or Excluded Species of Endogone.

Reference has been made above to the occurrence of spores of Endogone in the digestive tract of animals, and in this connection it may be mentioned that in one of these instances spores and mycelium were found in the stomach of a shrew, sent me by Mr. Judd from the vicinity of Washington, D.C. In this material, scanty but typical Endogone filaments bear a few very large spores, some of them 240 μ

in diameter, similar to those of *E. macrocarpa,* when young, but becoming quite opaque as they mature, owing to a blackening of the exospore. This cannot apparently be referred to any of the described species, although it is very similar to *E. tenebrosa.* The opacity of the spore, however, seems due rather to the formation of a black encrustation than to a gradual darkening of the contents such as takes place in *E. tenebrosa.*

A second type found in the digestive tract of a myriopod collected in Eastern Tennessee, appears also to belong to an undescribed Endogone. The hyphae and spores are typical of this genus, the latter brownish yellow, mostly longer than broad, the greater diameter about 38–45 μ, the walls not greatly thickened, peculiar from its slightly one-sided insertion on the sporiferous hyphae. Its size is very near that of *E. microcarpa,* but it differs in its much thinner wall, asymmetrical insertion and more elongate outline. On the other hand it differs from *E. fulva* in its smaller spores with relatively thicker walls.

A third form, which approaches more nearly to some of the variations of *E. macrocarpa,* was observed by Dr. Weston while working with water moulds in the Harvard Laboratory. It produced a rather scanty growth, consisting of a single subdichotomously branching hypha having all the characteristics of those peculiar to the genus. This grew in water about a fly, attacked by Saprolegniae, and produced abundant spores rather thin-walled, subspherical, pale brownish yellow, the larger 85–100 μ in diameter. It is quite probable that this represents a form of *E. macrocarpa,* modified by its growth under unnatural conditions.

E. Tozziana Sacc. & Cav. has been referred to Leucogaster, a disposition which is confirmed by an examination of a portion of the type.

SPHAEROCREAS Sacc. & Ell.

Type species

SPHAEROCREAS PUBESCENS Sacc. & Ellis.

(Figs. 79–82.)

Saccardo & Ellis (1882), p. 582.

Stigmatella pubescens Saccardo (1886), p. 680.
Sclerocystis pubescens von Höhnel (1910), p. 399.

This species was based on rather scanty material collected on leaves and sticks at Newfield, New Jersey, by Ellis; a portion of which has been examined in the Farlow Herbarium. For some inexplicable reason it was later associated in the fourth volume of the Sylloge, in the genus Stigmatella, with a second form, *Stigmatella aurantiaca* B. & C., a wholly different organism belonging to the Myxobacteriaceae, as I have formerly pointed out (Thaxter (1892), p. 402), where the close relationship of *S. pubescens* to Endogone is also referred to. Von Höhnel (1909), p. 127, includes in this genus his own *S. javanicum*, as well as the two species included in Ackermannia by Patouillard; although in a later paper (1910), p. 399, he transfers all of these to Sclerocystis B. & Br., reducing his own species, *S. javanicum*, to a synonym of *S. coremioides* B. & Br. Although this disposition seems correct, in so far as the others are concerned, it is certainly not justified in the case of the present species which has no characters which would indicate a near relationship to Sclerocystis.

Sphaerocreas pubescens is by no means an uncommon fungus, and is probably widely distributed. It has been repeatedly collected in moist woods and maple swamps at Kittery Point, Maine, and its vicinity, and has been also found at Intervale, N.H. by myself and at Chocorua by Dr. Farlow. It usually produces its fructification on dead leaves or twigs on the leaf cover, or on rotting branches, and in one instance was found in some quantity about old carrion. Less often it has been found on Sphagnum, like *E. pisiformis*, which it resembles very closely in general appearance; although it is usually much smaller, .2–2 mm., subgregarious, and paler in color when fresh, with a much more conspicuous external tomentum. The resemblance is so close, however, that Krieger appears to have included specimens of both in the miscellaneous gatherings which he has distributed in the Fungi Saxonici under *Endogone pisiformis*, No. 1651. In the Harvard copy at least, one of the two individuals examined proved to be *S. pubescens*, which, as far as I am informed, is the first European record of its occurrence; while the other was typical *E. pisiformis*, as I have interpreted it. It is at once distinguishable, however, from the small size of the subspherical, or broadly elliptical spores, Figures 81–82, which are 18×15–$25 \times 22\ \mu$, rarely larger, although Saccardo mentions a maximum of 30 μ. The spore-walls are relatively thick, 1.5–2.5 μ, and distinctly yellowish, while the contents appears to be hyaline or nearly so; usually with one or more large oil globules, associated with a variable number of small granules, but seldom uniformly and densely granular. They are produced acrogenously on very fine branching thick walled, refractive hyphae, 2 μ or less in diameter, with few if any visible septa, and are distributed indiscriminately throughout the compact mass, which becomes very hard on drying. On the surface of the spore-mass, which is subspherical, slightly umbilicate, and rather firmly attached to the substratum which it may partly envelope (Fig. 79), the hyphae gather in rather regularly disposed, compact, attenuated, radiate, discrete bundles, Figure 80, 60–100 μ in length and 12–30 μ at the base, while the apex may be less than 2 μ. The hyphal elements which form these bundles are so firmly and closely coherent that the individual filaments which compose them can only be distinguished with a high magnification or by crushing them apart. They form the conspicuous superficial tomentum which, although it may be less evident in older specimens, is very striking in younger individuals (Fig. 79).

Although the peculiarities of the hyphae and spores above referred to, might not be regarded as very convincing evidence for the separation of this species from Endogone, I have preferred to keep it distinct provisionally. The indiscriminate distribution of its spores in a solid mass would certainly forbid its inclusion in Sclerocystis, as represented by *S. coremioides* and *S. Dussii*, in both of which they are disposed "en une seule zone radiale," about a central columella.

The rather close resemblance between the general appearance of this species and that of *Endogone pisiformis*, above alluded to, might suggest that they were possibly stages in the cycle of a single fungus; *S. pubescens* representing the chlamydosporic condition of the zygosporic type. The character of the hyphae appears to be so different in the two cases, however, that such a connection seems very improbable, and although their habitats are similar, I have never seen them closely associated in Nature.

SCLEROCYSTIS B. & Br.

Berkeley & Broome (1873), p. 137.

Xenomyces, **Cesati (1879), p. 26.**
Ackermannia, **Patouillard, (1902), p. 180.**

Type Species

Sclerocystis coremioides B. & Br.

Berkeley & Broome (1873), p. 137, Plate X, fig. 56.

Xenomyces ochraceus **Cesati (1879), p. 26.**
Sphaerocreas Javanicum **von Höhnel (1908), p. 30, fig. 1.**

Von Höhnel, who examined the original types, vouches for the identity of his *Sphaerocreas Javanicum* and of *Xenomyces ochraceus* Ces., with *Sclerocystis coremioides* B. & Br., and an examination of the type material of Ackermannia which M. Patouillard has very kindly communicated seems to bear out his conclusion that this genus must also be regarded as a synonym. It appears to belong in the Endogoneae, the characters of its spores and hyphae being in general similar to those of Endogone. The three species seem to conform to a well marked generic type, and are distinguished from Endogone in producing numerous small sclerotium-like sporocarps in which a well defined sporogenous layer is very characteristic, the large spores lying side by side with their long axes directed radially from a central columella-like region.

Von Höhnel states that the sporocarps in his material, which was found in the Buitenzorg garden growing on bits of wood and sticks, were "zu einer festen porosen Maase verwachsen." They are represented as short-stalked, 500–600 μ broad, hard, dull yellow to grey-green, sometimes becoming superposed through distal proliferation. The closely woven hyphae are said to be septate, those of the short stalk 8 μ in diameter, those which occupy the central portion of the sporocarp about 4 μ, as are those which form a well defined outer layer; their free terminations showing no indication of special or characteristic modification. The spores as represented are placed side by side, as already described, immediately beneath this outer layer, and are long oval to long elliptical or even clavate, 60–90 $\times$ 20–50 μ. Their walls appear to be thin, and the contents is either finely granular or wholly lacking.

The type material of Berkeley and Broome in the Kew Herbarium was found in Ceylon and was regarded by Petch (1908) p. 116, as a sclerotium. Later, however, (1912) p. 282, he confirms the statement of von Höhnel as to its identity. The type of Cesati, now in the Herbarium of the Royal Gardens at Rome, was collected by Beccari in Borneo.

Sclerocystis Dussii (Pat.) von H.

(Figs. 83–85.)

von Höhnel (1910), p. 390.

Ackermannia Dussii Patouillard (1902), p. 181, figs. a–g.
Sphaerocreas Dussii von Höhnel (1909), p. 401.

This species, which was collected by Ackermann in Martinique and by Duss near Basse-Terre, corresponds very closely in general characters to the preceding species. It is said to form a superficial, golden yellow, more or less reddish "stroma," forming a crust or cushion, and covered with yellow giant cells 260–400 × 50–100 μ. The individual stromata are rounded or ovoid, solitary and scattered, or contiguous and confluent; each containing a "perithecium" (sporocarp), or two to three superposed, about one third of a millimeter in diameter, completely surrounded (entourés) by the stroma. The spores, (thèques) are rounded ovoid, brownish yellow, 70–130 × 35–100 μ, the walls thick; the sporogenous filaments 12–16 μ in diameter: they form a single radial zone lying side by side in a single layer with the long axis radially directed. The sporocarps ("perithecia") are hard and sclerotium-like, whitish, formed from interwoven hyphae which are thick-walled, colorless and 4–5 μ in diameter, surrounding the sporogenous layer, those on the surface terminating in characteristically modified broadly fusiform swollen extremities, or by a series of two or three such enlargements.

M. Patouillard has been so obliging as to send me two fragments of the type, one of which is comparatively young. From an examination of these specimens it would appear that the fungus arises in yellowish patches which may become variously confluent. This mycelial layer, crust or stroma, is made up of two distinct elements: relatively slender and thick-walled, branching, interlaced, aseptate hyphae extending radially and giving rise to the sporocarps: and large septate hyphae, the swollen segments of which are thin-walled and usually closely coherent, forming the covering of giant cells mentioned by Patouillard, as well as a rather scantily developed lysigenous matrix about the sporocarps, which appear to become completely free through its ultimate disappearance. On the surface of the crust or stroma, these cells, which appear very irregular in outline, though radially elongate, form a yellow pseudotissue, Figures 83–84; the whole at first continuous, but later, and on drying, becoming cracked and broken into irregular areas, with uneven elevations of the surface which correspond in a general way to the sporocarps lying immediately below them. Beneath this crust, the sporocarps are crowded, at maturity, in a loose dry mass, and may be supposed to be scattered separately when freed by its disintegration.

The younger specimen examined shows the sporocarp-origins still in process of formation immediately beneath the surface of the crust, as well as the more or less clearly defined relation of superposition which the older sporocarps continue for a time to bear to one another. Their temporary coherence is due to connecting wefts of smaller, relatively thick-walled, slender, intricately interwoven aseptate hyphae above mentioned, which form the second element of the mycelial crust, or stroma. How these wefts at first originate, it is not possible to determine from the material; but as they develop, a distal portion is distinguished from a basal, which forms a very short stout stalk, usually broader than long, but variably developed in different cases; and the broadening distal portion organizes the sporocarp proper. In this process the latter becomes differentiated into a central region, or columella, continuous with the stalk, and surrounded by a sporogenous layer from which the long oval to somewhat wedge-shaped or subclavate thick-walled brownish yellow spores diverge radially; lying side by side in a single layer which completely surrounds the columella, except in the basal region, where it remains continuous with the stalk (Fig. 83). A section of the sporocarp in this condition suggests a mushroom "button" in general outline.

The sporogenous layer, which occupies the surface of the columella, is thin and somewhat more dense. The spores, which radiate from it with great regularity, are rather uniform, with somewhat flattened extremities, and are attached by a pointed base, which is usually distinguished by a small well defined septum from its origin in the sporogenous layer (Fig. 85). Owing to the confused structure of the branching, densely woven elements which form the sporogenous layer, it has not been possible to determine the exact nature of these origins. Even in thin sections, I have been unable to determine satisfactorily such a simple and continuous relation, between the spores and the hyphae of the columella, as is shown by the figure of Patouillard in this species; or in that of *S. coremioides* drawn by Weese, in von Höhnel's Fragmenta, No. 174, fig. 1; which appears to be somewhat diagrammatic. Although it is not impossible that some form of conjugation may occur in this layer, which is hidden by the confusion of cut ends, loops and convoluted branches, I have seen nothing in the dried material that could be thus interpreted. The thin-walled spores with their basal septum suggest a resemblance to the corresponding type in Endogone illustrated by *E. fulva*. It may be said in this connection that the extraordinary definiteness of the spore-relations is not such as one would expect in a chlamydosporic type; but might possibly be interpreted as a higher development of the tendency to segregate somewhat specialized spore-groups, which has been above described in certain sexual species of Endogone, like *E. multiplex* or *E. tuberculosa*, and in the chlamydospores of *E. vesiculifera* and *E. fasciculata*.

The intersporal spaces are traversed by hyphae which grow radially from the columella and form, outside the spores, a layer about 40 μ thick of intricately woven filaments, which are also continuous externally with the similar hyphae of the stalk. After a given sporocarp has developed, this outer layer proceeds to organize one or more new wefts of hyphae, which may be terminal or sublateral or both, and these wefts in turn organize new sporocarps in a manner similar to that which has been just described (Fig. 84). The successively formed sporocarps may thus be arranged in series corresponding to the successive origins of these wefts; either in simple rows, or in a more complicated fashion, owing to divergence from the long or radial axis.

The stalk-portion which is often but slightly developed, shrivels and separates from its point of origin, as the maturing sporocarp becomes hard and sclerotic, and persists as an inconspicuous tuft of shriveled filaments; the lysigenous elements above referred to having also disappeared and freed the sporocarps from their lateral attachments.

After examining a number of sporocarps with some care, I have been unable to detect either in young or in more mature conditions the peculiar subfusiform superficial hyphal terminations figured and described by Patouillard, as above indicated. When fully mature and freed from its attachments, the hard sclerotic sporocarp is externally furfuraceous from the numerous broken projecting ends of the filaments which cover it. Were the peculiarly swollen terminations seen by Patouillard normally recognizable, their presence should readily separate this species from *S. coremioides*, but since it seems certain that they are not always present, it is possible that the two species are not specifically distinct, even though the presence of the large celled evanescent pseudotissue is not mentioned in descriptions of the East Indian form. The spore-measurements, as given by von Höhnel, are somewhat smaller, 60–90 × 20–50 μ; the average dimensions in the West Indian form, as I have determined them, being 95–115 × 20–50 μ; the extremes 60–120 × 34–55 μ, the shorter diameter being that of the distal end of the spore. The sporocarp measurements, on the other hand, as given for *S. coremioides*, are 500–600 μ, while those of *S. Dussii* which I have measured are 350–450 × 300–460 μ. Although the spores of the East Indian form are thus somewhat smaller and the sporocarps somewhat larger than in the West Indian species, the discrepancy does not appear to be very great, as compared with other instances of variation in the group.

Sclerocystis coccogena (Pat.) von H.

(Figs. 86–87.)

von Höhnel (1910), p. 390.

Ackermannia coccogena Patouillard (1902), p. 183, figs. h–j.
Sphaerocreas coccogena von Höhnel (1909), p. 401.

I am indebted to M. Patouillard for a very small fragment of this species, which was collected by Ackermann in company with *Endogone lignicola* Pat., growing on rotten wood, the locality being the Plateau des Rivières, Martinique, and the date of collection June, 1901. This fragment shows a small portion of the crust or covering which surrounds the sporocarps, and three or four of the latter, which are fully mature, but still attached to it.

Judging from the original figures and description, the "stroma" of this species is more definitely circumscribed and thicker (6–8 mm. by 4 mm. thick) than that of *S. Dussii*, resembling closely the spore-mass of some Endogone, and covered by a more even and clearly differentiated pseudoperidial layer. The small fragment of this layer which I have examined, is composed of rather loose coarse septate hyphae, the segments of which are irregular and do not form a coherent pseudotissue of giant cells as in the preceding species.

The sporocarps are similar in form and appearance to those of *S. Dussii*, but somewhat larger, 400–675 μ in greatest diameter. The spores, which are similar in their origin and arrangement, are somewhat smaller, their average dimensions being about 100 $\times$ 40–50 μ. The sporocarps themselves are at once distinguished, however, by the presence of numerous small spherical chlamydospores, Figure 87, similar to those of Endogone, which are formed terminally in the tomentum which covers the intricately woven superfical layer (Fig. 86).

In its thicker and more clearly circumscribed stroma, and more definite pseudoperidium, in its larger sporocarps and the production on their surface of spherical chlamydospores, and in the apparent absence of a lysigenous pseudotissue of large thin-walled hyphal segments, this species seems to be clearly distinguished.

GLAZIELLA Berk.

Berkeley (1879–80), p. 31, No. 8526.

Endogonella von Höhnel (1913), p. 294.

Type Species

GLAZIELLA VESICULOSA Berk.

(Figs. 88–94.)

Berkeley (1879–80), p. 31.

Xylaria aurantiaca Berkeley & Curtis (1868), p. 382, d.
Glaziella aurantiaca Cooke (1883), p. 83: Lloyd (1919), p. 30, fig. 1460.
Hypomyces alboluteus Ellis & Everhart (1893), p. 262 and 285.
Endogonella borneensis von Höhnel (1913), p. 41, figs. 4–5.

Owing to its large size and conspicuous orange yellow color, this species has been repeatedly observed in the American tropics, but the early misinterpretation of its characters has led to much confusion as to its probable position. It was first collected in Brazil by Glaziou, No. 8526, this type, of which through the courtesy of Miss Wakefield I have been able to examine a portion, being still in the Kew Herbarium. Later, material collected by Wright in Cuba, a portion of which has been examined in the Curtis Herbarium at Harvard, was redescribed as *Xylaria aurantiaca* by Berkeley and Curtis, it being assumed in both instances that it was an immature ascomycete, and that its peculiarly developed spores were young perithecia. In a similar fashion a gathering from Jamaica, communicated by Cockerell was subsequently described by Ellis as a new but sterile Hypomyces, which he named *H. alboluteus* E. & E. Still more recently it has again been described under a new generic name by von Höhnel, as *Endogonella borneensis* n. g. et n. sp., this being the first record of its occurrence in the Eastern Hemisphere.

This plant occurs in the form of a bladder-like, variably lobed, or even convolute, soon unattached sporocarp, Figure 88, 2 to 4 cm. long by 2 to 3 cm. broad and 1.5 to 3 cm. thick, more or less, when dry; much larger when fresh. The tough gelatinous wall 700–900 μ thick, is perforate below, the perforation entering a central cavity which is quite empty. On drying it becomes hard and brittle and loses much of its color, fading and becoming brownish with age, and exposure. The outer surface is more even, with a slight bloom, the inner pale yellowish and somewhat uneven. A section of the wall cut radially, Figures 89–90, shows a rather clearly defined distinction between an outer thicker, and an inner thinner dense layer and a looser broader sporogenous region which lies between them, but is not separated by any clean cut line of demarkation.

Although the fungus is usually described as immature or sterile, I have myself seen no specimens which are not fertile and but two that were even moderately young. Of these the youngest was found in 1891, near Kingston, Jamaica, partly buried in a very soft rotten log. The sporocarp was completely enveloped by a thin white separable universal membrane within which it was already free and perforate below. The hyphae composing this membrane are slender, thick-walled and septate. In this unexpanded condition, which was thick and somewhat flattened, the spores, although for the most part nearly mature, Figure 90, were crowded in an irregularly double layer, and the plant was set aside as a species of Endogone.

As the sporocarp matures and enlarges this crowded condition disappears, and the outer and inner layers, especially, evidently take an important part in the tangential increase, so that the crowded spores become more and more discrete as the wall stretches itself, so to speak, and eventually assume a more or less definite arrangement in a single layer. The hyphal elements which take part in this process are thick-walled with rather frequent septa, densely compacted in the two walls and somewhat looser in the middle region (Fig. 94). As the sporocarp increases in size the septa become very numerous and the filaments greatly enlarged, so that the inner and especially the outer layers appear to be made up of dense thin-walled parenchyma which gradually passes into the looser lacunose middle region, the hyphae of which are also thin-walled for the most part and submoniliform, Figure 93, with large interhyphal spaces.

Unfortunately no information is available as to the earlier stages in the formation of the sporocarp, or the initial processes of spore-formation. The sporogenous zone is evidently traversed at intervals by radial wefts of finer, closely septate hyphae, which are dense and have a dark granular appearance (Figs. 90–91). These wefts are the sporogenous centres above referred to, and conceal the spore-origins which traverse them. The spore-initials emerge from them as shown in Figure 91, a tangential section, in the form of long clavate structures, terminating apparently aseptate filaments which, although it has not been possible to trace them to their origin, appear to grow radially from the inner margin of the inner dense layer of the sporocarp. The tips of these clavate structures become greatly enlarged and the walls enormously thickened; the lumen being sometimes almost obliterated. These swollen terminations push out into the interhyphal spaces of the middle zone, curving outward or backward, and are transformed to the mature spores. Sporulation seems to be more or less simultaneous at a rather early stage of development, and the enlargement of the orange sporocarp, after this has taken place, seems, under favorable conditions, to be a relatively rapid process.

The mature spores are very large, 200×200–$415 \times 380\ \mu$, spherical to broadly ellipsoid, a thinner gelatinous exospore about 10 μ thick and continuous with the wall of the sporogenous filament, being at first clearly distinguished from the very thick, continuous, laminated, 20–30 μ thick (to 38 μ with KOH) endospore. The contents is rather dense and finely granular, or jelly like in appearance, with larger oily bodies here and there. The sporogenous filaments which can sometimes be traced a short distance even from mature spores, seem to be simple and show no signs of any structure which might be regarded as possibly conjugating elements; although it is by no means certain that such organs might not possibly be recognizable in much earlier stages then those which have been examined. Owing to secondary tangential growth of the sporocarp, the spores, as above mentioned, not only tend to become discrete, and to arrange themselves in a single layer; but the spaces in which they lie may become considerably enlarged, so that they may lie in a loculus of considerable size, quite free, or suspended by a few adherent radiating filaments, Figure 92.

The spores, when they have been seen at all, have been variously referred to as asci, perithecia, vesicles or even glands. In very well matured individuals which have been slightly shrunken by alcohol, the position of the spores may be indicated externally, even to the naked eye, by slight corresponding elevations, which have very much the appearance of perithecial ostioles; and it is therefore not so surprising that, in the original description of the type, they should have been spoken of as "pale perithecia filled with hyaline gelatine."

Von Höhnel in his description, gives the first indication which has been found in the literature, of what appears to be their true nature, and includes what he regarded to be the new genus Endogonella among the chlamydosporic types of the Endogoneae. Apart from the fact, however, that the plant is described and represented upsidedown, it answers in all particulars to *Glaziella vesiculosa* Berk. There is also absolutely no difference which I have been able to discover, between *Xylaria aurantiaca* B. & C. collected by Wright in Cuba, and Berkeley's type collected in Brazil by Glaziou. It seems quite remarkable, however, that there appear to be no other synonyms, and that, although several other species of Glaziella have been listed, the genus remains monotypic.

A final opinion with regard to its true relationships can hardly be formed until the nature of the primary vegetative and sporogenous hyphae is known. It seems not impossible that, like Sclerocystis, it possesses two distinct sets of hyphal elements, the one concerned in sporulation and without septa, the other copiously septate. Although in Sclerocystis the septate elements seem to be lysigenous, and are designed to free and scatter the sporocarps and spores; and in the present genus, on the contrary, their object appears to be to hold them firmly in a coherent mass which they render esculent and make conspicuous by raising it above the substratum through a mechanism remotely comparable to that of some phalloids, their ultimate function, namely an effective dispersal of the spores, may be assumed to be the same.

In both genera, the fact that one element in their structure does not conform to the ordinary phycomycetous type, since it is composed of copiously septate filaments, may be explained by their specialization for a definite purpose other than sporulation. In Sclerocystis at least, the sporogenous element corresponds in structure to that of the Mucorales, since its hyphae are more strictly continuous even than in Endogone. Whether this is also the case in Glaziella cannot be determined till very young material is available for study. The presence of an independent and continuous endospore, Figure 91, like that of the zygospores in Endogone, points to a further possibility of their sexual origin.

In any case Sclerocystis and Glaziella may be assumed to illustrate the highest degree of differentiation which is reached in the family, so far as it is known; and their structure and development is the more remarkable, when it is contrasted with the relatively simple conditions which obtain in other forms. They do not seem, however, to be nearly related, although, if the above suggestion as to the sporogenous element in Glaziella is correct, the dual nature of the elements which compose them and the definite segregation of the sporulating regions are characters in common. The wefts of finer filaments which in Glaziella are associated with spore-origins which might be regarded as corresponding to the sporocarp initials of Sclerocystis, do not, however, seem comparable; since the fine filaments which compose them become closely septate, and seem to arise directly from the inner layer.

It should be mentioned that all other fungi which have been included in Glaziella appear to belong among the ascomycetes: similarity in color, sterility, and gelatinous consistency having apparently been the only reasons for their inclusion in it. Among these forms the most important is *Glaziella splendens* Cooke (1882), p. 83, described by Berkeley & Curtis (1868) as *Xylaria splendens*, which, like "*Xylaria*" *aurantiaca*, was also collected in Cuba by Wright, is represented in the Curtis collection by one half of the type, the other half, which is figured by Lloyd (1919), p. 29, being in the Kew Herbarium. The Curtis half has been examined and sectioned with some care, and is certainly in no way related to Glaziella. Its characters are entirely similar to those of *Entonema liquescens* Möller (1901), p. 247 with figure, and it seems very probable that it may be the sterile condition which appears sometimes to be associated with this species, or at the least an immature stage. It is not hollow, has the same bright granular superficial crust and an inner gelatinous region composed of colorless hyphae with thick soft gelatinous walls, which is subtended by a contrasting black zone or line. There are absolutely no signs of spores or of developing perithecia, and one is inclined to agree with Möller that descriptions of sterile forms of this nature should be disregarded by mycologists.

Glaziella sulphurea Patouillard (1903), p. 292, judging from the description, is certainly not a Glaziella, and appears also to correspond very closely with the sterile condition of *Entonema liquescens*.

Glaziella ceramichroa (Berk. & Broome) Cooke (1882) is very surely not of this genus. Mr. Petch assures me that his reference (1910), p. 427, No. 61, of the species to Hypocrella is correct.

Glaziella abnormis (Berk.) Cooke (1882), in which asci and ascospores are described, must evidently be excluded; since it also appears to be very near to, if not identical with, *Entonema liquescens*.

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EXPLANATION OF THE PLATES.

Description of the Figures.

The figures are for the most part camera drawings which have been reduced more than one half in reproduction. Unless otherwise stated the original combination used is Zeiss obj. D and ocular 4, with table projection.

PLATE I.

Endogone pisiformis Lk.

Figures 1–5. Successive stages in the formation of zygospores. 1–4, obj. J, oc. 4; 5 obj. J, oc. 2.
Figure 6. Mature spores with gametes still attached.
Figure 7. Peculiarly differentiated hypha forming the superficial tomentum of a young individual.

Endogone multiplex Thaxter.

Figure 8. Mature spore showing hyphal envelope.
Figure 9. Mature spore with discrete gamete attachments, treated with KOH.
Figure 10. Larger and smaller spore-groups surrounded by envelope with incorporated humus-material.

Endogone tuberculosa Lloyd.

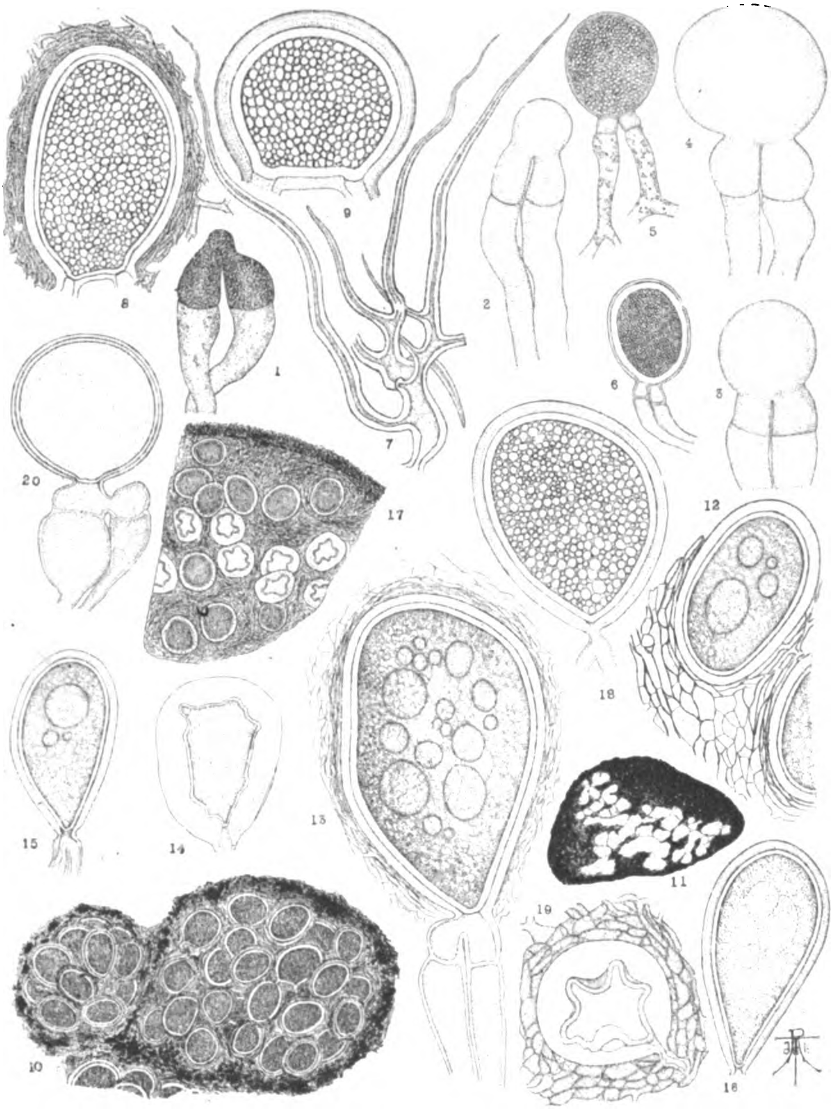
Figure 11. Section of sporocarp showing disposition of sporogenous areas, (light), with surrounding and penetrating earthy envelope (dark): magnified about seven times as reduced.
Figure 12. Portion of gleba showing spores and pseudoparenchymatous hyphal tissue.
Figure 13. Spore showing envelope and origin from gametes. Zeiss J, oc. 1.
Figure 14. Fully matured spore with greatly thickened exospore.
Figures 15–16. Spores of more elongate type.

Endogone incrassata Thaxter.

Figure 17. Section of a portion of sporocarp, showing peridium and gleba with fully mature and more immature spores.
Figure 18. Spore showing characteristic fatty contents and probably sexual origin. Zeiss D, oc. 12.
Figure 19. Fully mature spore with greatly thickened exospore and compacted surrounding hyphae.

Endogone lactiflua Berk.

Figure 20. Showing origin of zygospore from dissimilar gametes.



THAXTER—REVISION OF ENDOGONEAE

PLATE II.

Endogone fasciculata Thaxter.

Figure 21. Sorus of chlamydospores viewed externally. Zeiss obj. A, oc. 4.

Figure 22. Section of sorus of chlamydospores showing, above, a few adjacent young zygospores. Zeiss. obj. A, oc. 4.

Figures 23–26. Different stages in the formation of zygospores. Zeiss obj. J, oc. 2.

Figures 27–28. Separated chlamydospores.

Endogone vesiculifera Thaxter.

Figure 29. Partial section of a sorus showing origins and association of chlamydospores and thin-walled clavate vesicles. Zeiss obj. A, oc. 4.

Figure 30. Clavate vesicles and their origin.

Figures 31–32. Separated chlamydospores and their origins.

Endogone fuegiana Spegazzini.

Figure 33. Section of gleba showing discrete spore-groups. Zeiss obj. A, oc. 4.

Figure 34. Section of single spore-group, showing spore-origins.

Endogone microcarpa Tulasne.

Figures 35–37. Chlamydospores from Californian material.

Endogone arenacea Thaxter.

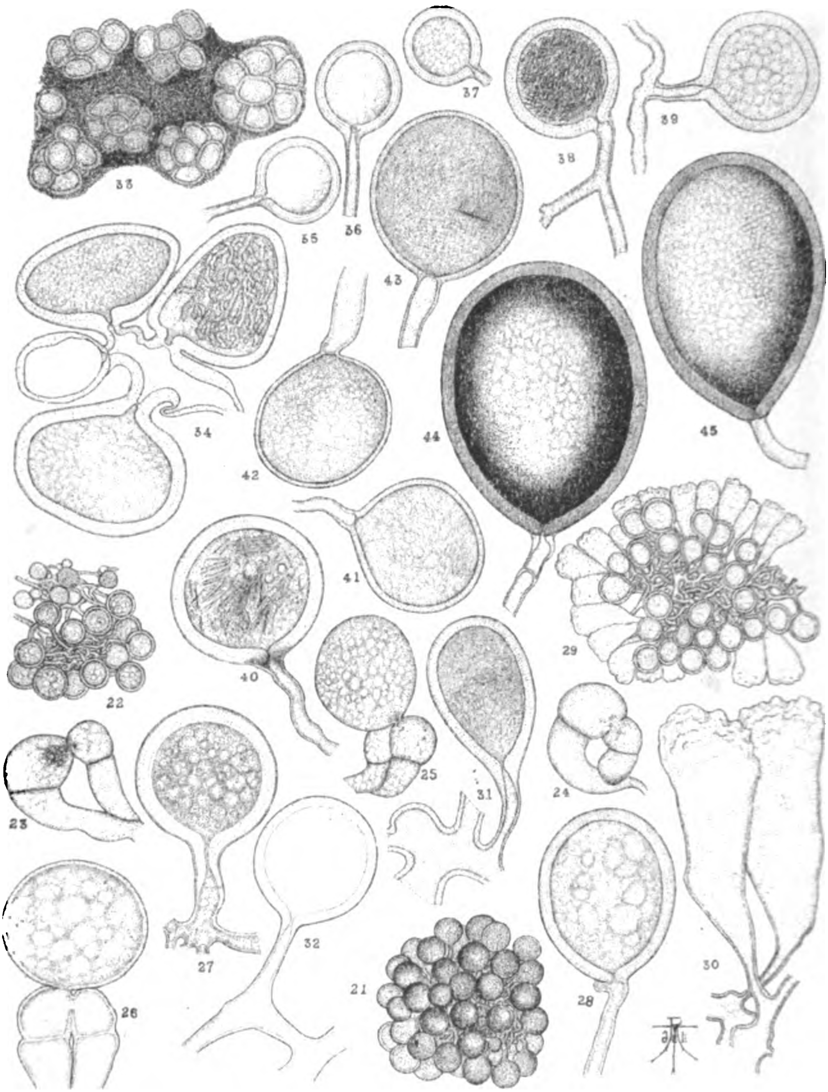
Figures 38–40. Three mature chlamydospores with their origins.

Endogone pulvinata Hennings.

Figures 41–43. Three chlamydospores from the type material.

Endogone borealis Thaxter.

Figures 44–45. Two chlamydospores.



THAXTER—REVISION OF ENDOGONEAE

PLATE III.

Endogone tenebrosa Thaxter.

FIGURE 46. Nearly mature chlamydospore, showing unbroken protoplasmic isthmus.

Endogone radiata Thaxter.

FIGURE 47. Section of sporocarp, showing part of surface and gleba, with radiately arranged chlamydospores. Zeiss obj. A, oc. 4.
FIGURES 48–51. Chlamydospores of varying form.

Endogone canadensis Thaxter.

FIGURES 52–55. Chlamydospores showing the slender sporophore.

Endogone fulva (Berkeley) Patouillard.

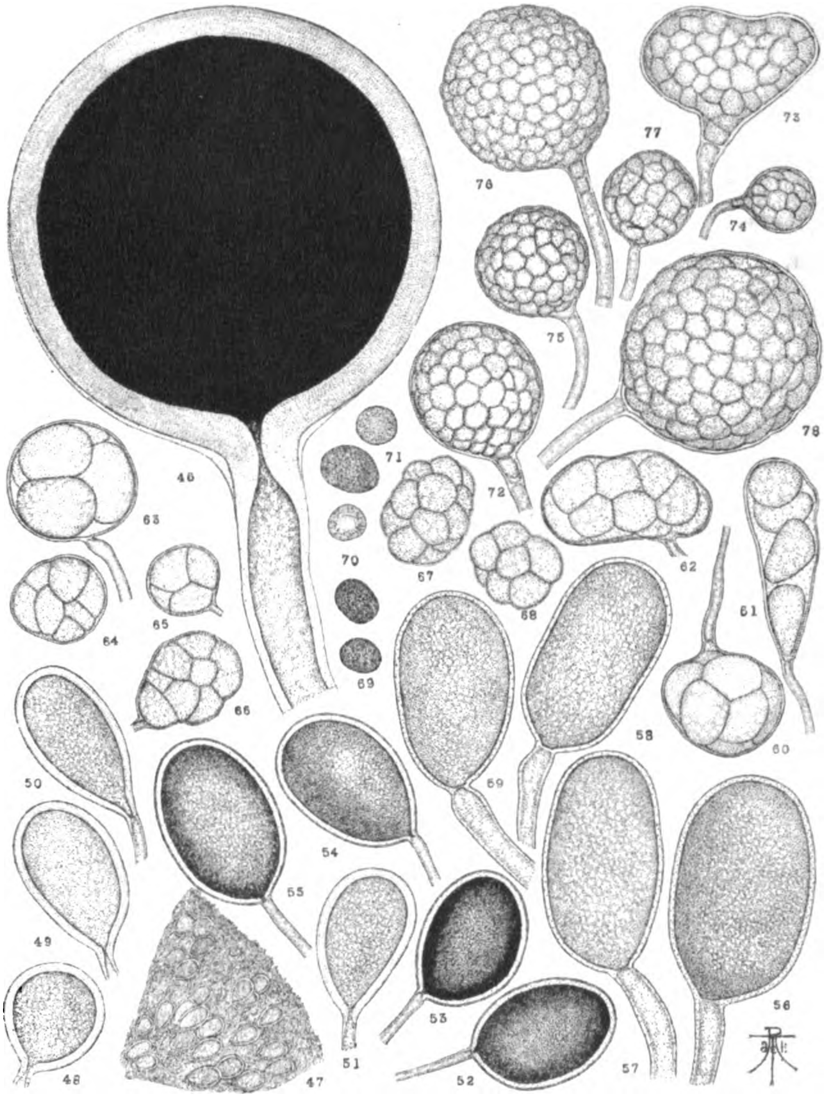
FIGURE 56. Chlamydospore from the Type of *E. Mölleri*.
FIGURE 57. Chlamydospore from the island of Grenada, B.W.I.
FIGURE 58. Chlamydospore from Florida.
FIGURE 59. Chlamydospore from Grenada, B.W.I.

Endogone reniformis Bresadola.

FIGURES 60–63. Sporangia from the Straits of Magellan.
FIGURES 64–66. Sporangia from Spegazzini's type of *E. argentina*.
FIGURES 67–68. Sporangia from the Berlin type of *E. reniformis*.
FIGURE 69. Single spores from the last.
FIGURES 70–71. Single spores from Spegazzini's type.

Endogone malleola Harkness.

FIGURES 72–74. Sporangia from the californian type of Harkness.
FIGURES 75–76. Sporangia from Torrends Fungi Selecti, No. 159, *E. Torrendii* Bres.
FIGURES 77–78. Sporangia from New Zealand communicated by Lloyd.



THAXTER—REVISION OF ENDOGONEAE

PLATE IV.

Sphaerocreas pubescens Saccardo & Ellis.

FIGURE 79. Spore mass seen in optical section seated on fragment of wood. Zeiss obj. A. oc. 4.

FIGURE 80. Hyphal bundles which radiate from the spore-mass.

FIGURES 81–82. Six spores showing their attachment to slender hyphae.

Sclerocystis Dussii (Patouillard) von Höhnel.

FIGURE 83. Sorus showing stalk arising from a second sorus below, with central columella and external hyphal layer, above which is a portion of the upper surface of the "stroma" showing the "giant cells." Zeiss obj. C, oc. 2.

FIGURE 84. Section including the surface of the stroma with its giant cells, and showing several different sori in situ. Zeiss obj. A, oc. 2.

FIGURE 85. Two spores isolated and showing basal septum.

Sclerocystis coccogena (Patouillard) von Höhnel.

FIGURE 86. Small portion of a sorus, showing a small part of the columella, spore zone and outer layer, the filaments of the latter producing several small spherical chlamydospores. Zeiss obj. A, oc. 2.

FIGURE 87. Small chlamydospores separated from surface of sorus-envelope.

Glaziella vesiculosa Berkeley.

FIGURE 88. Hollow sporocarp. A lobe cut away to show interior, the smaller black area below being the normal inferior perforation. Partly ventral view. Somewhat less than natural size as reduced.

FIGURE 89. Section of the sporocarp wall from a nearly mature individual in which the spores are becoming discrete. Zeiss obj. A, comp. oc. 3.

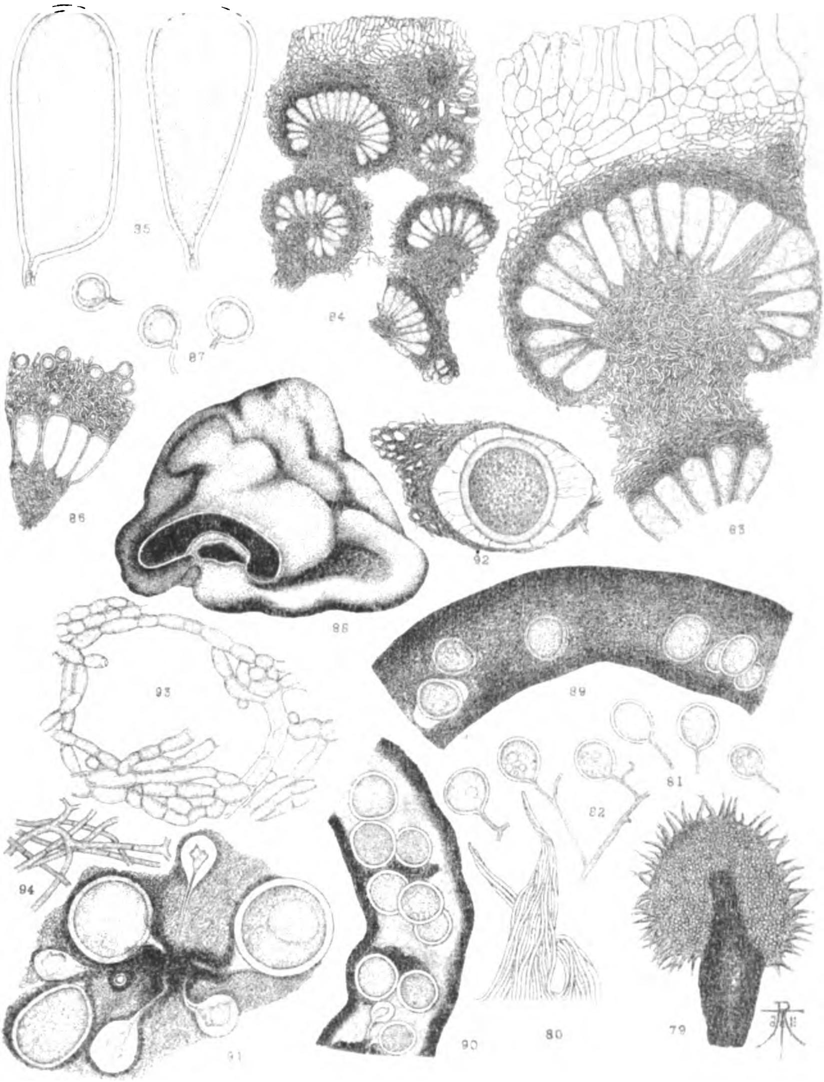
FIGURE 90. A similar section from a young individual, showing the dense superficial layers, the (black) hyphal wefts associated with the sporogenous filaments, and the crowded arrangement of the spores. Zeiss obj. A, comp. oc. 3.

FIGURE 91. Tangential section from same individual, showing sporogenous weft cut transversely, with several sporogenous hyphae emerging from it. bearing terminal spores in several stages of development. Zeiss obj. A, oc. 2.

FIGURE 92. Portion of fully mature sporocarp, showing a spore suspended by hyphae in a large chamber, a part of the lacunose middle layer shown at left. Zeiss obj. A, oc. 2.

FIGURE 93. Hyphae from the middle zone of a younger individual. Leitz water im. oc. 2.

FIGURE 94. Hyphae from the same region in an older individual. Same magnification.



THAXTER—REVISION OF ENDOGONEAE

results are at hand, however, which show the variation of such an effect with wave-length. All that can be stated is that the photoelectric effect on dust may cause part of the diminution of intensity and the deviation from the scattering curve as recorded in Figure 1.

Conclusion.—It seems that (1) the lower atmosphere is practically free from ozone, (2) molecular scattering alone will not account for the attenuation of the shorter wave-lengths from distant terrestrial sources, (3) absorption by oxygen and ionization of the air may explain the deviation from the curve for molecular scattering, (4) probably the isothermal layer of our atmosphere contains the ozone which sets the limit of the solar spectrum in the ultra-violet.

My thanks are due Professor Lyman for many suggestions and for his aid in securing suitable locations for conducting this work, and Dr. Stetson for the use of his apparatus in measuring the opacities.

Jefferson Laboratory,
Cambridge, February, 1922.

Proceedings of the American Academy of Arts and Sciences.

Vol. 57. No. 13.— June, 1922.

CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION FOR RESEARCH. No. 138.

THE ECHINODERMS OF THE CHALLENGER BANK, BERMUDA.

By Hubert Lyman Clark.

With One Plate.

RECORDS OF MEETINGS.

One thousand one hundred and fifth Meeting.

October 19, 1921.—Stated Meeting.

The Academy met at its House.

The President in the Chair. The Transactions of the last two meetings were read and approved.

There were twenty-two Fellows and one guest present:

The following letters were presented by the Corresponding Secretary:—from C. G. Abbot, W. C. Abbott, I. Babbitt, S. J. Barnett, J. S. Bassett, E. W. Berry, N. L. Bowen, J. C. Branner, C. D. Buck, F. Cajori, C. M. Campbell, L. L. Campbell, R. Cole, C. Day, W. F. Durand, R. A. Emerson, F. E. Farley, M. Farrand, W. S. Ferguson, E. C. Franklin, P. R. Frothingham, R. G. Harrison, W. E. Hocking, W. H. Howell, W. J. Humphreys, E. X. L. H. Hyvernat, F. E. Ives, C. F. Jenney, A. Keith, O. D. Kellogg, J. F. Kemp, F. Lawton, J. G. Lipman, J. L. Lowes, C. D. Maginnis, C. T. Main, W. J. Mayo, J. C. Merriam, E. D. Merrill, G. S. Miller, W. Patten, F. W. Peabody, W. L. Phelps, C. V. Piper, C. R. Post, H. N. Russell, F. Schlesinger, A. W. Scott, J. Stebbins, C. W. Stiles, W. S. Thayer, C. H. Walker, J. Warren, David White, Arthur Winslow, accepting Fellowship; from Hugo de Vries, G. H. Hardy, G. A. F. Molengraaff, accepting Foreign Honorary Membership; from A. W. Whitcher, presenting a framed photograph of Gainsborough's portrait of Sir Benjamin Thompson, recently bequeathed to Harvard College; from Miss Susanna Willard, presenting a silhouette of Rev. Joseph Willard, Vice-President and Corresponding Secretary of the Academy from 1780–1804; from the University of Virginia, presenting a medal commemorating its 100th anniversary; from the Wisconsin Academy of Sciences, presenting a medal commemorating its 50th anniversary; from the Rumford Historical Association thanking the Academy for a replica of the first Rumford medal; from the Reale Università di Padova announcing the celebration of its 700th anniversary; from R. S. Woodward giving an account of the centennial of the University of Virginia.

The Chair announced the death of the following Fellows: Joel Asaph Allen, Class II., Section 3: Charles Pickering Bowditch, Class III., Section 2: Eliot Channing Clarke, Class I., Section 4: Hiram Francis Mills, Class I., Section 4; and of the Foreign Honorary Member Julius von Hann, Class II., Section 1.

A biographical notice of Edward Charles Pickering, by J. H. Metcalf was presented by the Corresponding Secretary.

The President stated that a Conference of Orientalists was held at the invitation of Class III., Section 2, of the Academy, in the House of the Academy, on October 5th.[1]

The following communication was presented: Professor C. B. Gulick, "The Greek Novel."

The following papers were presented by title:

"The General Conditions of Validity of the Principle of Le Chatelier," by Alfred J. Lotka. Presented by Irving Fisher.

"The Grid Structure in Echelon Spectrum Lines," by Norton A. Kent and Lucien B. Taylor.

"Effect of Tension upon Resistance of Abnormal Metals," by P. W. Bridgman.

"Effect of Pressure on Thermal Conductivity," by P. W. Bridgman.

"The Failure of Ohm's Law in Gold and Silver at High Current Densities," by P. W. Bridgman.

"Waverley Researches in the Pathology of the Feeble-Minded," by W. E. Fernald, E. E. Southard, M. M. Canavan, O. J. Raeder, and A. E. Taft.

"The Early Evolution of the Reflecting Telescope," by Louis Bell.

The meeting was then dissolved.

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1 For the record of this meeting *see* p. 460.

One thousand one hundred and sixth Meeting.

NOVEMBER 9, 1921.— STATED MEETING.

The Academy met at its House.
The PRESIDENT in the Chair.
The Transactions of the last meeting were read and approved.
There were thirty Fellows and two guests present:
The following communications were presented:
Louis Bell, "The Early Evolution of the Reflecting Telescope."
Discussion followed by A. G. Webster, E. Thomson, H. Cushing, W. S. Bigelow, and others.
Desmond FitzGerald, "On a great Collection of Skeletons of Animals from Prehistoric Times at the Rancho la Brea, near Los Angeles, California."
Discussion followed by W. M. Davis.
A. G. Webster, "Hermann von Helmholtz and his Significance for a Century of Science."
C. R. Lanman exhibited specimens of Helmholtz's autograph.
The Meeting was then dissolved.

One thousand one hundred and seventh Meeting.

DECEMBER 14, 1921.— STATED MEETING.

The Academy met at its House.
The PRESIDENT in the Chair.
The Transactions of the last meeting were read and approved.
There were thirty-nine Fellows and two guests present:
The Corresponding Secretary reported acceptances of Fellowship from:
F. S. Converse, A. P. Davis, William Emerson, C. M. T. Loeffler, H. A. Pilsbry, A. A. Young, and a resignation of Fellowship from R. A. Cram.
The President announced the death of Charles Robert Cross, Class I., Section 2; Chairman of the Rumford Committee since 1898.
The Librarian reported the gift to the Library from Admiral

F. T. Bowles of a set of the Transactions of the Institution of Naval Architects (London), and a set of the Transactions of the Society of Naval Architects and Marine Engineers (New York), with the offer to continue during his lifetime the gift of the volumes of these sets as they appear.

The following communications were presented:

Edward P. Warner, Professor of Aëronautical Engineering in the Massachusetts Institute of Technology, "Recent Developments in Aëronautical Science." (With lantern slide illustrations.)

Robert DeC. Ward, "Some Meteorological Phenomena in Relation to Flight."

Alexander McAdie, "Aërography." (With lantern slide illustrations.)

The Meeting was then dissolved.

One thousand one hundred and eighth Meeting.

JANUARY 7, 1922.— OPEN MEETING.

An Open Meeting was held at the House of the Academy from four to six o'clock.

The PRESIDENT in the Chair.

There were about two hundred and twenty-five Fellows and guests, including ladies, present.

Mr. Harlow Shapley, Director of the Harvard University Observatory, spoke on "The Galaxy: Its Content and Dimensions," with lantern slide illustrations.

Tea was served in the Reception Room on the third floor.

One thousand one hundred and ninth Meeting.

JANUARY 11, 1922.— STATED MEETING.

The Academy met at its House, on a night made formidable by cold and storm.

The PRESIDENT in the Chair.

The Transactions of the last Meeting were read and approved.

There were fifteen Fellows present:

The Corresponding Secretary presented a letter of resignation of Fellowship from W. T. Councilman.

The following proposed amendment to the Statutes was referred to a Committee consisting of the Corresponding and Recording Secretaries:

Chapter III, Article 2, second and last paragraphs, strike out after the word Fellow, "having the right to vote."

On the recommendation of the Council it was

Voted, To make the following appropriation from the income of the General Fund:

$300 for General Expenses.

$300 for House Expenses.

The following communication was presented:

G. A. Reisner, "Archeological Methods Used by the Harvard-Boston Expedition." (With lantern slide illustrations.)

The following papers were presented by title:

"Some Hymenopterous Parasites of Lignicolous Itonididæ," by C. T. Brues.

"A Table and Method of Computation of Electric Wave Propagation and Transmission Line Phenomena," by G. W. Pierce.

"Artificial Electric Lines With Mutual Inductance Between Adjacent Series Elements," by G. W. Pierce.

"The Dioptrics of the Eye As Related to Pictorial Art," by Adelbert Ames, Jr., presented by Louis Bell.

The Meeting was then dissolved.

One thousand one hundred and tenth Meeting.

FEBRUARY 8, 1922.— STATED MEETING.

The Academy met at its House.

The PRESIDENT in the Chair.

The Transactions of the two last Meetings were read and approved.

There were twenty-seven Fellows present:

The Corresponding Secretary announced the receipt of the following biographical notices:

Charles Pickering Bowditch, by A. M. Tozzer.

Barrett Wendell, by Robert Grant.

The President announced the death of James Bryce, Viscount Bryce, Foreign Honorary Member, Class III., Section 3.

The President announced that an invitation had been received from the Académie Royale de Belgique, inviting the Academy to take part in the celebration of its 150th anniversary on May 24, 1922.

It was

Voted, To ratify the vote taken at the last meeting, no quorum being then present, appropriating $600 from the General Fund, to be applied, $300 to General Expenses and $300 to House expenses.

The special Committee for an amendment to the Statutes, Chapter III., Art. 2, second and last paragraphs, to strike out after the word Fellow the words "having the right to vote," reported, recommending that the amendment be adopted; and, a ballot having been taken, it appeared that the amendment was adopted, twenty votes being cast in the affirmative, and none in the negative.

The President stated that he had appointed E. H. Hall a member of the Committee on Biographical Notices.

The following communication was presented:

W. B. Cannon, "New Evidence for Nervous Control of Some Internal Secretions." (With lantern slide illustrations.)

The following paper was presented by title:

"Atmospheric Attenuation of Ultra-Violet Light," by E. L. Schaeffer, presented by Theodore Lyman.

The Meeting was then dissolved.

One thousand one hundred and eleventh Meeting.

MARCH 8, 1922.— STATED MEETING.

The Academy met at its House.

The PRESIDENT in the Chair.

The Transactions of the last Meeting were read and approved.

There were forty-three Fellows and several guests present:

The Corresponding Secretary announced the receipt of an invitation from the Royal University of Padua inviting the Academy to take part in the VII centenary of its foundation, May 14–17, 1922; and an invitation from the Société Asiatique de Paris to send delegates to take part in the 100th anniversary of its founding, July 10–13, 1922.

The Corresponding Secretary announced the receipt of the following biographical notices:

John Wallace Baird, by R. M. Yerkes.

William Gilson Farlow, by C. L. Jackson.

The President announced the death of Charles Leonard Bouton, Class I., Section 1.

On recommendation of the Council, the following appropriations were made for the ensuing year:

From the income of the General Fund, $8,300.65, to be used as follows:

for General and Meeting expenses	$1,300.00
for Library expenses	3,000.00
for Books, periodicals and binding	1,100.00
for House expenses	2,300.00
for Treasurer's expenses	600.00

From the income of the Publication Fund, $3,951.95, to be used for publication.

From the income of the Rumford Fund, $6269.65, to be used as follows:

for Research	$1,000.00
for Purchase and binding of books and periodicals	200.00
for Publication	600.00
for use at the discretion of the Committee	4,469.65

From the income of the C. M. Warren Fund, $1,842.44, to be used at the discretion of the Committee.

E. B. Wilson reported for the Committee on Membership, proposing amendments to the Statutes as follow:

(1) to amend Chapter II., Art. 2, by substituting "Two hundred and ten" for "Two hundred" in the third line.

(2) to amend Chapter IX., Art. 1, third paragraph, by omitting the words "of the same Class."

The President appointed the Nominating Committee for officers for the ensuing year, 1922–23, as follows:

G. D. Birkhoff, of Class I.

C. H. Warren, of Class II.

Frederic Dodge, of Class III.

The following communication was presented:

Adelbert Ames, Jr., "The Physiology of Vision and the Technique of Art."

The following papers were presented by title:

"Additions to the Hydroid Fauna of the Bermudas," by Rudolf Bennitt, presented by E. L. Mark.

"The Parasitic Worms of the Animals of Bermuda: I. Trematodes," by F. D. Barker, presented by E. L. Mark.

The Meeting was then dissolved.

One thousand one hundred and twelfth Meeting.

MARCH 11, 1922.— OPEN MEETING.

An Open Meeting was held at the House of the Academy from four to six o'clock.

The PRESIDENT in the Chair.

There were about one hundred Fellows and guests, including ladies, present.

Professor George Grafton Wilson, of Harvard University, spoke on "The Recent Conference on the Reduction of Armaments."

Tea was served in the Reception Room on the third floor.

One thousand one hundred and thirteenth Meeting.

APRIL 8, 1922.— SPECIAL MEETING.

A special meeting of the Academy was held at its House on Saturday, April 8, at four o'clock in the afternoon to receive Professor Hendrik Antoon Lorentz of the University of Leiden, a Foreign Honorary Member of the Academy.

A number of Fellows, and guests upon their invitation, were present.

The President introduced Professor Lorentz who spoke on some features in the work of the late Josiah Willard Gibbs, Professor of Mathematical Physics in Yale College, and Fellow of the Academy.

At the close of the address an hour was spent socially in the upper room where tea was served.

One thousand one hundred and fourteenth Meeting.

APRIL 12, 1922.—STATED MEETING.

The Academy met at its House at 8.15 P.M.

The PRESIDENT in the Chair.

There were thirty-six Fellows present:

The Transactions of the meetings of March 8 and April 8 were read and approved.

The President announced the death of John Wilkes Hammond, Fellow in Class III., Section 1.

The Corresponding Secretary reported the receipt of a biographical notice of William Thompson Sedgwick by Edmund B. Wilson.

The President announced the appointment of C. H. Haskins and A. E. Kennelly to represent the Academy at the 150th anniversary of Académie Royale de Belgique, on May 24th; and of C. H. Haskins and C. H. Moore at the 700th anniversary of the University of Padua.

The following amendments to the Statutes were adopted:

Chapter II., Article 2, last line, add the words "and ten" so as to read "nor shall there be more than Two hundred and ten in any one Class."

Chapter IX., Article 1, paragraph 3, strike out the words "of the same Class."

The following Communications were presented:—

Mr. Elihu Thomson. "Laboratory Products."

Mr. John L. Lowes. "A Neglected Note Book of Coleridge."

The following papers were presented by title:—

"A Revision of the Endogoneae." By Roland Thaxter.

"The Echinoderms of the Challenger Bank, Bermuda." By Hubert L. Clark. Presented by E. L. Mark.

"The Heat of Vaporization of Mercury." By F. W. Loomis. Presented by P. W. Bridgman.

"The Joule-Thomson Effect in Air." By P. H. Royster. Presented by P. W. Bridgman.

"The Ratio of the Calorie at 73° to that at 20°." By Arnold Romberg. Presented by P. W. Bridgman.

The Meeting was then dissolved.

One thousand one hundred and fifteenth Meeting.

MAY 10, 1922.— ANNUAL MEETING.

The Academy met at its House at 8.15 P.M.

The PRESIDENT in the Chair.

Thirty-seven Fellows were present.

The Transactions of the Meeting of April 12 were read and approved.

Professor J. R. Jewett was appointed a delegate to represent the Academy at the celebration of the 100th anniversary of the Société Asiatique de Paris, July 10–13, 1922.

The Corresponding Secretary reported that the Council had transferred William Emerson, with his consent, from Class I., Section 4 to Class III., Section 4.

The Corresponding Secretary presented the following biographical notices: — Joel Asaph Allen, by H. C. Bumpus; Eliot Channing Clarke, by G. F. Swain; Henry Lee Higginson, by M. A. DeW. Howe; Franklin Paine Mall, by W. T. Councilman; John Elliot Pillsbury, by W. H. Dall; Elmer Ernest Southard, by C. M. Campbell; Andrew Dickson White, by W. D. Bancroft; Edward James Young, by W. W. Fenn.

The following report of the Council was presented: —

Since the last report of the Council, there have been reported the deaths of seven Fellows: Joel Asaph Allen, Charles Leonard Bouton, Charles Pickering Bowditch, Eliot Channing Clarke, Charles Robert Cross, John Wilkes Hammond, Hiram Francis Mills; and two Foreign Honorary Members: Viscount Bryce, Julius von Hann.

Sixty-one Fellows and three Foreign Honorary Members were elected by the Council and announced to the Academy in May 1921. Two Fellows have resigned.

The roll now includes 569 Fellows and 67 Foreign Honorary Members (not including those elected in April 1922).

The annual report of the Treasurer, Henry H. Edes, was read, of which the following is an abstract:

General Fund.

Receipts.

Balance, April 1, 1921	$8,495.29	
Investments	4,691.00	
Assessments	3,480.00	
Admissions	100.00	
Sundries	257.65	$17,023.94

Expenditures.

Expense of Library	$4,815.44	
Expense of House	2,386.91	
Treasurer	505.96	
Assistant Treasurer	250.00	
General Expense of Society	1,596.61	
President's Expenses	64.25	
Income transferred to principal	367.23	$9,986.40
Balance, April 1, 1922		7,037.54
		$17,023.94

Rumford Fund

Receipts.

Balance, April 1, 1921	$3,938.64	
Investments	4,110.92	
Sale of Publications	49.90	$8,099.46

Expenditures.

Research	1,200.00	
Books, periodicals and binding	317.27	
Publications	392.58	
Sundries	73.50	
Income transferred to principal	186.89	$2,170.24
Balance, April 1, 1922		5,929.22
		$8,099.46

C. M. Warren Fund.

Receipts.

Balance, April 1, 1921	$4,226.62	
Investments	1,205.55	$5,432.17

Expenditures.

Research	$2,975.00	
Vault Rent, part	3.00	
Income transferred to principal	52.11	$3,030.11
Balance, April 1, 1922		2,402.06
		$5,432.17

Publication Fund.

Receipts.

Balance, April 1, 1921	$4,470.01	
Appleton Fund investments	1,843.68	
Centennial Fund investments	2,441.28	
Authors' Reprints	73.87	
Sale of Publications	227.34	$9,056.18

Expenditures.

Publications	$3,738.73	
Vault Rent, Part	10.00	

Income transferred to principal	$187.32	$3,936.05
Balance April 1, 1922		5,120.13
		$9,056.18

FRANCIS AMORY FUND

Receipts.

Investments	$1,533.75	

Expenditures.

Publishing statement	$55.60	
Interest on bonds bought	44.00	
Income transferred to principal	1,434.15	$1,533.75

The following Reports were also presented: —

REPORT OF THE LIBRARY COMMITTEE.

The Librarian begs to report for the year 1921–22, as follows:

During the year, 87 books have been borrowed by 19 persons, including 13 Fellows and 2 libraries. Many books have been consulted and used at the Library. All books taken out have been returned or satisfactorily accounted for, except three.

The number of books on the shelves at the time of the last report was 37,543. 868 volumes have been added, making the number now on the shelves 38,411. This includes 170 purchased from the income of the General Fund, 54 from that of the Rumford Fund, and 644 received by gift or exchange.

The expenses charged to the Library during the financial year are:

Salaries	$2,970.25
Binding: —	
General Fund	1,055.10
Rumford Fund	123.21

Purchase of periodicals and books: —	
General Fund	760.93
Rumford Fund	221.62
Miscellaneous	2.10
	$5,313.21

ARTHUR G. WEBSTER, *Librarian.*

May 10, 1922.

REPORT OF THE RUMFORD COMMITTEE.

The Committee met on October 19, 1921. Professor Charles R. Cross expressed himself as unwilling to be a candidate for re-election to the Chairmanship, the matter of organization was therefore postponed.

Professor Cross died on November 16, 1921.

At a meeting held December 14, 1921, Theodore Lyman was elected Chairman and Arthur G. Webster, Secretary.

The following grants in aid of researches in light or heat have been made during the academic year 1921–22:

October 19, 1921. To Professor Norton A. Kent, of Boston University, for the purchase of a Lummer Plate; Additional to former appropriation (235) $500

To Professor Harvey N. Davis, of Harvard University, in aid of his researches on the Improvement of the Design of Liquid Air Machinery (236) 300

April 12, 1922. To Professor Percy W. Bridgman, of Harvard University, for the continuation of his researches on the Thermal and Optical Properties of Matter under High Pressure; Additional to former appropriation (237) 500

To Professor Frederick A. Saunders, of Harvard University, in aid of his Spectroscopic Researches; Additional to former appropriation (238) 150

To Professor William Duane, Harvard University, for an Investigation on the Heat Energy of Electrons (239) . . . 300

Reports of progress in their respective researches have been received from the following persons: R. T. Birge, P. W. Bridgman, W. W.

Campbell, A. L. Clark, F. Daniels, P. F. Gaehr, R. C. Gibbs, H. L. Howes, L. R. Ingersoll, N. A. Kent, F. G. Keyes, C. A. Kraus, C. L. Norton, F. Palmer, Jr., J. A. Parkhurst, H. M. Randall, T. W. Richards, F. A. Saunders, W. O. Sawtelle, B. J. Spence, L. S. E. Thompson, O. Tugman, F. W. Very, A. G. Webster, D. L. Webster.

The following papers in the Proceedings have been published with aid from the Rumford Fund since the presentation of the last Report:

Awards of the Premium and Grants for Research in Light and Heat. Charles R. Cross, Vol. 56, No. 10.

The Grid Structure in Echelon Spectrum Lines. Norton A. Kent and Lucien B. Taylor, Vol. 57, No. 1.

The Effect of Pressure on the Thermal Conductivity of Metals. P. W. Bridgman, Vol. 57, No. 5.

The following papers have been approved for publication:

The Atmospheric Attenuation of Ultra-Violet Light, Dr. E. L. Schaeffer.

On the Ratio of the Calorie at 73° to that at 20°, Professor Arnold Romberg.

The Heat of Vaporization of Mercury, Professor F. W. Loomis.

The Joule-Thomson Effect in Air, P. H. Royster.

For nearly twenty-five years Professor Charles R. Cross acted as Chairman of the Rumford Committee. The members of the Committee wish to express their appreciation of his untiring devotion and to record their deep regret at his loss.

THEODORE LYMAN, *Chairman.*

May 10, 1922.

REPORT OF THE C. M. WARREN COMMITTEE.

The Committee had at its disposal at the end of the fiscal year in March 1921, $4,501.03. During the year ending March 31, 1922, grants to the amount of $2,875 were made. The balance on that date was $1,626.03.

Since the last annual report awards have been made as follows:

To Professor Henry Fay, Massachusetts Institute of Technology, $200 was granted June 1, 1921, for a research on the influence of nitrogen upon the case hardening of steels and the study of the heat treatment of beta-brasses.

To Professor H. H. Willard, University of Michigan, $300 was granted June 1, 1921, for the study of an electrometric method of determining the endpoint in volumetric analysis.

To Professor R. L. Datta, Calcutta, India, $400 was granted June 1, 1921, for a research on the determination of the temperature of explosion of endothermic substances.

To Professor D. A. MacInnes, Massachusetts Institute of Technology, $100 was granted June 15, 1921, for work on liquid junction potentials.

To Professor L. J. Desha, Washington and Lee University, $200 was granted September 23, 1921, for a study of the fluorescence of organic compounds.

To Professor V. K. Krieble, Jarvis Chemical Laboratory, Trinity College, $100 was granted February 21, 1922, for a study of asphalts.

To Professor F. R. Brunel, Bryn Mawr College, $200 was granted February 21, 1922, for work on the addition of halogen hydrides to unsaturated compounds.

To Professor C. James, New Hampshire College, $500 was granted May 4, 1922, to be applied toward an investigation on the ytterbium earths.

To Professor Charles A. Kraus, Clark University, $500 was granted May 4, 1922, to continue his work on the constitution of metallic substances.

Reports of progress have been received from Professors Brunel, Kraus, Conant, James, Fay, and MacInnes. The other recipients of grants have been asked to submit reports of their work.

The Chairman of the Committee is attempting to get as complete a collection as possible of reprints of the papers describing the work which has been assisted by grants from the Warren Fund in the past.

James F. Norris, *Chairman.*

May 10, 1922.

Report of the Publication Committee.

During the twelve months since the presentation of the last annual report, from April 1, 1921 to March 31, 1922, there have been published No. 3 of Vol. 14 of the Memoirs, Nos. 5–11, inclusive, of Vol. 56

of the Proceedings, and Nos. 1 to 10, inclusive, of Vol. 57. Costs of printing, happily, show a slight falling off from the excessive prices of preceding years, which is partly offset by the increased use of plates and line engravings in the published papers. The financial statement is as follows:

Receipts.

Balance, April 1, 1921	$5,882.10	
Appropriation	3,559.11	
Sales of publications	226.18	
Received for authors' reprints	75.03	$9,742.42

Expenses.

Engraving, printing, and binding	$3,450.92	
Cartage and mailing	277.97	
Committee's expenses	9.84	$3,738.73
Balance, April 1, 1922		$6,003.69

The above figures do not include the sum of $382.58 received from the Rumford Committee for publication of Rumford papers.

Respectfully submitted,

Louis Derr, *Chairman.*

May 10, 1922.

Report of the House Committee.

The House Committee submits the following report for 1921–22.

With the balance of $12.35 left from last year, an appropriation of $2,500, and $145 received from other societies for the use of the rooms, the Committee has had at its disposal the sum of $2,657.35. The total expenditure has been $2,531.91, leaving an unexpended balance on April 1, 1922, of $125.44. The expenditure has been as follows: —

Janitor	. . . .	$925.00
Electricity	A. Light	169.53
	B. Power	86.00

Coal	Furnace	953.21
	Water Heater	30.50
Care of Elevator		71.45
Gas		62.29
Water		8.80
Telephone		78.18
Janitor's Materials		10.78
Upkeep		106.47
Ash Tickets		29.70
Total Expenditure		$2,531.91

The amount $145 contributed by other societies for the use of the building leaves the net expense of the House $2,386.91.

Meetings have been held as follows: —

The Academy	
Stated meetings	8
Open meetings	3
Special meetings	4
American Antiquarian Society	1
Archaeological Institute	1
Colonial Dames	1
Colonial Society	4
Geological Club of Boston	2
Harvard-Technology Chemical Club	7
	31

The rooms on the first floor have been used for Academy Council and Committee meetings and also by the Trustees of the Children's Museum.

Respectfully submitted,

John Osborne Sumner, *Chairman.*

May 10, 1922.

On the recommendation of the Treasurer, it was

Voted, That the Annual Assessment be $10.00.

The annual election resulted in the choice of the following officers and committees:

GEORGE F. MOORE, *President.*
ELIHU THOMSON, *Vice-President for Class I.*
HARVEY CUSHING, *Vice-President for Class II.*
ARTHUR P. RUGG, *Vice-President for Class III.*
HARRY W. TYLER, *Corresponding Secretary.*
CHARLES B. GULICK, *Recording Secretary.*
[1] HENRY H. EDES, *Treasurer.*
ARTHUR G. WEBSTER, *Librarian.*

Councillors for Four Years.

EDWARD V. HUNTINGTON, *of Class I.*
CHARLES PALACHE, *of Class II.*
WILLIAM C. WAIT, *of Class III.*
KIRSOPP LAKE, *of Class III.*

Finance Committee.

HENRY P. WALCOTT, JOHN TROWBRIDGE,
HAROLD MURDOCK.

Rumford Committee.

THEODORE LYMAN, LOUIS BELL,
ARTHUR G. WEBSTER, PERCY W. BRIDGMAN,
ELIHU THOMSON, HARRY M. GOODWIN,
CHARLES L. NORTON.

C. M. Warren Committee.

JAMES F. NORRIS, WALTER L. JENNINGS,
HENRY P. TALBOT, ARTHUR D. LITTLE,
GREGORY P. BAXTER, LAWRENCE J. HENDERSON,
FREDERICK G. KEYES.

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[1] Died October 13, 1922.

Publication Committee.

Louis Derr, *of Class I.*
Herbert V. Neal, *of Class II.*
Albert A. Howard, *of Class III.*

Library Committee.

Harry M. Goodwin, *of Class I.*
Thomas Barbour, *of Class II.*
William C. Lane, *of Class III.*

House Committee.

John O. Sumner,
Wm. Sturgis Bigelow, Robert P. Bigelow.

Committee on Meetings.

The President, George H. Parker,
The Recording Secretary, Edwin B. Wilson,
Edward K. Rand.

Auditing Committee.

George R. Agassiz, John E. Thayer.

The Council reported that the following gentlemen were elected members of the Academy: —

Class I., Section 1 (Mathematics and Astronomy):
Walter Sydney Adams, of Pasadena, California, as Fellow.
Arthur Stanley Eddington, of Cambridge, as Foreign Honorary Member.
Class I., Section 2 (Physics):
Edwin Crawford Kemble, of Cambridge, as Fellow.
Class I., Section 3 (Chemistry):
Richard Chase Tolman, of Washington, as Fellow.

Class I., Section 4 (Technology and Engineering):
Gano Dunn, of New York, as Fellow.
Thomas Alva Edison, of New Jersey, as Fellow.
Class II., Section 1 (Geology, Mineralogy, and Physics of the Globe):
Emmanuel de Margerie, of Paris, as Foreign Honorary Member.
Austin Flint Rogers, of Palo Alto, as Fellow.
Class II., Section 2 (Botany):
William Henry Weston, Jr., of Cambridge, as Fellow.
Class II., Section 3 (Zoölogy and Physiology):
Nathan Banks, of Cambridge, as Fellow.
Thorne Martin Carpenter, of Boston, as Fellow.
Alfred Clarence Redfield, of Boston, as Fellow.
Class II., Section 4 (Medicine and Surgery):
Sir Thomas Clifford Allbutt, of Cambridge, as Foreign Honorary Member.
Stanley Cobb, of Ponkapoag, as Fellow.
Joseph Lincoln Goodale, of Boston, as Fellow.
Robert Williamson Lovett, of Boston, as Fellow.
Class III., Section 1 (Philosophy and Jurisprudence):
William McDougall, of Cambridge, as Fellow.
Class III., Section 3 (Political Economy and History):
Edward Channing, of Cambridge, as Fellow.
George La Piana, of Cambridge, as Fellow.
Henri Pirenne, of Ghent, as Foreign Honorary Member.
Class III., Section 4 (Literature and Fine Arts):
Arthur Kingsley Porter, of Cambridge, as Fellow.
Paul Joseph Sachs, of Cambridge, as Fellow.
Charles Henry Conrad Wright, of Cambridge, as Fellow.

Monsieur J. Cavalier, Professor in the School of Science of the University of Toulouse and Rector of the University, addressed the Academy.

The following paper was presented by title: "The Phenomena of Polymegaly in the Sperm-Cells of the Family Pentatomidae." By Robert H. Bowen. Presented by Edmund B. Wilson.

The Meeting was then dissolved.

October 5 and 6 and 7, 1921.— Special Meeting of Orientalists.

A Special Meeting was held at the House of the Academy, beginning on Wednesday morning, the 5th of October, at ten o'clock, when the Academy received the Delegates from the Société Asiatique of Paris and the Royal Asiatic Society of London, deputed to confer with the members of Class III of the Academy upon matters concerning the promotion of Oriental studies.

A series of joint meetings of Orientalists was begun at London in 1919, and continued at Paris in 1920. This meeting of 1921 was held as a continuation of that series. In July, 1922, the French Society will celebrate the centenary of its foundation, and in 1923 the British Society will follow suit. Not until 1924 would another opportunity recur for holding such a meeting in America.

Accordingly, pursuant to a vote of the Council of the American Academy, an invitation was sent on April 13, 1921 to the Société Asiatique, the Royal Asiatic Society, and the Societá Asiatica Italiana, to meet with the members of Class III of the Academy, on the 24th of June, 1921, or at such later time as might appear more convenient, and at the House of the Academy, in the city of Boston.

The invitation was authorized by the Council of the Academy at the instance of several gentlemen,— Americans, Orientalists, friends of the Orient and of Oriental learning,— whose names follow: Dr. William Sturgis Bigelow, of Boston; Professor James H. Breasted, of the University of Chicago; Mr. Charles Dana Burrage, of Boston; Professors Albert T. Clay and Charles C. Torrey, of Yale University; Dr. Arthur Fairbanks, of the Museum of Fine Arts, Boston; Professors James R. Jewett, Charles R. Lanman, George Foot Moore, and James H. Woods, of Harvard University; Professor Duncan B. Macdonald, of Hartford Theological Seminary.

The invitation was most cordially and promptly accepted,— on behalf of the French Society, by its President, Mr. Émile Senart, Member of the Academy of Inscriptions and Belles-Lettres of the Institute of France, and on behalf of the English Society, by its

President, Lord Reay (deceased August 1, 1921), of the British Academy. For the Italian Society, its President, Professor Pavolini of Florence, wrote that the Ministry held out hopes that the sending of a Delegate might be sanctioned. Unfortunately, these hopes were not realized.

The French Society's Delegates were: Paul Pelliot, Member of the Academy of Inscriptions and Belles-Lettres of the Institute of France, Professor of Chinese at the Collège de France; Alexandre Moret, Director of Studies at the École Pratique des Hautes Études, Conservator of the Musée Guimet in Paris.

The Delegates of the English Society were: Dr. Arthur Ernest Cowley, Fellow of Magdalen College, Oxford, Librarian of the Bodleian Library; Dr. Stephen Langdon, Professor of Assyriology at Oxford; Herbert Weld-Blundell, Esq., of London (Queen's College, Oxford); Mr. Henry Lee Shuttleworth, of Delhi, of the Indian Civil Service.

Upon the Academy's invitation, the American Oriental Society sent the following Delegates: its President, Reverend James Buchanan Nies, of Brooklyn Heights, New York; Dr. William Sturgis Bigelow, of Boston; Professor James H. Breasted, of the University of Chicago; Charles Dana Burrage, Esq., of Boston; Professor Albert T. Clay and Professor Edward Washburn Hopkins, of Yale; Professor A. V. Williams Jackson, of Columbia University; Professor Charles Cutler Torrey, of Yale.

All these were present, except Professor Clay.

The non-resident Delegates, during their stay, were the guests of The Omar Khayyam Club of America. With two or three exceptions, they were lodged and entertained by the Omar Club at the House of the Harvard Club of Boston.

The Delegates were received by the following Fellows of the Academy; the President of the Academy, Professor George Foot Moore; his immediate predecessor, Professor Theodore William Richards; the Corresponding Secretary of the Academy, Professor Harry W. Tyler; the Recording Secretary of the Academy, Professor James Hardy Ropes; President Lowell of Harvard; Mr. John Ellerton Lodge, of the Boston Museum of Fine Arts; Mr. Edward Sylvester Morse, of the Peabody Museum, Salem; Dr. Francis H. Williams, of Boston; Professors James Richard Jewett, Kirsopp Lake, Ephraim Emerton, Charles R. Lanman, David G. Lyon, Clifford Herschel Moore, George Andrew Reisner, and James Haughton Woods, of Harvard.

Sessions of Wednesday, October 5, 1921.

President Moore opened the sessions by welcoming to the Academy the Delegates of the Oriental Societies, and spoke briefly of the purpose and spirit of the joint meeting.

Professor Pelliot responded on behalf of the visitors. Moreover, as bearer of an official message to the Academy, he read a letter addressed to President Moore by M. Senart, of the Institute of France, as President of the Société Asiatique. The letter tells of the satisfaction of the Society at the establishment of relations of sympathy and coöperation with the Academy, and of its hope for long and fruitful maintenance of these relations. In particular, it tells of the proposed celebration in the early days of July, 1922, of the hundredth anniversary of the founding of the French Asiatic Society, and expresses the hope that the Academy will take part on that occasion.

Professor Hopkins, of Yale, in response to a call from the Chair, gave a brief account of the recent progress of American studies in the literature of India.

Professor Torrey, of Yale, in like manner, spoke of the progress of Semitic studies, with some account of the collections of Semitic antiquities in the Museums at Philadelphia, Yale, Harvard, Princeton, and New York (collection of J. Pierpont Morgan).

Professor Reisner, of Harvard, reviewed the work of American philologists and archæologists in the Egyptian field, and mentioned the notable collections of Egyptian antiquities in American Museums.

Professor Lyon, of Harvard, finally, gave some account of the Harvard Semitic Museum, and of the Harvard Excavations at Samaria.

The assembled company then proceeded in motor-cars to the Boston Museum of Fine Arts. The Director, Dr. Arthur Fairbanks, being detained at home by illness, the visitors were received by the Acting-Director, Mr. Hawes. They were the guests of the Museum at luncheon. In the afternoon, they were conducted, some through the Egyptian Rooms by Dr. Reisner, and others through the Japanese Rooms by Mr. John Ellerton Lodge. The Delegates and their hosts dined together at the Harvard Club.

During the afternoon, Dr. Reisner gave an account of his twenty-two years of archæological research in Egypt, illustrated by the objects now on exhibition, of which the most notable are: 1. Eleven sculptures in the round of Chephren, Mycerinus, Shepseskaf, and other members of the royal family of the Fourth Dynasty; 2. Two sculptures in the round of prime importance, and many lesser statues and reliefs of the Old Empire; 3. The statue of the Lady Sennuwy, and the painted wooden coffin of the monarch Dehuti-nekht, both of the Middle Ethiopian Monarchy (900–300 B.C.), the other half of which is in Khartum.

Sessions of Thursday, October 6, 1921.

President Moore called the assembly to order at ten o'clock.

Dr. Arthur Ernest Cowley, of Oxford, Librarian of the Bodleian Library, spoke upon the Hittite hieroglyphic inscriptions. He believes that they belong to the ninth and eighth centuries B.C., and that their language is connected with that of the inscriptions of Van, the ancient Armenian tongue.

Dr. Cowley laid stress on the distinction between these and the earlier cuneiform Hittite texts. We cannot assume without proof that the language of the Carchemish inscriptions is the same as that of the cuneiform tablets of Boghaz-keui. Nor can we even be sure that the signs always have the same values and conceal the same language at Tyana and Marash, for instance, as at Carchemish. Still we may continue to call the inscriptions Hittite, since the Assyrians spoke of the king of Carchemish as Šar mat Ḫatti, and since the king of Carchemish also called himself by a similar title, Lord of Ḫana, ruler of Ḫattina, according to Dr. Cowley's decipherment. Ḫana, at the confluence of the Ḫabur with the Euphrates, and Ḫattina, the district to the west of Carchemish, are mentioned together in the Cappadocian texts just published by Mr. Sidney Smith for the British Museum. These places were on the caravan route to Babylon, and Carchemish was bound to keep possession of them.

In the inscriptions of Carchemish three successive kings are named, and the last inscription mentions a name which is deciphered as Sarduris. This must be Sarduris II of Van, who had various dealings with the Hittites, as is known from the Vannic inscriptions. His date, and therefore the date of the last of the three kings of the Carchemish inscriptions, is about 750 B.C. Other indications corroborate this conclusion, so that the dates of this group of texts may be taken to fall between 850 and 750 B.C. They are thus contemporary with the inscriptions of the neighboring kingdom of Van, with which also there seems to be some linguistic connexion. Several comparisons were made with Vannic grammar and vocabulary.

Mr. Alexandre Moret, of Paris, Conservateur of the Musée Guimet, speaking in French, then followed. The title of his paper was: L'accès de la plèbe aux droits religieux et politiques en Egypte.

A visit to the splendid Egyptian galleries of the Boston Museum of Fine Arts shows what a contrast there is between the funerary monuments of the Old Empire and those of the Middle Empire. Among the former, the superb statues of king Mycerinus and his family are most notable; among the latter, the magnificent coffins of private individuals. This change implies nothing less than a religious and social revolution. Under the Old Empire (3000 to about 2600 B.C.), the king admits to religious and administrative functions only his relatives, friends, courtiers. And to them alone he accords participation in the funerary rites which assure survival in the other world. In Egypt, as in Greece and Rome, religious rights blend with political rights. To play a rôle in society, one must take some part in the religious rites of which the king, son of the gods, god himself, is the sole dispenser among men.

Beginning with the Middle Empire, about 2000 B.C., all is changed. The funerary monuments, by their character and increasing number, make it evident that every man has meantime gained access to the much-valued religious and funerary rites. Every man, no matter whether he be a plebeian or of the royal family, whether favored by the king or not, may now possess a tomb, a coffin, a stele, may have the attributes of a king in the other world, and may claim as such to bear a sceptre and to wear a crown and the royal apparel. These things are depicted upon the sides of the coffins of common people. The sacred rites which were formerly known only to the king are now known to all. From a religious point of view, society has become quite democratized.

Political and civil rights also have in the meantime been won by the common people. This appears from the steles and from the papyrus-texts of administrative and literary contents. These show that the royal administration now concedes to every man the right to enter upon a public career, to hold land for burial-places, and to use and dispose of royal lands (subject to the king's right of eminent domain), and the right to independent commercial and industrial activity (not, for instance, in the royal workshops alone), and to have recourse to the royal tribunals of justice by right of petition, formerly accorded only to the higher classes. Society has been levelled under a monarchy which, although of divine right, has become democratized.

This rise of the common people, in the period between the Old and the Middle Empires (say from 2800 to 2000 B.C.), was not brought about without violent crises, which, as in Greece and Rome, wear the aspect of a social revolution. A description of these changes may be found in the texts which Professor James H. Breasted has commented upon and coördinated in the seventh chapter of his *Development of Religion and Thought in Ancient Egypt*. The beautiful coffins of the time of the Middle Empire attest the results of the social and political struggles involved.

Dr. Stephen Langdon, Professor of Assyriology at Oxford, presented the results of his studies upon the Babylonian Poem of the Righteous Sufferer. His reconstruction of the poem upon the basis of tablets from Niniveh and Sippur and Assur shows striking resemblances to the Hebrew Book of Job.

Dr. Langdon announced the recovery of several new texts which supply missing sections of the Babylonian poem. It now appears that the poem consisted of four books, each of about 120 lines, written in strophes of ten lines each. The book was written by an orthodox poet of the ninth century B.C., as an apology or defence of traditional theology against the current pessimism and skepticism of the time. This legend of a righteous and orthodox man unjustly afflicted with poverty and disease, originated in Sumer, and was known as early as the twenty-fifth century.

The Righteous Sufferer was a resident of Nippur, named Lalur elimma, "Good is the protection of Enlil." The Semitic poem, as now reconstructed from texts of the late period, utilizes some old Sumerian legend which has not been recovered. The poem contains a detailed statement of the pessimism of the day, and the orthodox reply thereto. The Righteous Sufferer challenges the justice of God and the ways of providence. The good suffer and the wicked prosper. Strict observance of the rituals availed not. The priests of the mysteries and divination failed to avert the afflictions sent by the gods. This righteous man had committed no sin, and yet he was daily visited by divine punishment. Death is therefore preferable to life, and labor in the service of religion is futile.

After a long account of the current pessimism as illustrated by Lalur elimma's bitter complaint, the poet refers to the orthodox theory of rewards and punishments. Affliction is a certain indication of sin. If the sufferer has committed no offense against God, then his ancestors must have done so. The orthodox theory of original sin is expounded, and emphasis is laid on man's ignorance and God's impenetrable wisdom. Across the gulf between God and man, only prayer and ritual elicit a reply. Faith in the orthodox rituals finally triumphs over skepticism, and the Righteous Sufferer receives a revelation by divination, and sees that his virtue will soon receive its reward. He is restored to health and prosperity, and the poem ends with a long hymn of praise to Marduk, god of Babylon, who intervened and delivered the believer. This later element of the poem shows that the work was finally issued from the school of the priesthood at Babylon, who redacted all the older poems in like manner to glorify their patron deity Marduk.

The *Poem of the Righteous Sufferer* forms one section of Mr. Langdon's volume, *Babylonian Wisdom*. This will contain also the recently recovered *Dialogue of Pessimism* and the *Books of Proverbs*.

On behalf of The Omar Khayyam Club of America, its President, Mr. Lanman, presented to each one of the Delegates a copy of Mr. Burrage's three volumes, to wit: 1. his "Twenty Years of The Omar Khayyam Club of America," (Boston, 1921); 2. his "Exact Facsimile of the rare and famous first edition of Edward FitzGerald's Rubaiyat of Omar Khayyam, the Astronomer-poet of Persia. Translated into English verse. London: Bernard Quaritch, Castle Street, Leicester Square, 1859"; and 3. his miniature edition of "The Rubaiyat of Omar Khayyam of Naishapur."[1] In presenting these gifts, the speaker said, for substance, somewhat as follows:

It would indeed be a doubtful compliment to give to you, in the House of this venerable Academy, the works of a sot and a materialist. Such a one, as Mr. Burrage observes,[2] many people suppose that Omar was. This belief is far from the truth. Like Demokritos of Abdera, Omar was one of the most learned men of his day, and with that learning went a deep religious conviction and feeling which we may not lightly deny. If any incline to doubt it, we may well ask them, Why did Cowell, who was the Professor of Sanskrit at Cambridge and Edward FitzGerald's teacher and friend,— Why did Cowell urge his pupil to the work of translating the Quatrains and aid him in the doing? For Cowell was one of the most devout Christians that ever combined learning with unaffected piety.

You, gentlemen, who, representing the Asiatic Societies, are today here present as duly accredited Delegates to the American Academy, are the guests of the Omar Club. It would ill comport with the dignity of the Academy if she should turn you over to the hospitality of a Club organized for mere conviviality. Happily, such is not the case. Its members do indeed set store by good-fellowship; but they have endeavored,— notably through the

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[1] This last is about 1 inch by $2\frac{3}{4}$ in size, and is an edition of twenty copies bound in full blue morocco, hand-tooled in gold, with inlays of red and green morocco, with jade jewel inset, and put in a case, and privately printed by the Rosemary Press for the Omar Club. A leaf following the title reads: "Dedicated by The Omar Khayyam Club of America to its guests, the Delegates of the *Société Asiatique*, *Royal Asiatic Society*, *Società Asiatica Italiana*, and *American Oriental Society*, as a souvenir of their Joint-meeting with the *American Academy of Arts and Sciences* at Boston, in October, 1921."

[2] "Twenty Years," page 17.

labors of Eben Francis Thompson,[1] the founder of the Club,— to earn the respect and gratitude of scholars and men of letters, by making possible a right estimate of Omar as mathematician and teacher and poet, and by setting in a true light the relations of FitzGerald's consummate poetry to its Persian original.

In his "Quatrains from the Greek," Walter Leaf speaks of "the pathos of human life, its vanity and vexation, its brevity and uncertainty, with the background of 'the veil through which we cannot see' and the recurrent refrain, 'Let us eat and drink, for tomorrow we die.'" He adds that "the genius of FitzGerald has given us . . . what is, for our own day, a classical form for this poignant theme."

FitzGerald himself, in his once despised first edition (page xiii), says of Omar's poetry: "Any way, the Result is sad enough: saddest perhaps when most ostentatiously merry: any way, fitter to move Sorrow than Anger toward the old Tentmaker, who, after vainly endeavouring to unshackle his Steps from Destiny, and to catch some authentic Glimpse of Tomorrow, fell back upon Today as the only Ground he got to stand upon." FitzGerald's presentation of what seemed to him the essential features of Omar's philosophy of life has attained (as witness the editions and translations — for number, they pass belief) a popularity in which some would see a sign of the decadence of the age. Rather, let us look at Omar,— as that man [2] would have us do of whose loving labors and of whose gladness in gladdening others these books are the fruit,— let us look at Omar as one who would teach us the lessons of courage and hope and contentment and self-reliance, as one whose lessons, superimposed upon "the will to believe," shall teach us to make the most of the present through love of home and of country and of God.

At the close of the Session, the company took luncheon at Young's Hotel, and spent the afternoon visiting places of historic interest in the environs of Boston, such as Lexington and Concord. In the evening, it met again, informally, in Cambridge, at the house of Professor James R. Jewett of Harvard University.

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[1] In his "Quatrains of Omar," collected and translated, and in his "FitzGerald's Omar," with a Persian text and close prose translation.

[2] Burrage, in "Twenty Years," page 101.

SESSIONS OF FRIDAY, OCTOBER 7, 1921.

The meeting was called to order at ten o'clock.

Dr. Nies, President of the American Oriental Society, gave an account of the Society's plans for the establishment of a School of Living Oriental Languages, and of its recent steps for enlarging its resources with a view to more extensive publication of works upon the Orient.

Professor Pelliot spoke upon Native and Foreign Scholarship in the field of Sinology, with an account of his explorations in Chinese Turkestan from 1906 to 1909, and in particular of the Grottoes of Touen-houang, and of the vast importance of their contents for the future investigation of the history of China.

Mr. Shuttleworth described a hill-festival in the Western Himalayas, and illustrated his description with pictures from his collections.

At the close of the formal Sessions, the afternoon hours were left unassigned, in order that the guests might use them for further study of the Egyptian and Japanese Galleries of the Museum, and for other similar visits.

A farewell gathering was held in the evening, in the Æsculapian Room of the Harvard Club. Here dinner was served, Mr. Lanman presiding. Brief addresses were made by President Lowell of Harvard, by Dr. Cowley of Oxford, by Professor Pelliot of Paris, by Mr. Burrage of Boston, and by Professor George Foot Moore of Harvard. The dominant note of these utterances was that of satisfaction over the opportunity which such meetings offer for personal acquaintance among the workers in these fields, and for mutual sympathy and encouragement.

BIOGRAPHICAL NOTICES.

JOHN WALLACE BAIRD (1869–1919).

Fellow in Class II, Section 3, 1916.

In the untimely death of John Wallace Baird American science and particularly his science, psychology, have suffered a grievous loss. Born at Motherwell, Ontario, May 21, 1869, of Scottish parents, Baird early learned the virtues and rewards of self-reliance, devotion to duty, coöperation and loyalty.

He was one of twelve children, all of whom lived to celebrate the fiftieth anniversary of the marriage of their father and mother. Charles Baird, the father, was an industrious and successful Canadian farmer of sterling worth in home, church and state. The mother, Agnes Browning, possessed exceptional patience and wisdom and rare skill as singer of old ballads and narrator of family and neighborhood traditions.

Undoubtedly our colleague's essential education was gained at home. His formal education began in the "little red school house" and ended in the university. His undergraduate work in the University of Toronto was interrupted by years of school teaching, which enabled him to pay his own way. After graduating from the university in 1897, he devoted himself almost uninterruptedly to the study of psychology, first in his own university and subsequently in Leipzig. For two years he was a fellow in psychology at the University of Wisconsin and for one year at Cornell, where in 1902 he was granted the degree of doctor of philosophy. Thereupon he was appointed assistant in psychology at Cornell. After one year, this appointment was followed by that of research assistant in psychology, Carnegie Institution of Washington. At the end of his year's work as Carnegie research assistant, Baird accepted his first major academic appointment, that of instructor in psychology, Johns Hopkins University. This was followed after two years' service by appointment to an assistant professorship in psychology in the University of Illinois. In 1910, at the end of his fourth year in Illinois, he accepted an assistant professorship in psychology at Clark University, where, in 1913, he was given the status of professor of psychology.

Ever a serious minded and diligent student, Baird, at first alternately and later simultaneously, studied and taught until he had mastered his chosen profession of teacher and investigator in psychology and had achieved academic position of eminence, influence and rare opportunity for service. His appointment as Carnegie research assistant in psychology gave him an excellent chance to demonstrate his originality, resourcefulness and enthusiasm for research. The result of the year's work was a report on "The color sensitivity of the peripheral retina," which stands as his most important publication, for subsequently he gave himself increasingly to his advanced students and his research was conducted largely by and through them.

During nine years of fruitful service to Clark University, John Wallace Baird achieved immortality by training many able students for psychological research. With a genius for friendship he pursued his path of duty and opportunity single mindedly, whole heartedly and with entire forgetfulness of self. Honors he never sought; praise he shrank from. His students learned to respect, admire and love him because of his devotion to their interests and the obvious sincerity of his belief in constructive work in teaching and research.

In April, 1917, despite physical unfitness for the strain of work in Washington, Baird undertook to assist in directing the work of the committee for psychology of the National Research Council. In December he was compelled to go to the Johns Hopkins Hospital for treatment of a recurring malady and there, on February 2, 1919, in his fiftieth year, his life ended.

The career of our lamented colleague is a consistent lesson in unselfish and loyal devotion to family, friends, country and human welfare. His life clearly was sacrificed to duty, for his fatal illness with its frequent and long continued periods of acute pain resulted from a terrible ordeal of strength in which he saved others from a watery grave. The sort of sacrifice which he willingly made at a time of exceptional anxiety and risk, he continued to make throughout his life, defying bodily suffering and manfully doing his full duty until the end.

John Wallace Baird's life of generous service has received fitting tribute of respect and affection in a memorial volume bearing his name published by Clark University.

ROBERT M. YERKES.

ARLO BATES (1850–1918).

Fellow in Class III, Section 4, 1900.

Arlo Bates was born in East Machias, Maine, December 16, 1850, the son of Dr. Niran, and Susan (Thaxter) Bates. The strange name that was given to him was due to a family idiosyncrasy which originated with his grandfather, who had a theory that every man should have "a name that is all his own and nobody else's." So he called his son Niran; Niran upon the same principle begat Arlo, and Arlo in his turn begat Oric. You will search the dictionaries of biography and mythology in vain for any of these names, they are merely combinations of letters with no significance, but unique and calculated to impress the bearer from birth with the sense of individuality which was a family characteristic, not least strongly marked in the subject of this memoir.

He was educated at Bowdoin College, where he received the degree of S.B. in 1876, A.M. in 1879, and Litt.D. in 1894. Already while an undergraduate his strong instinct for literature as a profession began to manifest itself, he became the editor-in-chief of the college paper called The Bowdoin Orient, and thus started on his chosen career. Shortly after his graduation he resolved to try his fortune in Boston, and moved to the city in the same year, to remain a resident of it until his death on August 26, 1918.

His first venture in Boston was a paper which he named The Broadside. This led an unprosperous existence during the years 1878–79 and was then abandoned. In the following year he was made editor of the Sunday Courier, a journal which had once been highly esteemed, when it numbered among its contributors many of those whose names were associated with the North American Review, but had sadly degenerated when he took hold of it, and was living then chiefly on its name and advertisements. Its proprietor gave him a free hand in the conduct of his own columns, with the result that during the thirteen years that he remained its editor he was able to restore it to something of its old prestige. It became again one of the accepted Sunday papers, and was extensively read for its editorials, literary reviews and notes on current topics, for all of which he was responsible and most of which he wrote himself.

The period of his connection with the Courier was, in one respect at least, the happiest and also the most unhappy in his career, both the result of his marriage. In 1882 he was married to Miss Harriet L. Vose, of Brunswick, Maine, who under the name of Eleanor Putnam was a well-known magazine writer, and the author of a book on Old Salem. Their union was a singularly ideal and sympathetic one, sharing as they did to the full their intellectual tastes as well as their devotion to each other, but after only four years of this companionship she died, and to the end of his life he never ceased to mourn her. She left him one child, Oric, to whom his affection was transferred and centered more and more as the boy grew up.

His literary career began soon after his arrival in Boston. His first attempts were not successful in finding a publisher, but not discouraged by this experience he persevered, and in 1881 published his first novel, "Patty's Perversities." For the next twenty-seven years he continued a fairly regular output of novels, poems and essays, in spite of his arduous professional labors. "Who's Who in America" for 1916–17, the last volume issued before his death, gives a list of twenty-three titles with their dates, ending with "The Intoxicated Ghost" in 1908. While these books won and held for him the respect of his literary associates they did not achieve the wide popularity for which he had hoped, and it was doubtless the disappointment at this result which led him to abandon writing during the last ten years of his life. He was slow to recognize that his real strength lay not in fiction or poetry but in essays. Of these he published only three volumes, the two series of "Talks on Writing English" and the "Talks on the Study of Literature," all of which are of permanent value, and delightful reading because of his critical ability, his high standard of purity in the use of our language, and his exhilarating freedom of thought and expression.

In 1893 he resigned his position on the Sunday Courier to accept the professorship of English literature in the Massachusetts Institute of Technology. There he entered enthusiastically upon the most difficult task of his life and the one in which he achieved his greatest success. To imbue a lot of young students who went to the Tech to fit themselves for the most practical professions, with little or no time, as they thought, for "ornamental" studies, to imbue them with the sense that ability to express themselves in clear sound English should be an essential element of their training, and that the basis of this should be a knowledge and appreciation of the masters of their language, was no easy matter in that atmosphere of practical work, but he did it. The testimony of many students who sat under him, as well as the commendation of his associates on the faculty, leave no doubt of that, and it is still further shown by the fact that many who were not regularly connected with the Institute enrolled themselves as special students in order to have the advantage of his teaching. During the twenty-three years that he occupied his position he maintained it on a level with the best teaching in any of our universities, and he had the satisfaction of knowing that the seed he had planted was bearing good fruit. But there was another and a darker side to the picture. The period of his service in the Tech was the most turbulent in its history. Controversies arose, spread and would not down. Questions of policy, administration, the possible union with Harvard, every kind of problem that can disrupt a governing board, were discussed, not always with academic calm. Divisions of opinion were sharp and sometimes bitter. Into these he threw himself whole-heartedly, strong as always in his convictions, and vehement in his expression of them. Even before the clouds rolled away he found himself in a minority, out of sympathy with the new spirit that was growing in the institution in spite of his efforts, distrustful of its changes, and unwilling as always to compromise. Disheartened at the outcome as well as by the slight prospect of continued usefulness under the conditions that had thus been brought about, he retired in 1915, shortly before the Tech moved from Boston to Cambridge, and three years before his death.

Thus another disappointment was added to his life, and if I seem to dwell unduly upon these it is because they are essential to a knowledge of his character and its development. Highly sensitive as he was, and of a temperament that was naturally prone to melancholy, he was less fitted than a more robust personality would have been to withstand these slings and arrows, for as such he regarded them.

No account of Arlo Bates would be complete without at least a reference to his association with the Tavern Club, where for twenty years it is hardly too much to say that he was the life and soul of the club, contributing to an extent equalled by few others towards the distinctive character which gave it its reputation. Always ready to prepare a skit, a burlesque, a miracle play, or any kind of original entertainment, and equally ready to take part in any or all of them, the "moroseness" which many who did not know him well regarded as characteristic was there shown to be merely skin deep and easily punctured. Some of his wittiest and most brilliant work was done in the plays which he wrote for the club, and it is a pity that there was so small a public to enjoy them. But those who had the privilege will never forget it nor the affection in which they held him.

He was elected a Fellow of this Academy March 14, 1900, and a Member of the National Institute of Arts and Letters in 1904. An account of his life and work, with tributes from various sources, was published in the Technology Review for November, 1918, Vol. XX, pp. 615 ff.

EDWARD ROBINSON.

CHARLES PICKERING BOWDITCH (1842–1921).

Fellow in Class III, Section 2, 1892.

Charles Pickering Bowditch was born in Boston on September 30, 1842, and died in Jamaica Plain on June 1, 1921. He was the son of Jonathan Ingersoll Bowditch and Lucy O. (Nichols) Bowditch. He entered Harvard College in 1859 and was graduated in the Class of 1863 after having been suspended for his participation in some college pranks. He received the Master's Degree in 1866.

As a member of the Presidential party he witnessed Lincoln's First Inauguration on March 4, 1861. He served in the Civil War as 2d Lieutenant, 1st Lieutenant, and Captain in the 55th Massachusetts Volunteer Infantry and later he was Captain in the 5th Massachusetts Volunteer Cavalry of which his brother, Henry, was a Major.

He spent the year 1865 in the oil regions of Pennsylvania and from 1865 to 1872 he was in charge of the Estate of William W. Wadsworth at Geneseo, New York, and from 1866 to 1872 he was Trustee of the Estate of Allen Ayrault at the same place. He returned to Boston in 1872 and, except for periods of travel in Europe, the Orient, Mexico, Central America, and California, he resided in Boston until his death.

Mr. Bowditch's grandfather, Nathaniel Bowditch, was the Fifth President of the American Academy, serving from 1829 to 1838 and succeeding John Quincy Adams as President. His father, J. Ingersoll Bowditch, was Treasurer of the Academy from 1842 to 1852. Mr. Charles P. Bowditch was elected a member of the Academy in 1892 and was its Treasurer from 1905 to 1915 and President from 1917 to 1919.

He was also a member of the following societies: Boston Society of Natural History, serving as Vice President from 1895 to 1907, the American Geographical Society, the American Antiquarian Society, and numerous American and European Anthropological organizations. His historical-genealogical interests are shown in his membership in the Massachusetts Historical Society, the Bostonian Society, the Colonial Society of Massachusetts, and the New England Historical-Genealogical Society. Each of these institutions is indebted to him for generous support. His varied interests are shown by this list of organizations of which he was a member. As a man of affairs he was an officer in many corporations and numerous benevolent enterprises and a Trustee of many estates.

After a pleasure trip to Mexico and Yucatan in 1888 his main avocation was the investigation of Central American antiquities and, more especially, the Maya system of hieroglyphic writing. In this study he was the most outstanding figure in America. His book, "The Numeration, Calendar Systems, and Astronomical Knowledge of the Mayas" (1910), was the most important book published up to that time on the Central American hieroglyphic writing. He added much to the knowledge of this subject and blazed a trail which will always remain open to future students of this subject. This book, together with numerous pamphlets, show the results of an acute mathematical mind and most painstaking study. He was a worthy foe to speculative theories and his deductions are based on mathematical calculations and sound common sense.

Mr. Bowditch's connection with the Peabody Museum of Harvard University was a long and a close one. From 1888 to the time of his death he was its greatest benefactor. In 1894 he became a Trustee of the Museum and always took the greatest personal interest in the welfare of the institution and its varied activities. His patronage of the Central American work of the Museum covered many sides. He financed and planned annual expeditions to the Maya field, beginning in 1891 and continuing in an almost unbroken series down to the present time. The scientific results of these expeditions were published, for the most part at Mr. Bowditch's expense, in six folio volumes of Memoirs and several volumes of Papers. The collections acquired by these expeditions now fill the greater part of two large halls in the Museum. He brought together a large library of the books and manuscripts relating to Mexico and Central America which he gave to the Museum in addition to over fifty thousand pages of photographic reproductions of early manuscripts and rare books on the history and languages of these countries. He established Fellowships in Maya research in the Archaeological Institute of America and in the Peabody Museum. He was in great part responsible for the establishment of the teaching of Anthropology in Harvard University.

Mr. Bowditch's patronage of the study of Central American antiquities was a patronage based on personal investigations, study, and an intimate knowledge of this field. American Anthropology has perhaps no other case where an effort in one field of interest has been so long continued, so intense, and so productive of results.

A list of the published and unpublished articles written by him together with a list of his editorial work is printed in the American Anthropologist (N.S.) v. 23, 1921.

ALFRED M. TOZZER.

SETH CARLO CHANDLER (1846–1913).

Fellow in Class I, Section 1, 1883.

Seth Carlo Chandler was born in Boston, Mass., on September 16, 1846, the son of Seth Carlo and Mary (Cheever) Chandler. He died on December 31, 1913, after a career of remarkable achievement.

As a boy he developed early, showing a fine combination of mental and practical capability. While still attending the English High School of Boston, he was chosen to perform some computations for Professor Benjamin Pierce of Harvard University. This circumstance seems to have developed Chandler's mathematical bent and led him, after graduation in 1861, to become the assistant to the distinguished astronomer, Dr. B. A. Gould. While the other lads of sixteen years may have pursued collegiate courses, Chandler found in Gould his university. Here was the beginning of a life-long friendship. Under Gould he worked with the title of Aid in the U. S. Coast Survey from

1864 to 1870. On October 20, 1870, he married Caroline Margaret Herman of Boston, who with several daughters survives him. Dr. Gould was now in the Argentine Republic, founding the national observatory at Cordoba. Chandler had declined Gould's invitation to go with him, possibly having in view his impending marriage. Feeling now the need of more lucrative employment than afforded by science, he became life insurance actuary from 1870 to 1885. Here his mathematical ability discovered various interesting laws. For example, he derived an accurate formula showing the distribution by age of applicants for life insurance.

In 1881 he moved to Cambridge and took part in the work of the Harvard Observatory. In 1886 he became a private investigator, or as he called it an "amateur" astronomer. In 1904 he removed to Wellesley Hills, Mass., where he lived until his death. To give any adequate account of his scientific work is impossible in this sketch. It is to be found in more than 200 papers published chiefly in the Astronomische Nachrichten, the Astronomical Journal, and the Annals of the Harvard College Observatory.

His Almacantar, an equal-altitude instrument floating in mercury, gave results of greatest precision, and furnished him with the first intimations of changes in latitude. To the series of masterly papers by Chandler on the variation of latitude, appearing in 1891–1894, Professor H. H. Turner has rendered a magnificent tribute in his book "Astronomical Discovery." No better bird's-eye view can be found of this great discovery, so contrary to the accepted opinions of the astronomical world at that time. Chandler's courage and sound practicality are shown in these words, written in 1893. "It should be said, first, that in beginning these investigations last year, I deliberately put aside all teachings of theory, because it seemed to me high time that the facts should be examined by a purely inductive process; * * * and that the entangled condition of the whole subject required that it should be examined afresh by processes unfettered by any preconceived notions whatever. * * * I am not much dismayed by the argument of conflict with dynamic laws, since all that such a phrase means, must refer merely to the existent state of the theory at any given time."

Facts won against theory. With great industry he skillfully coördinated thousands of observations and proved conclusively that the changes in latitude occurred according to two superposed oscillations, one of fourteen months, the other of a year. With this key he unlocked many mysteries of the past. His work harmonized the Washington observations for latitude. He "added a hundred feet to Bradley's monument," and showed that Pond's apparent errors attested the excellent quality of his observations. Variation of latitude explained also the difficulties experienced by Airy with his Reflex Zenith Tube.

For these researches he received in 1895 the Watson Gold Medal of the National Academy of Sciences, and in 1896 the Gold Medal of the Royal Astronomical Society. The latter was given also in consideration of Chandler's work on variable stars. These objects were a favorite study of his. Three successive catalogues of variables, prepared by him, may be mentioned.

He was editor of the Astronomical Journal from 1896 until, on account of ill-health, he resigned and became associate editor in 1909. Among his numerous activities, Chandler was interested in the transmission of astronomical intelligence by telegraph. He devised the "Chronodeik" for determining the time. He studied cometary orbits, and made computations which led to the discovery, in coöperation with the Harvard Observatory, of the position of the small planet Eros on photographs made at the observatory four years before the planet was known to exist.

Chandler was elected a Fellow of this Academy in 1883. For his Almacantar he was awarded in 1884 the medal of the Massachusetts Charitable Mechanics Association. He received in 1891 from De Pauw University, Indiana, the honorary degree of LL.D. He was a member of the National Academy of Sciences, and a Foreign Associate of the Royal Astronomical Society. He was a member of various other scientific associations. For many years he served efficiently on the Gould Fund Committee of our Academy; also on the Bache Fund Committee of the National Academy.

Dr. Chandler possessed what may seem unusual in a scientific mind, keen business judgment. He was able to gauge the underlying financial conditions accurately, so much so that his advice in such matters carried weight with friends engaged directly in the business world. This was of great service to him in his affairs relating to trusteeship, the duties of which he faithfully and efficiently discharged.

He was very fond of good music of the old school, Beethoven, Gounod, and Verdi being among his favorites. He did not care for the modern composers, and never listened to their work if he could avoid it. A forced hearing of Debussy or Brahms, it is said, was sure to bring forth some humorous but scathing criticism at the finish. His contention was that music was for the pleasure of the senses only, and attempts to make it appeal to the intellect were disastrous. He read almost everything except the modern novel. This he was apt to class as "cheap stuff." Relaxing from his scientific work, he would become absorbed in some other subject, for instance the American Civil War, and read volume after volume about it. In French, Dumas was a favorite author. In such periods of reading Renan's Life of Christ, Saint Paul, and the Bible would come in close succession. Tales of adventure, detective stories, history, and biography as related to history appealed to him.

Dr. Chandler was devoted to his family and their interests. Although the family dinner table was a large one and he might be much preoccupied, having been called several times before responding, it is said that he never failed to notice instantly the absence of any member of the family group. He found much satisfaction in restoring his grandfather's homestead at Strafford, Vermont, where he enjoyed the long summer vacations with his family. The writer, who spent a summer some years ago at Strafford, remembers the delightful way Dr. Chandler entered into the community life. His daughters with others had become interested in dramatics, and he took great pleasure in conveying the "band of strolling players" as he called them, over to neighboring villages to give performances.

He was fond of driving, preferring horses to an automobile. "Not to own a machine" he said, "from being a proof of aristocracy had now become a mark of respectability." He enjoyed books on magic and gave sleight of hand exhibitions at Strafford. The "Old City Wizard" was a title bestowed upon him in those days. It was a pastime of his at Strafford to design, make, and sail beautiful little yachts, two or three feet in length. He used to say that he was sailing the exact model of one of the "Cup Defenders" years before she was launched. To a clever local mechanic who assisted him in some parts of the construction, it was a wonder that the models should have the exact displacement predicted by their designer. Such was his life at Strafford. To live out of doors in the country with his family was an ideal vacation for him.

Dr. Chandler was an entertaining conversationalist, and a delightful companion, an English astronomer remarking that it was worth crossing the Atlantic to visit him. He was cordial and constant in friendship, so unpretentious that many who met him only in later life were unaware that he had any claim to distinction. To those who know his work, "he has left the remembrance not so much of mere talent as of positive genius." The creative power of his intellect combined with courageous and unflagging industry produced a record of notable achievements, linking his name inseparably with the history of astronomy of his time.

EDWARD S. KING.

ELIOT CHANNING CLARKE (1845–1921).

Fellow in Class I, Section 4, 1887.

Mr. Clarke was born in Boston on May 6, 1845, and was the son of the Rev. James Freeman Clarke and Anna H. Clarke. His father was one of the most distinguished Unitarian clergymen of his day, a leader in thought, and the author, among other things, of a book entitled "Ten Great Religions," which occupies a high place in theological literature.

The first ten years of Mr. Clarke's life were passed in Boston and in Meadville, Pa., the home of his mother's family. In 1855 his parents settled at Jamaica Plain. He was educated in the public schools, preparing for college at the Eliot High School, and was graduated from Harvard College in 1867. He was Chief Marshal of his class. He took some special studies at the Massachusetts Institute of Technology in 1867–68, and in February, 1868, he began his career as a civil engineer on the bridge then building over the Mississippi River at Quincy, Illinois. His uncle, Thomas Curtis Clarke, was a noted civil engineer, a member of the firm of Clarke, Reeves & Co., Bridge Builders, of Phoenix, Pa., and in 1896 President of the American Society of Civil Engineers. Mr. Clarke's firm was building the bridge over the Mississippi River and the work offered a good opportunity for the nephew to begin his engineering experience. Later, he was engaged upon other engineering works, viz., the bridge over the Mississippi River at Hannibal, Mo., and other structures built by the Phoenix Co., the Chicago Water Works Tunnel, and the Chicago Sewerage System.

In the dull times which followed the Panic of 1873, Mr. Clarke returned to Boston to take further special studies at the Institute of Technology in 1875–76. In July of the latter year he was appointed Engineer in charge of a survey for a main drainage system for Boston. The project was adopted and construction was begun in 1877. It was carried through to completion in 1884 under the supervision of Mr. Clarke, who published a description of the work in 1885. At this time he was recognized as one of the leading sanitary engineers of the United States. In 1885 he received the Norman Medal of the American Society of Civil Engineers for his paper entitled "A Record of Tests on Cement Made for the Boston Main Drainage Works." In the work which this paper describes a great deal of cement had been used and Mr. Clarke had made some novel and valuable experiments. Among other things, he was one of the first to prove and to advocate the importance of fine grinding of cement, showing that the coarse grains had very little cementing quality. In 1884 he became Chief Engineer of the Massachusetts Drainage Commission, which was appointed to design methods of preventing pollution of the waters of the Charles, Mystic and Blackstone River basins.

Shortly after this time he gave up his strictly engineering work to become the Manager of mill properties at Lowell, to which his attention was devoted for a number of years. His retirement from engineering was a distinct loss to the profession.

Mr. Clarke was a man of wide interests. He was a Fellow of the American Academy of Arts and Sciences, and its Treasurer for eleven years. He was a member of the Massachusetts Natural History Society, the Massachusetts Horticultural Society, the Colonial Society, and the Corporation of the Massachusetts Institute of Technology. He served also as a trustee of the Massachusetts School for the Feeble Minded, as trustee and vice-president of the Provident Institution for Savings, as director of the State Street Trust Co. and of other companies. He was interested in astronomy and prepared a work on that subject.

Mr. Clarke was married in 1878 to Alice V. Sohier, by whom he had five children, three of whom survive him. He is also survived by one sister, Miss Lillian Freeman Clarke.

Mr. Clarke was a man of great engineering ability, a clear thinker, an efficient organizer, a good administrator and a most lovable man. Before leaving his chosen profession for business, he had reached a position of preeminence in it, and in his business career he showed the same rare qualities. A host of friends mourned his passing.

George F. Swain.

WILLIAM GILSON FARLOW (1844–1919).

Fellow in Class II, Section 2, 1874.

It is certainly presumptuous for one not a botanist to write about Dr. Farlow, but an intimate friendship of over 50 years makes it possible for me to speak of him as a man, and I hope I have succeeded in presenting an adequate picture of his botanical achievements by constructing a mosaic from the facts and opinions of the four experts, who have written about him already.

By the death of William Gilson Farlow the Academy has lost one of its most distinguished fellows, since it was his rare good fortune to begin his scientific work, when a great body of material had been collected by such pioneers as Curtis in fungi, Harvey in algae, Tuckerman in lichens, Sullivant in mosses and many others, and the science in this country had reached the point where it needed some man with breadth and grasp enough to draw all these scattered parts into a connected whole. In Farlow it found the genius, enthusiasm and character needed for this great work and the training, which developed and supplemented these natural gifts. He occupies, therefore, the same commanding position in cryptogamic botany that Asa Gray holds in the development of our knowledge of flowering plants.

He was born in Boston December 17, 1844, the son of John Smith Farlow, a prosperous public-spirited citizen and Nancy Wight (Blanchard) Farlow, both of Massachusetts parentage. From his father he inherited strong tastes for botany and music. In fact, John K. Paine, then recently established in Cambridge, urged him to become a professional musician, but the call of botany was too strong and music sank into a delightful recreation after his exacting scientific work.

He was educated at the Quincy Grammar School and English High School in Boston, followed by a year at the Boston Latin School, which with his training in Harvard College made him a sound classical scholar. In after years he was fully alive to the greater breadth of view and roundness of intellect given him by this training in the humanities, as well as to the value of his Latin and Greek in giving him a mastery of the force of scientific terms and names.

It is said that his attention was first drawn to botany when a boy by finding hepaticas in the woods near his father's place in Newton. However this may be, he made such rapid progress in the science that by his senior year we lower classmen spoke of him with bated breath as a prodigy of botanical learning.

This progress was largely due to the fact that in college he encounttered the first of his two great teachers — Asa Gray, who gave him a solid foundation for his later professional studies; but instead of embarking on these at once after his graduation in 1866, following the advice of Asa Gray, he took up the study of medicine, and after studying anatomy for a year under Jeffries Wyman — a man whose casual talk was a liberal education, he entered the Harvard Medical School in November, 1867, and graduated from it in 1870, securing before his graduation the coveted appointment of surgical interne at the Massachusetts General Hospital under that great surgeon, Henry J. Bigelow; so that he took up his higher botanical studies with a much more rounded general education than falls to the lot of most scientific men, the effect of which could be traced throughout his life in his unusual sanity and breadth of view.

In 1870 he began his special botanical education by serving for two years as assistant to Asa Gray, whose inspiring teaching gave him a comprehensive knowledge of flowering plants and made him thoroughly familiar with the systematic outlook on the science, while his example helped to make him a botanist in the broadest sense of the word, instead of a mere specialist on the cryptogamic side.

Even at this early day he had selected his line of work — the cryptogams — and began to study the algae in the herbarium at Cambridge and also in the field at Woods Hole, where in 1871 he joined a scientific party under S. F. Baird, publishing in this year his first paper "Cuban Seaweeds."

At that time there were no facilities in America for studying cryptogams, so again following the advice of Asa Gray, he decided in 1872 to go to Europe, where he spent the better part of two years in study at Strassburg under Anton De Bary, then the first authority on fungi in the world. He was very fortunate in working under two such men as De Bary and Asa Gray, each a master in his own field, and these lay so far apart, not only in matter, but in methods of treatment, that he gained from them a remarkably broad and comprehensive grasp of the science.

It was indeed a new botanical atmosphere into which he was plunged at Strassburg. Systematic botanists were spoken of scornfully as "hay collectors," and with the zeal of new converts most German botanists prided themselves on their ignorance of flowering plants. De Bary himself was not free from this sort of narrowness, but Asa Gray had impressed the importance of systematic work and flowering plants so thoroughly on Farlow, that he did not allow himself to be swept off his feet even by the flood of new ideas with which he was continually deluged by De Bary and the eminent students he had drawn about him, such as Graf Solms and Rostafinsky.

When I heard Farlow talk with De Bary in the Strassburg laboratory I heard two naturalists discussing the question on equal terms, except for the greater knowledge and experience of the older man — a very striking contrast to the state of almost abject pupilage, in which we chemical students were kept by our German professors. The difference lay, of course, in the students, not in the professors.

During his stay with De Bary he grew familiar with the whole field of work in the morphology and development of fungi and in plant anatomy, and toward the end of the time published a paper on "An Asexual Growth from the Prothallus of Pteris cretica," which was attacked so heartily that he returned to America with his reputation made.

Although the larger part of his time in Europe was spent with De Bary, he gave shorter periods of study to lichens under J. Mueller at Geneva, and to algae with Bornet and Thuret at Antibes, and travelled extensively, visiting most of the celebrated botanists and herbaria.

These years of study in Europe brought his education to an end and made him, so far as is known, the only American cryptogamic botanist capable at that time of doing original work himself and of teaching it to advanced students. More than this, they put the finishing touch to the cultivation of the qualities that made him great — his strong and piercing intelligence, his phenomenal memory, the discriminating judgment and devotion to truth that made him refuse to accept conclusions, until they were absolutely established, and the unusual breadth of view so often mentioned already. To these must be added an insatiable love of work, as well as great and constantly increasing stores of learning.

As soon as he reached America his appointment as Assistant Professor of Botany in Harvard University put him in a position to make the most of these treasures and to raise cryptogamic botany in the United States from a mere sketchy appendix in a general course of botany to the rank of an independent study.

For five years he was stationed at the Bussey Institution of Harvard University, although his teaching was in the college, and then in 1879 he was transferred to Cambridge as Professor of Cryptogamic Botany.

These five years at the Bussey Institution, however, had an important influence on his life as well as on the botanical development of the country, since they called his attention to the fungous diseases of plants, and he threw himself into work in this virgin field with such energy that he is acknowledged as one of the founders of phytopathology in the United States — a study which has since reached such proportions that we lead the world in it at present.

His papers in this field are numerous and important. Among them may be mentioned studies of potato rot, grape mildew, black knot, onion smut, gymnosporangia, fungous diseases of hollyhocks, roses, and even of salted codfish.

After he was settled in Cambridge, his plans included the establishment of a herbarium and a library of cryptogamic botany, in addition to the teaching and research properly belonging to his professorship.

The first step towards his herbarium had been taken even before his return from Strassburg, as then Asa Gray bought for him the famous collection of fungi made by the Rev. M. A. Curtis. To this nucleus were added later many other famous collections, which had been either bequeathed to the University, or purchased by him. Conspicuous among them were Tuckerman's lichens, Sullivant's, James's, and Kennedy's mosses and hepatics, Faxon's sphagna, and, quite as important as these, his own rich collections of fungi and algae. His father's wealth enabled him also to make his library — like his herbarium — the fullest and best in the country and both were always open to botanists qualified to use them.

As a lecturer he was peculiarly happy, for to the authority of a master he added a clear style, the faculty of bringing essential points into strong relief, and a humorous quality, which riveted the attention of his hearers; but even more important than his lectures was his work with students in research. Here his inspiring personal teaching — for he never left them to an assistant — developed many distinguished students, among them such masters as Roland Thaxter, William Trelease, W. A. Setchell, Kingo Miyabe, and Herbert M. Richards.

His scientific papers average three a year for the whole forty-five years of his active work, if the papers of his students are included, as they should be. They have been characterized as "clear, concise and accurate," the well-considered careful utterances of a master, who never yielded to the temptation of rushing into print, and also was chary of establishing new species out of the great wealth of material at his disposal, since he had a profound contempt for bad species, as shown by his caustic remark about the manufacturers of them.

"If a difference can be *imagined*, it is a new species; if it can be seen, it is a new genus."

Nearly two-thirds of his papers have to do with fungi, including the studies of plant diseases already mentioned; while somewhat less than one-third deal with algae, many of these with the contamination of water supplies by them. One published in 1879 was reprinted 38 years later by Professor Whipple as "one of the classics of state sanitation." It is written in a popular style, as are several of his papers on fungi.

Other useful papers consist of reports on the cryptogams collected by various exploring expeditions and of lists of cryptogams found in special localities. For example, a list of the seaweeds of the New England coast, (published in 1881) "included keys, descriptions, critical notes and plates," and according to Professor Riddle "still remains our only scientific manual of the seaweeds of this region."

Quite as important as this was the "Provisional Host Index of the Fungi of the United States" published in 1888 and 1890 with Mr. A. B. Seymour, which has proved of the greatest use to working botanists.

Of his bibliographical papers the most important was a "Bibliographic Index to North America Fungi," of which the Introduction and the first 312 pages, prepared in collaboration with Mr. A. B.

Seymour, were published by the Carnegie Institution in 1905 and although the publication has not been continued, the collection of data went on until his death. This was perhaps his greatest work, and some idea of its magnitude can be obtained from the facts that at the time of his death the Index included about 350,000 references, and the 312 pages published in 1905 brought the alphabetical list of that day only through Badhamia. As the work is essentially finished, it is hoped that it may soon be published since it will be of untold value to specialists.

Another great work left unfinished at his death is an account of selected species of American fleshy fungi, which was to be illustrated with over 100 colored plates. These have been executed under his direction with the utmost care, but although the plates were finished, the pressure of his other botanical undertakings prevented him from even beginning the descriptions of the species. It would be well, if these descriptions could be supplied by another hand, so that what promises to be a classical work may see the light.

In discussions of nomenclature he threw his powerful influence in favor of sane and stable methods for naming fungi, thus helping to check the extreme radicalism of many American botanists, and preserving relations with the better men abroad.

But his papers alone — important as they are — did not make him "the creator of cryptogamic botany in the United States." It was the man himself — his personality, his breadth, his wise conservative judgment, his learning, his helpfulness, and his devotion to truth.

American botanists were brought in contact with this commanding influence by means of his papers and his students and even more effectively by his direct personal intercourse with them, for he was always ready to give them generous help, and they were more than ready to make use of it, submitting their puzzles to his excellent judgment, reinforced as it was by a reading that covered the whole literature of the cryptogams, not in abstracts, but in the original sources; borrowing specimens from his herbarium for comparison; asking him to look up references in journals hard to find outside his library; or even sending him specimens to determine. Some idea of the volume of this work is given by the fact that a single correspondent confesses to 100 letters in his own handwriting.

His achievements earned wide recognition. Two genera were named after him, Farlowia among the algae, and Farlowiella among the fungi, beside a great number of species. Harvard, Wisconsin and Glasgow conferred the degree of LL.D. upon him, and Upsala on the two hundredth anniversary of the birth of Linnaeus, at which he represented our Academy, crowned him with laurel as one of its Doctors of Philosophy.

He was elected a fellow of our Academy in 1874, and was also a member of the National Academy of Sciences, the Philosophical Society, the Linnaean Society and the French Academy of Sciences. He served as president of the New England Botanical Club, of which he was one of the founders, of the American Association for the Advancement of Science; of the American Society of Naturalists, and of the Botanical Society of America.

There is little more to be said about his life, which was passed in Cambridge during term time, with the exception of a few journeys to Europe and botanical excursions on this continent, the most important being one to Mexico and California with Asa Gray in 1885. Most of his work in the field, however, was done in the White Mountains, where his vacations were usually passed. A walk in the woods with him introduced one to a new world largely microscopic, but full of interest and even beauty.

In 1900 his marriage to Miss Lilian Horsford made his life one of complete happiness until his death on June 3, 1919.

Farlow's most striking characteristic — apart from those I have already pictured in connection with his scientific work — was the humor which permeated and irradiated all he said, making even his common talk amusing and delightful, but this humor is so elusive that it evaporates between the pen and paper, so that I can refer to no example of it. On the other hand, his wit often flashes out in all but his most serious papers.

His hatred of affectation, sham and superficiality was intense, and his outspoken denunciation of them, driven home by his incisive humor, made so deep an impression that many, who knew him mostly by hearsay, thought him a sarcastic pessimist, but his friends saw that his attacks were directed only against those who deserved them, and knew that the real nature of the man was affectionate and kindly, making him the most staunch and faithful of friends, and the sympathetic helper even of those who had no claim on him whatever.

It is pleasant to think of his declining years when full of well-earned honor, happy in his troops of friends, happier in his family with his mind undimmed by any weakening of his faculties, he was able to continue in active work until the brief sickness, which brought the end.

CHARLES LORING JACKSON.

JULIUS VON HANN (1839–1921).

Foreign Honorary Member, Class II, Section 1, 1902.

Nearly half a century ago Julius von Hann began to take his place as the universally acknowledged leader of meteorological science, and for many years previous to his death he stood out head and shoulders above his fellow-workers. He grew up with and himself was, as it were, a large part of the rapid modern development of meteorological science. He was able, through his intense application and industry, and because of his great intellectual powers, not only himself to contribute largely to the advance of his science but also to keep closely in touch with all the work which was being done by investigators and writers everywhere. For years his many contributions to the *Meteorologische Zeitschrift*, often modestly signed J. H., were never-failing evidence of his truly extraordinary grasp of his subject and of the universal range of his reading. He was, as fully as any one human being can be, a living encyclopedia of his chosen science. And this is in no sense to be taken as suggesting that his mind was merely a storehouse of dry, hard facts. He was very human. He saw the many and varied relations of meteorology and climatology to human life and activities, and he was always on the lookout for opportunity to emphasize these relations. His writings were always clear, vivid, and interesting. His "Handbuch der Klimatologie," for example, which inevitably has to deal largely with "dry" statistical details, is enlivened throughout by carefully selected, vivid, first-hand descriptions of weather types and of human or botanical responses to the climatic environment.

His fellow-workers who remain are dynamic or physical meteorologists, or climatologists, or are specializing in this or that subdivision of their science. This is a natural and inevitable situation at the present stage of our knowledge of the atmosphere. It cannot well be otherwise. But it leaves a great gap which no one man can ever fill, because meteorology has now grown to such an extent that specialization is the rule, and no single mind will ever again master all of its details. Hann's "Lehrbuch der Meterologie" is the one absolutely indispensable textbook in that science. Upon his "Handbuch der Klimatologie" all studies of climatology must, for years to come, be based. This extraordinary grasp of the whole wide range of his science he maintained practically till the day of his death.

Somehow, when a man like this passes away, a bare statement of the essential facts of his life and a list of his contributions to science seems unnecessary and futile. Yet there is something singularly significant in the fact that this man, living a very simple life, with very few changes of residence, extended his interests and his reading to all parts of the world. He knew the geographical and climatological conditions of almost every corner of the globe as intimately as if he had himself lived there. Hann — for thus, and not as von Hann, he will oftenest be recalled — was born March 23, 1839, near Liuz, in Austria. He began his professional life as a school-teacher. At the age of twenty-nine he entered the Central-Anstalt für Meteorologie in Vienna. From 1874 to 1897 he was its Director, an office from which he retired at the age of fifty-eight. For many years he was a professor at the University of Vienna, first of Physical Geography and later of Physics. His work for meteorology did not cease when he ceased to be Director. He went to Graz as Professor at the University, and there, in the Physical Institute, he wrote his "Lehrbuch der Meteorologie," whose three editions bear the dates 1901, 1906, 1915. A fourth edition, supervised by Süring, is now in course of preparation. In 1900, Hann returned to Vienna as Professor of Cosmical Physics, a position which he held until his retirement in 1910. The "Handbuch der Klimatologie" he wrote while in Vienna. The three editions of this book bear the dates 1883, 1897, 1908–1911. These two books are Hann's monumental publications. It is almost literally true that no student of meteorological science can do a day's work without referring to them. Throughout his long editorship and joint editorship of the *Meteorologische Zeitschrift* (1866–1920) he steadily contributed to the pages of that journal a series of articles and notes which are invaluable, for in these he revised, summarized, commented upon, and put into permanent form a vast body of meteorological and climatological material. In 1906, in commemoration of forty years of his editorship, a special *Hann Band* of the *Zeitschrift* was issued. Two other major publications are the *Atlas der Meteorologie*, forming Part III of the *Berghaus Physikalischer Atlas* (1887) which was for years the standard meteorological atlas of the world, and *Die Erde als Gauzes; ihre Atmosphäre und Hydrosphäre* (1st edition, 1872; 3d edition 1880; 5th edition 1896).

Hann was the recipient of many honors, and was made a member of many learned societies, both in Europe and abroad. He was the first foreigner to receive the Symon's Gold Medal of the Royal Meteorological Society (1904).

Hann died in Vienna, October 1, 1921, in his eighty-third year. No more fitting tribute could possibly be written of him than that contained in the notice of his death sent out by his former colleagues in Vienna. "Ein Leben ununterbrochener Geistesarbeit und reinster Forschung im Dienste der Wissenschaft ist abgeschlossen. Aber ungezählte Fäder führen von Hann's Werken in alle Länder der Erde und wirken in seinem Sinne fort."

R. DeC. Ward.

HENRY LEE HIGGINSON (1834–1919).

Fellow in Class III, Section 4, 1912.

The "Life and Letters of Henry Lee Higginson," by Professor Bliss Perry, published in the autumn of 1921, affords so full and accessible a record of the career and character of this Fellow of the Academy that anything beyond a brief summary would be superfluous for the present purpose.

Two conspicuous anomalies in the life of so eminent a citizen of Boston and son of Harvard were that he was born in New York (November 18, 1834) and that he was a member of Harvard College for only a few months in the freshman year of his class of 1855. He was, however, of pure New England descent, and when he was in his fourth year his family left New York and provided him with that Boston background which he was to adorn for more than eighty years. The brevity of his connection with Harvard, for which he was prepared at the Boston Latin School, was due to a weakness of his eyes. Of the ten years between his leaving college and the outbreak of the Civil War, more than five were spent in two visits to Europe, and a year and a half, in the interval between them, as a clerk in the counting house of S. & E. Austin, Boston merchants. The second of his European visits, from 1856 to 1860, was devoted largely to the study of music, pursued to the extent of physical injury, and also to the end of reaching the reluctant decision that his talents would not justify his becoming a professional musician. It was then, however, that he determined, if he could ever compass it, to enrich the lives of his countrymen with music as his own life had been enriched by the music of Vienna and other European cities.

The disappointed student returned to America only a few months before the outbreak of the War of Secession. His immediate future could not long remain uncertain. As an officer, first of the Second Massachusetts Infantry and then of the First Massachusetts Cavalry, he proved himself an admirable soldier. Serious wounds received in June, 1863, incapacitated him for much of the second half of the war. In December, 1863, he married Ida Agassiz, daughter of Louis Agassiz.

In the years immediately following the war he sought his fortune, in company with his young wife, through oil in Ohio and cotton in Georgia, but without success. In 1868 he joined the Boston banking and brokerage firm of Lee, Higginson & Co., with which he was conspicuously identified for the remaining fifty-one years of life.

By 1881 his labors and good fortune enabled him to realize the dream of his young manhood through establishing the Boston Symphony Orchestra. This he maintained, at a very large personal cost, for thirty-seven years. The fortune which he spent upon it was the measure of his devotion to his city, his country, and his kind. But it was not expressed through this interest only, for his gifts to Harvard College, through a long period of years, gifts devoted primarily to the happiness and health of the student body, placed him among the great benefactors of that institution. To friends and others in need he was constantly holding out a helping hand. Though his name is most associated with the art of music and with education, he gladly furthered many another good cause, local and national. He was withal a strongly individual figure, outspoken in praise and blame, much swayed by his affections, endowed with many of the most lovable human qualities. By his death on November 14, 1919, in Boston, his community lost its foremost figure, and his country a pattern of the highest patriotism.

M. A. DeWolfe Howe.

FRANKLIN PAINE MALL (1862–1917).

Fellow in Class II, Section 3, 1901.

Franklin Paine Mall, 1862–1917, was born in Iowa of German extraction, his father being one of the 1848 immigrants. Nothing is known of his boyhood education, which was mainly in a boarding school near his home. He studied medicine in the University of Michigan, and received the M.D. degree in 1883, before attaining his majority. He then went to Germany and spent several years in study at Heidelberg and Leipzig, at the latter place in the laboratories of Ludwig and His, these being men of the highest rank in science and who exerted a great influence on his life. From 1886 to 1889 he was Fellow and Instructor in Pathology at the Johns Hopkins University under Professor Wm. H. Welch, from 1889 to 1892 Adjunct Professor of Vertebrate Anatomy at Clark University, and from 1892 to 1893 Professor of Anatomy at the University of Chicago. He returned to Baltimore in 1893 as Professor of Anatomy in the newly formed Johns Hopkins Medical School, which position he held to his death, being also the Director of the Carnegie Institute of Embryology which was established at the Johns Hopkins Medical School in 1912.

Such are the brief facts concerning the official career of the man who, in the great renaissance of medicine during the last fifty years probably did more in America than any other man to make possible this rebirth and growth. He was a great teacher, as such bringing to medical teaching the ideal that knowledge is to be sought not in lectures or books but by the study of nature, the student acquiring primary knowledge by independent work which might be extended by reading and at the same time receiving training in scientific methods which would increase individual power. This method at the time of its induction was novel, was resisted by both students and faculty, but was steadily carried out in his laboratory, and has become the accepted method of the best teachers.

He was a great scientific investigator. As such his work was thorough, he touched no subject on which his investigations did not throw light and in most cases he left the subject standing clearly, the obscurities gone. He was a leader, not a follower, his researches were carefully planned, he used all methods of approach, and was fertile in devising new methods of work. His individual contributions, of which there are more than a hundred, and the five hundred contributions from his laboratory, rank with highest contributions to the science of anatomy.

He was active in the promotion of opportunities for the advance of medical science in all places, his advice was always sought and valued, and his influence has been very great in the advance of medical teaching in this country and elsewhere. He led a simple and quiet life, was a good citizen, a good friend.

For account of his life and work see Johns Hopkins Hospital Bulletin, Memorial Service held May, 1918; Anatomical Record, January, 1918.

W. T. COUNCILMAN.

SIR WILLIAM OSLER (1849–1919).

Fellow in Class II, Section 4, 1897.

In 1849 William Osler was born in Tecumseh, Ontario, Canada, the son of Reverend F. L. Osler. Beginning his medical training at the University of Toronto, he continued it at McGill whence he received his M.D. degree in 1872. Two years were spent in study abroad at London, Berlin and Vienna. Returning to Montreal in 1874 he was made Lecturer on the Institutes of Medicine at McGill, and shortly afterwards was given the Professorship. This began what continued to be, for him, the chief interest of his life, for Osler, more than anything else, was a teacher, first at McGill, later at the University of Pennsylvania, then at Johns Hopkins, where he exerted his greatest influence as an inspiring leader of an increasingly large group of students, and finally at Oxford, where, as Regius Professor of Medicine, he held a unique position of influence on both American and British medicine.

In the early days of his medical career Osler was a diligent student of pathology and contributed important studies in this field, notably on blood platelets. Chiefly, however, in this period he was laying a foundation for his future clinical work in a thorough and extensive knowledge of pathological anatomy gained from making post mortem dissections with the enthusiasm of a keen minded, enthusiastic, indefatigable worker. This interest in pathological anatomy he never lost and his knowledge of it proved an ever ready help in his subsequent career as a clinician, teaching the principles of the practice of medicine in the wards of the various hospitals where he served.

At the Johns Hopkins Hospital he inaugurated what was to prove, perhaps, the most important contribution to methods of teaching medicine of the half century in the latter days of which we now live, namely, the learning of medicine by laboratory practice rather than by lecture and recitation, for he made of the hospital wards the laboratory of clinical medicine in which the same observational methods were pursued as in the laboratories of natural science and the facts of pathological anatomy and physiology were correlated with the phenomena of disease as seen in the individual patients. Into this laboratory method he brought the humanizing and inspiring influence of a personality keenly interested in helping and stimulating his fellows and one by nature endowed with a winsomeness, charitableness and humor that made of him for students and patients a lifelong friend. Though a laboratory, yet the wards were always clearly recognized as the place in which each individual patient must receive the best possible professional care and the kindly considerative aid that is due to a fellowman in distress. Though laboratory director, Osler in his wards was ever the true physician.

In all of his very numerous contributions to medical science and practice as well as in his textbook of medicine, Osler shows a very distinctive and delightful literary style. He is direct, simple and logical. Examples that illustrate and clarify are chosen with great discretion. In his addresses quotations evince both his knowledge of the best in literature and his ability to emphasize or impress his point by apt quotation. Always greatly interested in the historical background of medicine, he makes much use of historical reference in his writings. There is ever the quaint turn of his humor or some epigrammatic line to enliven the description or discussion. His words are chosen with great charm of diction and still it is rare that his meaning is at all ambiguous. In almost all of his writing there is a personal element that, for those who knew him, recalls vividly to memory the picture of the man and his personality. His publications were numerous and varied, for the most part dealing with clinical medicine. A bibliography published in 1919 shows 730 titles. His practice of medicine has remained, since the first edition in 1892, the most popular textbook for English speaking students as well as having been translated into French, Spanish and Chinese. A new edition has been issued at three year intervals and through this book Osler exerted a tremendous influence on the practice of medicine for in it Osler's personality dominates in a truly remarkable way when one considers the difficulty of introducing any personal note inherent in a textbook necessarily condensed when covering such a voluminous topic as the practice of medicine.

Through all the years of his activities as investigator, teacher, medical writer and hospital chief, Osler remained an active consultant, aiding fellow physicians in the solution of difficult problems in diagnosis. His own optimism radiated courage to the patients and his delightful personality and charm endeared him to innumerable physicians who brought him patients. He was most intolerant of unkind criticism of others and would never allow patients or physicians to bring to his ears unsavory gossip of fellow practitioners. He believed in and practiced direct honesty in dealing with patients and physicians, but a different opinion or advice from him never carried with it the sting of a rebuke nor the implication than an unjustifiable error had been made by another.

Osler was distinctly more a scholar than almost any of his medical contemporaries. He had a deep interest in the classics. That he, a physician, should be president of the British Classical Association, as he was in 1919, was a recognition by scholars of his classical learning. His interest in medical history has already been referred to. In this connection he was, all of his life, an ardent bibliophile and his library of early editions of medical classics and allied topics was a veritable treasure house. To many his name recalls the picture of a delightful eagerness and radiant charm of manner as, standing in his library either earlier in Baltimore or later in Oxford, with one of his choice volumes in hand, he turns the pages and talks of the author or his writings. This is the mental picture rather than that of physician, for, however much his life's work was that of inspiring medical men, he seemed peculiarly in his proper setting by his beloved books. He took a great interest in both the Bodleian Library at Oxford and in the Oxford Press, and to both he gave much thought and time, serving each in official capacity.

When Osler left Baltimore for Oxford he was almost universally conceded to be the leading man in American medicine. At Oxford he merely widened his sphere so as to become the leader for British as well as American medicine. He died Dec. 29, 1919 of complications following pneumonia. The death of his only son in the World's War and the strain incident to his own activities in connection with the problems of the sick and wounded were important contributing factors. In his lifetime he moulded in many very important ways medical thought and medical teaching. He was greatly beloved by students, fellow practitioners and patients by reason of his human friendliness and his kindliness. To his students and medical colleagues he was ever an inspiring leader stimulating to diligence in medical work and exemplifying in himself what the ideal physician and medical teacher and writer should be.

HENRY A. CHRISTIAN.

WILHELM PFEFFER (1845–1920).

Foreign Honorary Member in Class II, Section 2, 1897.

The death of Professor Wilhelm Pfeffer, on the 30th of January, 1920, removed one of the outstanding figures of the scientific world. With the exception of Strasburger, he probably influenced the work of the last generation of American botanists more deeply than any other man.

The last two decades of the nineteenth century was a period of remarkable development of botany in America. A number of factors contributed to this, but undoubtedly the most important was the influence of the work of the great German botanists of the previous twenty years. Through the translation of Sachs's famous text-book and several other important German works American botanists were introduced to the results of the investigations of the morphologists and physiologists, who made Germany at that period the leader in botanical science. Up to this time, one may almost say that physiology and comparative anatomy in botany, did not exist in America. As a result of this newly aroused interest, many of the younger botanists looked forward to studying in Germany.

It is true that a small number had found their way abroad in the seventies, but it was not until a decade later that the real exodus to the German laboratories began. For ten years or more there were always to be found American students in the principal botanical laboratories of Germany, especially in Strasburger's laboratory in Bonn and in Pfeffer's at Leipzig. These young Americans applied themselves to the acquirement of the latest methods of research, particularly in the field of histology and cytology with Strasburger, and physiology under Pfeffer's direction. It is hardly necessary to point out the results of this training on the subsequent development of botanical teaching and research in America.

Pfeffer was almost the last of that remarkable band of distinguished investigators who for nearly half a century made Germany the center of botanical progress in Europe.

The writer spent the summer semester of 1887 in Pfeffer's laboratory in Tübingen, just before he removed to Leipzig where the rest of his life was spent.

The old Suabian town of Tübingen is most picturesquely placed in the beautiful Neckar Valley, south of Stuttgart, and near the northern border of the Black Forest. This region is one of the most attractive in Germany, and the quaint old town, and the amiable South German people, who still clung to their picturesque customs and peasant costumes, made it a most satisfactory abiding place — aside from the scientific advantages of the University.

The laboratory was at this time one of the best equipped in Germany. It boasted a line of distinguished botanists as directors, two of whom, Von Mohl and Hofmeister, were worthy predecessors of Pfeffer. Under Pfeffer's able direction the facilities for work in physiology were probably at that time unequalled.

Sachs, at this period had practically ceased active work and Pfeffer was generally recognized as his legitimate successor.

Pfeffer was an indefatigable worker but found time to supervise carefully the work of his students and to give them the benefit of his valuable criticism and assistance. At this time he was but forty-two years old but looked older, his tall, thin and somewhat bent figure and strongly marked features making him seem older than his years.

Pfeffer was born, the son of an apothecary, in the village of Grebenstein near Cassel, March 9, 1845. He studied at Göttingen where he took his doctorate in 1865, in Marburg, where he afterward taught as docent, in Berlin and Würzburg, in the latter University working under Sachs.

In 1873 he was appointed professor extraordinarius in Bonn, and four years later went as full professor to Basel, where he remained only a year, after which he went to Tübingen. He held the position in Tübingen until his final removal in 1887 to Leipzig where he remained until his death in 1920.

In Leipzig he developed the great laboratory which for more than thirty years was the Mecca for students of plant physiology from all parts of the world. Throughout his long career in Leipzig he was generally recognized as the first physiologist of his generation.

While Pfeffer's name is primarily associated with strictly physiological problems, as a young man he published several morphological papers of considerable importance. Especially valuable was a paper on the development of the gametophyte and embryo of Selaginella, a paper that for a long time was the most important contribution to the subject.

It is, however, upon the very numerous and important contributions to plant physiology that his fame rests. These cover an extensive range of subjects, some of fundamental importance, not only biologically, but to physics and chemistry as well. His remarkable investigations in osmotic pressure have strongly influenced the work of subsequent workers in pure physics and chemistry, and their great importance has been fully recognized by these investigators. Pfeffer's extensive studies on plasma membranes and the phenomena of irritability include many papers of the first importance. During his stay in Tübingen he inaugurated a series of publications "Untersuchungen aus dem botanischen Institut zu Tübingen" modelled on the similar publication issued from the botanical Institute in Würzburg under the direction of Sachs. This publication ceased on Pfeffer's departure from Tübingen.

Pfeffer's best known work is his great text-book, Handbuch der Pflanzenphysiologie. This was translated into English and was for many years the standard work on the subject.

Pfeffer's name is also associated with the well-known periodical, Pringsheims Jahrbücher für wissenschaftlicher Botanik. After the death of Pringsheim, this was issued for several years under the joint editorship of Pfeffer and his distinguished colleague, Strasburger.

Shortly before the outbreak of the war Pfeffer's old students were invited to contribute to a "Festschrift" to celebrate the fiftieth anniversary of his doctorate and his seventieth birthday. The volume appeared in 1915, but the circumstances of the war resulted in the absence of many names which under normal conditions would certainly have appeared in it.

Pfeffer survived the horrors of the Great War, in which he lost his only son, and saw the collapse of the great German empire, in whose upbuilding he and his scientific colleagues played such an important rôle. He had the satisfaction, however, of knowing that their work would survive the downfall of the imperial government and that his name will always rank high in the annals of science.

DOUGLAS HOUGHTON CAMPBELL.

EDWARD CHARLES PICKERING (1846–1919).

Fellow in Class I, Section 1, 1867.

In the death of Edward Charles Pickering after a service of forty-two years as Director of the Harvard College Observatory, the American Academy loses an interested and important Fellow and the Science of Astronomy one who was at his death the dean of astronomical research in America.

He was born in Boston, Massachusetts, July 19, 1846, of a distinguished and highly cultivated New England family. In 1865, he graduated from the Lawrence Scientific School with the degree of S.B. He was immediately thereupon appointed instructor in mathematics in that institution, but the following year he became assistant instructor in Physics in the Massachusetts Institute of Technology, and two years after was made Thayer Professor of Physics.

From the very outset of his teaching his peculiar bent of mind was revealed and the work of research and organization which constituted his great contribution to modern science was begun.

He planned and put into practical shape for use in systematic class instruction the experimental laboratory method in the teaching of

Physics which did much to make the Institute of Technology famous and has since been accepted and adopted universally as an indispensable method of instruction in that subject. To the laboratory which he had organized and built up the Corporation of the Institute in 1872 at his suggestion gave the name Rogers Laboratory of Physics and the additional title of Director of the Laboratory was conferred upon him.

In the autumn of 1876 he was called to become the Director of the Harvard College Observatory, and accepting this invitation he entered upon the duties of the position in February, 1877. His selection by President Eliot seemed at the time a radical innovation for Professor Pickering was a physicist rather than an astronomer of the old school. However, the appointment was justified for it presaged the trend of the New Astronomy along the lines of Physics, a development in which Professor Pickering has borne a most honorable part.

As Director of the Observatory he showed great administrative ability and secured a large financial support for his projects, the endowment growing from a few hundred thousand to a million dollars.

Instead of venturing into the realm of speculative and picturesque astronomy, he was content to be what he called himself "a collector of astronomical facts," the interpretation of which he was perfectly willing to leave to the future. The posthumous value of the work of such men as Herschel and Argelander appealed especially to him and shaped the large investigations that he undertook, whose importance could not be completely revealed perhaps for centuries.

Immediately upon his appointment to his new position, Professor Pickering chose as his particular field of labor the photometry of the stars. Soon after the introduction of the dry plate, in general photography, he was led to investigate its applicability to the study of the stars and their spectra in which work he was a pioneer. He also realized the great value of the objective prism in stellar spectroscopy and made constant use of it in his studies of stellar spectra.

The Observatory under Professor Pickering has made its largest contribution to astronomy in four fields.

(1) Photometry. With the aid of the meridian photometer invented by him, he devised a scale of photometric magnitudes, determining these for eighty thousand stars upon a basis of more than two million observations.

(2) A scale of photographic magnitudes. It was shown later that these are convertible into visual magnitudes through reference to the spectral types of the stars.

(3) A system of classification of variable stars. Light curves have been determined for a large number of these and many thousand measures of their brightness have been made on a uniform scale for all of the sky.

(4) A system of classification of stellar spectra which has been universally adopted. The new Henry Draper Catalogue contains estimates based on this system of all stars to approximately the ninth magnitude, about 200,000 in number.

Through the establishment of an observatory at Arequipa in 1891 after two years of preliminary study it became possible to include measurements made on the stars throughout the southern heavens within the scope of the work of the Harvard College Observatory.

At a later date, 1911, an observing station was established at Mandeville, Jamaica, which has been devoted particularly to the study of the moon and the planets.

In this short article it is not possible to go into detail or even to mention the great variety of investigations carried on to a successful completion. The volumes of the Harvard Annals, more than eighty of which were published during Professor Pickering's directorship, can alone give any idea of this.

A word, however, should be said of the "photographic library" which now contains over a quarter of a million photographic plates that together weigh one hundred and twenty tons. Through the use of short focus lenses and automatic following apparatus there has been kept a "sky patrol" the results of which furnish the history of all the stars down to the tenth magnitude and measuring back for many years. By its use whenever any noteworthy stellar change is discovered the plates will reveal its past history and character, while otherwise one might have to wait years to understand the nature of the phenomena.

Through the use of this library, it is possible to find the history of the stars, as one turns back the pages of a book already printed. What may still be hoped for from this crystallized past of the heavens is shown by what it has already done in recording the extraordinarily favorable position for observation of the minor planet Eros at its opposition in 1893, though it was not actually known even to exist until several years later in 1898.

Professor Pickering strongly believed in associative work. To him is due the organization of the American Astronomical Society in 1898 (originally called the Astronomical and Astrophysical Society) which now has a membership of over three hundred from all parts of the country, and has been of great service in stimulating research and promoting acquaintance among astronomers. The American Association of Variable Star Observers, a body composed chiefly of amateur observers of these objects, also originated with him.

Professor Pickering was elected a Fellow of the Academy in 1867 at the age of twenty-one years and is said to have been the youngest member ever chosen. He was averse to holding office, probably because during many years regularity of attendance at the meetings of the Academy would have interfered with his professional duties. He was a member of the Council from 1878 to 1884 and a member of the Committee on the Library from 1877 to 1883. He rendered great service to the Academy and to scientific research through his unprecedentedly long and devoted work as a Member of the Rumford Committee. This began in 1869 and continued up to the time of his death with a break, however, from 1890 to 1892, during which interval he was awarded the Rumford Premium "for his work on the photometry of the stars and upon stellar spectra." He contributed to the Proceedings twenty-six papers, three of them in collaboration with others. There will shortly be published by the Academy a memoir containing the results of researches upon the photometry of faint stars, carried on at various observatories with the use of a form of photometer devised by him for this especial service.

In 1874, Professor Pickering married Miss Lizzie Wadsworth Sparks, a daughter of the Reverend Jared Sparks, the historian, and a former President of Harvard University. Mrs. Pickering died in 1906.

To those who had the privilege of a personal acquaintance with Professor Pickering his great mind will always seem secondary to his greater heart, his generous friendship and his social charms.

He was never a narrow specialist interested only in his own branch of science. All astronomy, indeed all science, received his interest and encouragement. His broad sympathy included such dissimilar interests as mountain-climbing and music. He was the founder of the Appalachian Mountain Club and its first President. Music was an inspiration to him in his work, not as a relaxation alone, but as a stimulant which helped him to solve mathematical and physical problems.

He had the cooperative mind and the highest unselfishness actuated all his relations with his fellow astronomers. He subordinated his own individuality in his work and even the interests of his observatory to the good of astronomical science. All the honors of the astronomical world were showered upon him and upon the institution which under his direction had led the queen of sciences into new triumphs in untrodden fields.

As it was said of Sir Christopher Wren that his monument was Saint Paul's Cathedral, so the monument of Professor Pickering is found in the ninety volumes of the Annals of Harvard College Observatory, a monument which as long as man looks up at the heavens and wonders and interprets, should be an honorable and enduring one.

JOEL H. METCALF.

JOHN ELLIOTT PILLSBURY (1846–1919).

Fellow in Class II, Section 1, 1893.

This distinguished naval officer and oceanographer was born at Lowell, Massachusetts, December 15, 1846, the son of John Gilman and Elizabeth Wimble (Smith) Pillsbury. His early education was received in the public schools, and at the age of fourteen he was appointed a page in the House of Representatives.

In 1862 he received from President Lincoln a nomination to the U. S. Naval Academy, from which he graduated in 1867, being commissioned ensign in 1868 and lieutenant in 1872.

He married Florence Greenwood Aitchison of Portland, Maine, August 26, 1873. Elsie Greenwood, later wife of Edward B. Richardson of Brookline, Mass., was the only issue of this marriage.

In 1875 he was ordered to the Hydrographic Office of the Navy Department and the following year detailed to the U. S. Coast Survey, where he gave ten years of service and placed his name permanently on the roll of those who have materially added to our knowledge of the secrets of the Ocean.

The investigation of the Gulf Stream was undertaken by the Survey on account of its importance to navigation as well as its scientific interest, and work was begun in 1883 with the schooner *Drift*, which, as her name implies, proved inadequate for the purpose and was replaced by the Coast Survey Steamer *Blake*.

New methods of current measurement and improved instruments for recording observations were devised by Pillsbury, and by the aid of the recently introduced steel cable anchorage was had, sometimes at a depth of over two miles. Observations extended from Tobago on the southeast to Hatteras on the north, and the movements and temperatures of this important current were definitely fixed over a great part of its course. Among the interesting new results of the work were the determination of daily fluctuations in the rate of flow more or less coincident with the tidal action, and the contribution of wave effect, driven by the trade winds, in increasing the movement of the stream.

During the Spanish War he commanded the dynamite cruiser *Vesuvius* and participated in the attack on San Juan, Porto Rico. He was later promoted to Commander and through the various grades to Rear Admiral and Chief of the Bureau of Navigation in 1908. He received all the medals for service and efficiency in the line of duty which under the law are granted by the Navy Department.

In 1909 he became a member of the Board of Managers of the National Geographic Society, Vice-president in 1915, and President of the Society in April, 1919; dying on December 30th of the same year.

A summary of his lifework is given in the Bulletin of the National Geographic Society of October 16, 1919. An account of the Gulf Stream work and results was given by Admiral Pillsbury in the National Geographic Magazine of August, 1912, and in Hydrographic Office publication No. 110, in 1894. A memoir on Charts and Chart-making was published in Proceedings of the U. S. Naval Institute No. 29, in February, 1894. An excellent portrait of the Admiral appeared in the National Geographic Magazine, volume 37, p. 341, in April, 1920.

The Admiral was elected a member of this Academy, April 12, 1893.

William Healey Dall.

ARTHUR SEARLE (1837–1920).

Fellow in Class 1, Section 1, 1877.

Arthur Searle, who died October 23, 1920, was born in London on October 21, 1837. His father, Thomas Searle, was an American citizen and a descendant of Governor Thomas Dudley of Massachusetts. His mother, Anne Noble, came from Derby, England, being English by birth as well as by ancestry. Thomas Searle seems to have been naturally fitted for the life of a scholar and a man of letters, but the restricted means of the family deprived him of a college education, and forced him into mercantile business at an early age. At the time of his marriage in 1834 he was a partner in a firm of London bankers. It was during this sojourn in England that his son Arthur was born in 1837, and two years later his other son George. As a consequence of a commercial panic Thomas returned in 1840 to America with his family to look after business interests. His wife soon died, and two years after in 1843 he himself passed away, leaving the care of the two boys to his elder brother and a sister in Brookline, Mass.

Both boys were sent early to private schools in Brookline and Roxbury, partly for the reason as Searle afterward suspected, to make life easier for their elders, not accustomed to such lively youngsters. The last school days were passed at the Brookline High School. Entering Harvard College at the age of fourteen years, he was graduated in 1856, as the second scholar of his class. In 1859 he received his Master's degree. Arthur, though only six years old at the time of his father's death, had found in him a companion and an instructor. Under such influence, the scholarly aspirations of the father seemed to have been as seed to find fruition in the son's life. The boy had an alert mind. At the age of seven he began his habit of psychological introspection by the discovery, while meditating on some subject, that it was he himself who was thinking. Thus, he became aware of the personal identity that was Arthur Searle. Before this time he had made his first experiment in physics, namely, as to the effect of centrifugal force acting on a bit of wood placed inside the whirling rim of his aunt Becky's spinning wheel. At eleven years he was interested in the revolution in France, and began to have political opinions, which were always conservative. But anything of a scientific nature fascinated him. The electric telegraph, anaesthetic surgery, the discovery of the planet Neptune in 1846, all appealed to his mind. Nor was he less gifted in other respects. His avidity for knowledge gave him even then the reputation of being a "walking dictionary." As a schoolboy, mathematics could be easily acquired while feeding his rabbits, and at college he found that he had already performed the chemical experiments which were being taught from a text-book without any provision for laboratory practice by the students. All branches of knowledge inside or outside the college curriculum interested him intensely, and he studied them all eagerly and thoughtfully. His first article was published in the Harvard Magazine, while he was still a student. It was on the plurality of worlds, and seemed prophetic of his future career, as he had no thought at the time of making astronomy a profession.

It was twelve years after graduation that Searle found his calling. The intervening time was a course in the university of life. Ill health led him to engage in farming for a time. Teaching, statistical work, and experience in a broker's office, all were tried. He also joined in a project to raise sheep in California, but the scheme after a brief trial was abandoned. Before returning home from California, he filled temporarily the place of an absent professor at Santa Clara.

In 1868, his brother George, who had been employed at the Harvard Observatory, resigned to study for the Catholic priesthood, and Arthur was asked to take his place. This he did, little thinking that at last he had found a permanent place with congenial occupation. The following year he was appointed Assistant, to be promoted to Assistant Professor in 1883, and Phillips Professor of Astronomy in 1887. In 1912 he became Phillips Professor Emeritus. Besides his Observatory work he also conducted astronomical courses at Radcliffe College from 1891 to 1912. He was married in 1873 to Emma Wesselhoeft, daughter of Dr. Robert Wesselhoeft of Boston. Mrs. Searle died in 1914. Two daughters survive their parents.

His earliest work at the Harvard Observatory was as a computer and observer. In the latter capacity he made observations of stars, double stars, nebulae, satellites of the planets, asteroids, and comets. These observations are contained in the Annals of Harvard College Observatory, Volumes 11, 13, 14, and 33; also in the Proceedings of this Academy, Volume 16. In 1889 he published in the Annals of

Harvard College Observatory, Volume 19, Part 1, the results, which he had gathered, of the early meteorological observations made at the Observatory from 1840 to 1888. Among these were included various miscellaneous observations relating to the aurora, lightning, meteors, earthquakes, and to some extent to the zodiacal light.

The zodiacal light was the subject of his first independent investigation. Beginning in 1874 he continued his observations of the zodiacal light and the Gegenschein until 1895, when the increasing use of electricity for street illumination made such work impossible in Cambridge. The results of these observations are contained in the Astronomische Nachrichten, Volumes 99, 102, 109, 116, 124, and 126, *Proceedings*, Volume 19, *Memoirs*, Volume 11; and the Annals of the Harvard College Observatory, Volumes 19, Part 2, and 33, Nos. 1, 2, and 3. Summaries of information written by him on the subject appear in the Monthly Weather Review and elsewhere.

The several lines of his inquiry dealt with the permanence, position, and magnitude of the ordinary western zodiacal light; the normal distribution of light in the zodiac and vicinity; and the position, parallax, and brightness of the Gegenschein. His studies led him to favor the hypothesis ascribing the phenomenon to light reflected from small meteoric bodies. He published a statement on the "Meteoric Theory of the Gegenschein" also in the English periodical, Observatory, August, 1899. Although he considered the meteoric hypothesis as the most probable explanation, he felt that his series of observations should be extended to reach a definite conclusion. The research should include the orbital movements and the light of asteroids and periodic comets. He expressed the hope that younger observers more favorably located might carry out his plan.

In the Observatory he was frequently engaged in the business management, particularly during the interim between Director Winlock's death and Professor Pickering's appointment. It was at this time that he published Volume 8 of the Harvard Annals, containing his account of the history of the Observatory from 1855 to 1876, with a description of the buildings, instruments and of work done. The volume included also a series of illustrations of Sun, planets, and other celestial objects, which had been drawn mostly by Trouvelot during Winlock's directorship.

Professor Searle spent ten years — from 1888 to 1898 — in making the meridian circle observations for the Zone Catalogue of 8337 Stars between 9° 50′ and 14° 10′ of South Declination in 1855 for the Epoch 1900.0. The results fill Volumes 62, 65, 66, 67, and 70 of the Harvard Annals. The Catalogue itself, contained in Volume 67, was published in cooperation with the Astronomische Gesellschaft. The reduction and publication of these observations with the superintendence of other computers consumed most of his time and energy until he retired in 1912. With his customary modesty, he regarded this not as a personal undertaking but as a large piece of routine work. Nevertheless, the various investigations related to meridian circle observations, which he undertook in the course of the work, show his skill and ingenuity in meeting such problems. They are indicated in the Introduction to the Catalogue just mentioned. Reference may be made here to "Results of Accessory Series of Observations made with the Meridian Circle," and "Comparison of Results obtained with different Forms of Apparatus in Meridian Observations," in the Annals of Harvard College Observatory, Volumes, 33, No. 11, and 41, No. 7. In 1908 he published in the Harvard Annals, Volume 60, No. 1, "Geometrical Methods in the Theory of Combining Observations." In the Annals, Volume 29, No. 6 are his observations of β Persei, and surrounding comparison stars.

Besides various articles in periodicals, he published "Outlines of Astronomy" in 1874, followed by a second edition in 1875. In 1910 his "Essays I–XXX" appeared, which, among other topics, discussed "Space and Time," interesting in the light of the theory of relativity.

Professor Searle became a Fellow of the Academy in 1877, at the same time with Professor Charles R. Cross, who recently died. His scientific papers presented to the Academy, and not already mentioned are in the *Proceedings*, Volumes 19, 24, and 55. The last paper "Orbits Resulting from Assumed Laws of Motion" was a result of an extensive investigation begun in 1882, forming an important part of a treatise which he had practically completed at the time of his death. Reading this paper on the balanced effect of "inward" and "outward" forces on a moving body, one is carried back to the initial physical experiment, which he performed in his childhood with the help of his aunt's spinning wheel.

He was very much of a mathematician, and when any question of the sort arose he was consulted. The results will be found in various places. For example, in the paper on "Stellar Photometry" published in the *Proceedings*, Volume 11, the discussion of the path described by stars at various declinations in the field of a telescope when the axis is not properly adjusted, was prepared by him. Not only a mathematician, he was proficient in many languages. "A Note on the Battle of Pharsalus" was the result of re-reading Caesar's "De Bello Civili," which he did for recreation. He amused himself in writing verse both in Latin and in English. One of these poems written at the time of his wife's death has been published since his own death; the Latin version in the Harvard Graduates' Magazine, the English version in Popular Astronomy.

Professor Searle was the most modest of men. His extremely retiring disposition probably accounts for his not accepting Dr. Gould's invitation in 1869 to go as his assistant to Cordoba. Later, he might have been appointed director of another observatory, if he had been willing. His life flowed in a quiet stream. It was as he would have it. The turmoil of strenuous life did not attract him. From youth he was not keen for even the ordinary pleasures of society, and yet he had many warm friends, and a host of acquaintances. All who knew him well, were delighted with his conversational powers. His sense of humor and the merry twinkle in his eye as he recounted some episode were passports to friendliness. His philosophical studies made his thinking clear. When he spoke, it was as one having the authority of careful thought. In discussing any subject he had a succinctness of expression which swept away all intricacies and left the matter in outlines readily understood. He was of a type, not so common at the present day, of a scholarly gentleman, versed in many branches of learning, and keenly susceptible to the delights of music, of art, and the manifestations of nature.

EDWARD S. KING.

WILLIAM THOMPSON SEDGWICK (1855–1921).

Fellow in Class II, Section 3, 1886.

William T. Sedgwick, a Fellow of the American Academy of Arts and Sciences since 1886, died suddenly in Boston, January 25, 1921, at the age of 66 years, while still in the full tide of his activities as a teacher, investigator and public servant. The son of William and Anne Thompson Sedgwick, and a descendant of Robert Sedgwick, who settled in Boston in 1636, he was born in West Hartford, Connecticut, December 29, 1855, and throughout his life cherished the traditions of his New England origin and training. He graduated with high rank from the Sheffield Scientific School of Yale University in 1877, and taught physiological chemistry in Chittenden's laboratory in 1878–1879. In 1879 he became Fellow and subsequently Assistant in Biology at Johns Hopkins University, where he took the degree of Ph.D. in 1881, and in the same year married Mary Catherine Rice of New Haven. He received the honorary degrees of Sc.D. (Yale, 1909) and LL.D. (University of Cincinnati, 1920) and was a member of many learned societies, serving as president of the American Society of Naturalists, of the American Public Health Association, and of the Society of American Bacteriologists, of which he was one of the founders and the first president. He was a member of the International Health Board of the Rockefeller Foundation, of the Advisory Board of the United States Hygienic Laboratory, of the Public Health Council of Massachusetts, of the Royal Sanitary Institute of Great Britain, and of other important organizations. He served as president of the Boston Civil Service Reform Association in 1900 and of the State Association in 1901; and from 1897 down to the time of his death was curator of the Lowell Institute of Boston.

In 1883 he became Professor of Biology in the Department of Biology, later known as the Department of Biology and Public Health, at the Massachusetts Institute of Technology, then under the presidency of Francis Walker. As head of that department he began a service in the teaching of general biology and in the public health movement in America that continued for nearly forty years and brought distinction alike to himself and to his institution, rendering his laboratory one of the important centers of biological work in America. The culmination of the honors that he received came in the last year of his life when he served as exchange professor to the universities of Cambridge and of Leeds, and also as a representative of the Intitute of Technology, Harvard University, the American Public Health Association and the U. S. Public Health Service at the International Health Conference at Brussels. In both capacities, as foreign observers have testified, his lectures and addresses made a deep impression, carrying a message from America to the older world that went far beyond the merely technical aspects of his subject.

He was the author of important technical papers and general addresses, too numerous to be listed here, and also of several larger works. Among these may be mentioned the "General Biology," a text-book published jointly with E. B. Wilson (1886), "The Human Mechanism," published with Theodore Hough (1906), "A Short History of Science," with H. W. Tyler (1917), and above all the "Principles of Sanitary Science" (1902) which at once took its place as the standard work on the subject and assured Sedgwick's position as one of the foremost leaders in this field.

Sedgwick's life was the uneventful one of a teacher and investigator, happy in his work, in his friends and in a home life singularly congenial and rich; but even its bare outline impresses us with his versatility and the wide range of his interests. He was a born teacher, one who loved his work and kept always in view a higher ideal than merely to impart information. He knew how to inspire his students and followers with his own buoyant eagerness, thoroughness and tenacity of purpose. He taught them to think straight, aim high and work hard. A sane and good humored optimism was inseparable from his personality; and not less characteristic were the sturdy common sense and shrewd sense of humor with which he was wont to illuminate the dry technicalities of his subject, driving home the underlying principles by the use of homely and telling illustrations that made them living realities never to be forgotten. In these respects Sedgwick was indeed a teacher unrivalled, as many generations of "Tech" students can bear witness. He made comrades of his students, and they gave to him affectionate and enduring friendship. Alike by precept and by the example of his own life he taught them that man does not live by bread alone; that the student of science fails to attain his largest measure of success if his mind be not kept open to the larger world of literature, art and human fellowship. His students felt towards him an almost filial regard and learned to look to him in their later lives for wise and helpful counsel. In this respect he has with good reason been compared to Dr. Arnold, but as one of his former students has finely said: "The master of Rugby was far off on the snowy heights. Sedgwick was in the midst of the rush of life and he held us by the hand."

Though Sedgwick's early inclination was towards the study of physiology and medicine he later gravitated irresistibly into sanitary science and conservation of the public health; and in this field he was one of the earliest and most prominent pioneers. He and his many pupils contributed more than any other to the emancipation of these subjects from medicine in the narrower sense, and their recognition as important independent branches of applied biology which offer the widest opportunities for public service outside the practise of medicine. To this end he contributed by important studies on epidemics, largely in connection with the work of the State Board of Health of Massachusetts and the Lawrence Experimental Station, by the work of numerous students trained in his laboratory, and especially by the publication in 1902 of his authoritative work on Sanitary Science and the Public Health, referred to above, which has recently been characterized by competent authority as still the best existing epitome of the subject.

The interest in public welfare displayed in these various activities, was but one side of a larger interest in educational and civic problems that drew him into many other forms of public service. He was a valued member or trustee of many public institutions, in and outside of Boston. He played a prominent part in the struggle for civil service reform in Massachusetts in 1900–1901, and then and later delivered many public addresses on subjects connected with the general welfare. During his long service as curator of the Lowell Institute he became widely known to the citizens of Boston, winning general esteem by the breadth of view and enlightened regard for the public interest with which for so many years he administered his important trust.

As one who had the privilege of intimate friendship with him for more than forty years, the writer may be permitted finally to emphasize Sedgwick's high minded and noble character. He was a man of vision, of lofty ideals, of faith in the eternal fitness of things. No man was less self-seeking or more appreciative of others. He was kindly, generous and human, with a gift for friendship that made him the center of an always enlarging circle of friends and enriched his life with widely varied human interests. To those friends he gave a loyalty and ever ready helpfulness that knew no change with the passing years. He exemplified the best traditions of his profession as an inspiring teacher and a leader of research in his chosen field. His friends and colleagues rejoiced in his achievements as if they had belonged in part to them; and they will cherish the memory of his happy and useful life.[1]

EDMUND B. WILSON.

ELMER ERNEST SOUTHARD (1876–1920).

Fellow in Class II, Section 4, 1911.

Dr. Elmer Ernest Southard died in New York City on February 8, 1920 after a very brief illness at the age of forty-three.

When stricken down by the fatal infection he was busily engaged in making a series of communications dealing with his special field of work. He was at the height of his power, and his accomplishments might well be considered an earnest of still richer productivity in the years to come. A man of incessant industry, with a keen and alert intellect, restlessly searching after the solution of age-long problems, he had a personality which won him many warm friendships, and a talent for inspiring his associates and pupils.

After an education in the public schools of Boston, and at Harvard College, he graduated from the Medical School in 1901. Immediately after graduation he began to occupy himself with that sphere of investigation with which he later continued to identify himself so closely. He became early associated with the pathological work of the Massachusetts State Hospitals for the Insane. In 1909 he became Bullard Professor of Neuropathology. In 1912 he was appointed Director of the newly established Boston Psychopathic Hospital (at that time called the Psychopathic Department of the Boston State Hospital).

The value of a scientific worker is only in part to be estimated by the published results of his personal investigations. Equally important may be the influence of the worker on associates and pupils, on the community where he lives, on the whole body of professional workers who are working in the same field as himself. The influence

[1] The writer desires to acknowledge his indebtedness to appreciative reviews of Professor Sedgwick's life and work by two of his former pupils, Samuel C. Prescott (*Technology Review* for April, 1921) and C. E. A. Winslow (*Journal of Bacteriology*, May, 1921).

of Dr. Southard radiated widely throughout the country, and there are many serious workers in widely scattered centers who owe their inspiration to him. His publications witness to the great industry of the man. They cover a wide field; they furnish an important body of material which has been incorporated in the general body of knowledge which pertains to the disciplines of neuropathology and of psychiatry.

In the Southard Memorial Number of the Bulletin of the Massachusetts Department of Mental Diseases a complete bibliography of his works is published, with brief comments and abstracts (pp. 30–199). The earlier communications deal with the more technical aspects of morbid processes, involving the central nervous system. They are detailed studies of tissue reactions which do not involve the categories of the personality. In later communications the reactions of the individual began to play a prominent rôle, and the question of the correlation of structural damage with functional inefficiency became a central problem. One of the fundamental problems of psychiatry, namely, the basal conditions underlying mental deterioration, or, to put it in another way, the etiology of so-called "dementia praecox," was a subject of much careful investigation on his part. These are problems of very complex nature, and in the formulation of his views Dr. Southard not only brought together much interesting material, but showed a lightness of touch and ingenuity of expression which charmed those who read his papers.

The position of Director of the Psychopathic Hospital brought with it new responsibilities and interests which reflected themselves in his investigations. He had passed from the investigation of mere tissue to the study of the morbid activity of the individual, and now he was brought to deal with the problems of mental health in relation to community life. Work of this type necessarily brings a great variety of problems, and in regard to them Dr. Southard showed his usual keen insight and fertility of resources. He contributed papers on the treatment of special types of disease, on hospital organization, on the training of special types of workers, on the relationship of the hospital to the community, on the possibility of making available to industrial organizations the principles which had been worked out in the limited sphere of the hospital.

The war brought its special problems in his field, and he contributed a book on "Shell-shock and Other Neuropsychiatric Problems presented in 589 Case Histories from the War Literature, 1914–1918."

Throughout all these various interests the fundamental fascination of the problem of the relation of structure to function continued, and in a boldly outlined program for research into the basis of feeble-mindedness he sought to determine the "minimum brain machinery with which speech and thought processes get performed." Two Research Series have already been published — the Waverley Researches in the Pathology of the Feeble-Minded (Memoirs of the American Academy of Arts and Sciences, Vol. XIV, No. II. May, 1918; Vol. XIV, No. III. December, 1921). In these researches Dr. Southard and his associates have furnished a standard for scientific work in this field.

A general survey of the work done by Dr. Southard shows a growing breadth of vision and steadily maturing genius, and those working in this special field of medicine realize what a tragedy it was when it was suddenly deprived of one of its most brilliant workers.

C. M. CAMPBELL.

BARRETT WENDELL (1855–1921).

Fellow in Class III, Section 4, 1889.

Barrett Wendell, a member of this Academy since 1889 was born in Boston on October 23, 1855, and died on February 9, 1921. He was the son of Jacob and Mary Bartoldi (Barrett) Wendell. His first American ancestor on the paternal side was Evart Jansen Wendell, who came from Emden in East Friesland to New York about 1640 when some twenty-five years old.

Entering Harvard with the Class of 1876, he was compelled by ill health to leave college for a year; consequently he took his A.B. degree with the Class of '77. During 1877–8 he attended the Harvard Law School and later was a student in offices both in New York and Boston, but his early intention to follow the law was abandoned for the work of teaching and writing English following his appointment in 1880 as an instructor at Harvard. He served the college in this capacity until 1888 when he was made an Assistant Professor. Ten years later he becamc a full Professor of English, holding that office until 1917, when he resigned and was chosen Professor Emeritus.

During the thirty-seven years of his incumbency at Harvard Wendell was a positive constructive force in the fields of English composition and of comparative literature. He substituted for more formal methods a frank incisiveness of speech and an independence of outlook that challenged the interest and won the affection of nearly three generations of students. The class room audiences who found his controversial attitude engaging never failed to perceive the wisdom, genial humor and passionate sympathy with what is best in literature that underlay his marked and sometimes whimsical peculiarities. To the individual seeking aid he gave of himself in generous measure never to be forgotten by the recipient.

From the outset of his career as a teacher of English Wendell was also an industrious writer. Before the appearance in 1891 of his "English Composition," a text book widely adopted, he had published two novels, "The Duchess Emilia" in 1885 and "Rankell's Remains" in 1887. In the following order appeared "Cotton Mather," 1891; "Stelligeri and Other Essays Concerning America," 1893; "William Shakespeare, a Study in Elizabethan Literature," 1894; "A Literary History of America," 1900; "Raleigh in Guiana," "Rosamond" and "A Christmas Masque," 1902; "The Temper of the Seventeenth Century in English Literature" (his lectures at Trinity College, Cambridge, England, in 1902–1903–1904), 1904; "History of Literature in America" (written in collaboration with Chester N. Greenough), 1904; "Liberty, Union and Democracy — the National Ideals of America," 1906; "The France of Today," 1907; "The Privileged Classes," 1908; "The Mystery of Education," 1909; and finally, in 1920, "The Traditions of European Literature," the second and concluding volume of which was interupted by his final illness.

Of these writings it may be said that his "Cotton Mather" and his "English Composition" stand out in the product of his early period; his "A Literary History of America," which contradicted the judgments of the sages and aspersed some idols, has become with time a standard treatise; his "The France of Today" (delivered originally as Lowell lectures) opened the eyes of Americans to their ignorance of French racial characteristics. The sympathetic insight displayed in this book was so deeply appreciated by the French people that since his death one of the lecture halls of the Sorbonne has been renamed after him "Hall Barrett Wendell." "The Traditions of European Literature," only one volume of which he completed, was a labor of love, the fruition of years of discerning scholarship. The period from Homer to Dante was covered by the first volume and the second would have brought the survey down to modern times.

During his sabbatical vacations Wendell visited Europe at various times. In 1902–3 he represented Harvard University at the 300th anniversary of the Bodleian Library at Oxford, and was Clark lecturer at Trinity College, Cambridge, England. In 1904–5 he was the first of the annual lecturers on the Hyde foundation at the Sorbonne and other French universities. In 1911 he went around the world, traveling in India, China and Japan.

Wendell was a member of the American Academy of Arts and Letters, of the Massachusetts Historical Society, and a Fellow of the American Academy of Arts and Sciences. He received from Columbia University in 1913 the honorary degree of Doctor of Letters (Litt. D.); Harvard University conferred upon him the same degree in 1918, and Strassburg University, France, that of Doctor of Laws (LL.D.) in 1920.

Wendell was a man of pronounced individuality, warm in his sympathies, singularly loyal in his attachments, and free from littleness. He never concealed his convictions, which were often critical of modern tendencies and points of view. If he seemed to champion the past at the expense of the present, it was because of his insistence on standards and his veneration for the summits not the table lands of tradition. His conversation had the charm of freedom from the commonplace.

Wendell was married on June 1, 1880, to Edith Greenough of Quincy, who, with two sons and two daughters, survive him.

ROBERT GRANT.

ANDREW DICKSON WHITE (1832–1918).

Fellow in Class III, Section 2, 1868.

Andrew Dickson White was born in Homer, November 7, 1832, and died in Ithaca, only twenty-five miles from his birth-place, on November 4, 1918. As a student at Hobart College and later at Yale, he was impressed with the inadequacy and narrowness of the college training of those days. This feeling increased as he grew older and when Ezra Cornell consulted him as to the best employment of some of his wealth for the public benefit, Mr. White soon succeeded in inspiring him with the vision which became embodied in Cornell University. Mr. White became of course the president of the new university and it is to him that we owe the placing of scientific and technical courses on a level with the humanities. The development of Cornell has been a striking illustration both of the power of the ideal and of Mr. White's wisdom. It is scarcely an exaggeration to say that Cornell has been successful in so far as it has followed the ideals of its first president. The principles that he laid down over fifty years ago are likely to guide the course of the University for years to come.

People wondered in the early days why a man like Goldwin Smith should leave Oxford and come to Cornell; but it was the spirit of protest in him that made him love Cornell to the end of his life. At Cornell they were trying to do something new and worth while. It was the spirit of Andrew D. White that appealed to Goldwin Smith.

Though Mr. White's real reputation will rest on the work that he did in starting Cornell University as the embodiment of an ideal, this was by no means the whole of his work. His "History of the Warfare of Science with Theology in Christendom" and his Autobiography are the two works which the general public knows, and it is sometimes forgotten that he was one of the founders of the American Historical Association and its first president. While president of Cornell University he was also professor of Modern European history. In 1887 he presented to Cornell his historical library and it was only fitting that the combined departments of history and political science should be known officially as "The President White School of History and Political Science."

Mr. White's diplomatic career was varied and honorable. He was minister plenipotentiary to Germany from 1879 to 1881 and to Russia from 1892 to 1894 and later ambassador to Germany from 1897 to 1902.

A man of means and a wonderful host, he kept open house in Ithaca. Distinguished visitors to this country always visited Mr. White and no one who lived in Ithaca during the last years of Mr. White's life can fail to realize what he meant to the social life of the faculty. The

University very properly bears the name of its founder, Ezra Cornell; but no one will question that it was Andrew D. White who put the breath of life into the young institution.

Honors of course came to Mr. White in profusion. He enjoyed them keenly but they did not change him. A list of these, with other information, may be found in an obituary notice published in the Memoirs of the New England Historic Genealogical Society, Vol. 73, p. LX (1919).

Wilder D. Bancroft.

American Academy of Arts and Sciences

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RECORDING SECRETARY.

Charles B. Gulick.

TREASURER.

Harold Murdock.[1]

LIBRARIAN.

Arthur G. Webster.

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	Terms expire 1924.	
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	Terms expire 1925.	
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	Terms expire 1926.	

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COMMITTEE ON MEETINGS.

The President,
The Recording Secretary,
George H. Parker, Edwin B. Wilson, Edward K. Rand.

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[1] Appointed Nov. 8, 1922, to fill the term of H. H. Edes, deceased.

LIST

OF THE

FELLOWS AND FOREIGN HONORARY MEMBERS.

(Corrected to October 20, 1922.)

FELLOWS.— 579.

(Number limited to six hundred.)

Class I.— *Mathematical and Physical Sciences*.— 195.

Section I.— *Mathematics and Astronomy*.— 46.

Charles Greeley Abbot	Washington, D. C.
Walter Sydney Adams	Pasadena, Cal.
George Russell Agassiz	Boston
Raymond Clare Archibald	Providence, R. I.
Solon Irving Bailey	Cambridge
Edward Emerson Barnard	Williams Bay, Wis.
George David Birkhoff	Cambridge
Ernest William Brown	New Haven, Conn.
William Elwood Byerly	Cambridge
Florian Cajori	Berkeley, Cal.
William Wallace Campbell	Mt. Hamilton, Cal.
Julian Lowell Coolidge	Cambridge
George Cary Comstock	Madison, Wis.
Leonard Eugene Dickson	Chicago, Ill.
Philip Fox	Evanston, Ill.
Fabian Franklin	New York, N.Y.
Edwin Brant Frost	Williams Bay, Wis.
Frank Lauren Hitchcock	Belmont
Edward Vermilye Huntington	Cambridge
Dunham Jackson	Minneapolis, Minn.

Oliver Dimon Kellogg	Cambridge
Edward Skinner King	Cambridge
Carl Otto Lampland	Flagstaff, Ariz.
Joseph Lipka	Cambridge
Joel Hastings Metcalf	Portland, Me.
George Abram Miller	Urbana, Ill.
Clarence Lemuel Elisha Moore	Watertown
Eliakim Hastings Moore	Chicago, Ill.
Frank Morley	Baltimore, Md.
Forest Ray Moulton	Chicago, Ill.
Henry Bayard Phillips	Boston
William Henry Pickering	Cambridge
Charles Lane Poor	New York, N. Y.
Roland George Dwight Richardson	Middletown, Conn.
Henry Norris Russell	Princeton, N. J.
Frank Schlesinger	New Haven, Conn.
Harlow Shapley	Cambridge
Vesto Melvin Slipher	Flagstaff, Ariz.
Frederick Slocum	Middletown, Conn.
Virgil Snyder	Ithaca, N. Y.
Joel Stebbins	Madison, Wis.
William Edward Story	Worcester
Henry Taber	Worcester
Harry Walter Tyler	Newton
Robert Wheeler Willson	Cambridge
Frederick Shenstone Woods	Newton

Class I., Section II.— *Physics*.— 56.

Joseph Sweetman Ames	Baltimore, Md.
Samuel Jackson Barnett	Washington, D. C.
Carl Barus	Providence, R. I.
Louis Agricola Bauer	Washington, D. C.
Louis Bell	Newton
Percy Williams Bridgman	Cambridge
George Ashley Campbell	New York, N. Y.
Leslie Lyle Campbell	Boston
Emory Leon Chaffee	Belmont

Daniel Frost Comstock . . . . Cambridge
William David Coolidge . . . . Schenectady, N. Y.
Henry Crew . . . . Evanston, Ill.
Harvey Nathaniel Davis . . . . Cambridge
Arthur Louis Day . . . . Corning, N. Y.
Louis Derr . . . . Brookline
William Johnson Drisko . . . . Winchester
William Duane . . . . Boston
Alexander Wilmer Duff . . . . Worcester
Arthur Woolsey Ewell . . . . Worcester
William Suddards Franklin . . . . Cambridge
Harry Manley Goodwin . . . . Brookline
George Ellery Hale . . . . Pasadena, Cal.
Edwin Herbert Hall . . . . Cambridge
Hammond Vinton Hayes . . . . Boston
John Charles Hubbard . . . . New York, N. Y.
Gordon Ferrie Hull . . . . Hanover, N. H.
Charles Clifford Hutchins . . . . Brunswick, Me.
Frederic Eugene Ives . . . . Philadelphia, Pa.
James Edmund Ives . . . . Washington, D. C.
William White Jacques . . . . Boston
Edwin Crawford Kemble . . . . Cambridge
Norton Adams Kent . . . . Cambridge
Frank Arthur Laws . . . . Boston
Henry Lefavour . . . . Boston
Theodore Lyman . . . . Brookline
Thomas Corwin Mendenhall . . . . Ravenna, O.
Ernest George Merritt . . . . Ithaca, N. Y.
Albert Abraham Michelson . . . . Chicago, Ill.
Dayton Clarence Miller . . . . Cleveland, O.
Robert Andrews Millikan . . . . Chicago, Ill.
Harry Wheeler Morse . . . . Los Angeles, Cal.
Edward Leamington Nichols . . . . Ithaca, N. Y.
Ernest Fox Nichols . . . . New Haven, Conn.
Charles Ladd Norton . . . . Boston
George Washington Pierce . . . . Cambridge
Michael Idvorsky Pupin . . . . New York, N. Y.
Frederick Albert Saunders . . . . Cambridge

John Stone Stone . . . . New York, N. Y.
Maurice deKay Thompson . . . . Brookline
Elihu Thomson . . . . Swampscott
John Trowbridge . . . . Cambridge
Arthur Gordon Webster . . . . Worcester
David Locke Webster . . . . Palo Alto, Cal.
Edwin Bidwell Wilson . . . . Brookline
Robert Williams Wood . . . . Baltimore, Md.
John Zeleny . . . . New Haven, Conn.

Class I., Section III.—*Chemistry.*—51.

Wilder Dwight Bancroft . . . . Ithaca, N. Y.
Gregory Paul Baxter . . . . Cambridge
Arthur Alphonzo Blanchard . . . . Cambridge
Marston Taylor Bogert . . . . New York, N. Y.
Bertram Borden Boltwood . . . . New Haven, Conn.
William Crowell Bray . . . . Berkeley, Cal.
Russell Henry Chittenden . . . . New Haven, Conn.
Arthur Messinger Comey . . . . Cambridge
Charles William Eliot . . . . Cambridge
Henry Fay . . . . Boston
George Shannon Forbes . . . . Cambridge
Edward Curtis Franklin . . . . Palo Alto, Cal.
Frank Austin Gooch . . . . New Haven, Conn.
Lawrence Joseph Henderson . . . . Cambridge
Charles Loring Jackson . . . . Boston
Walter Louis Jennings . . . . Worcester
Grinnell Jones . . . . Cambridge
Frederick George Keyes . . . . Cambridge
Elmer Peter Kohler . . . . Cambridge
Charles August Kraus . . . . Worcester
Arthur Becket Lamb . . . . Cambridge
Irving Langmuir . . . . Schenectady, N. Y.
Gilbert Newton Lewis . . . . Berkeley, Cal.
Warren Kendall Lewis . . . . Boston
Arthur Dehon Little . . . . Brookline
Charles Frederic Mabery . . . . Cleveland, O.

Forris Jewett Moore	Boston
George Dunning Moore	Worcester
Edward Williams Morley	West Hartford, Conn.
Edward Mueller	Cambridge
Samuel Parsons Mulliken	Boston
Charles Edward Munroe	Forest Glen, Md.
James Flack Norris	Boston
Arthur Amos Noyes	Pasadena, Cal.
William Albert Noyes	Urbana, Ill.
Thomas Burr Osborne	New Haven, Conn.
Samuel Cate Prescott	Brookline
Ira Remsen	Baltimore, Md.
Robert Hallowell Richards	Jamaica Plain
Theodore William Richards	Cambridge
Martin André Rosanoff	Pittsburgh, Pa.
Stephen Paschall Sharples	Cambridge
Miles Standish Sherrill	Brookline
Harry Monmouth Smith	Brookline
Julius Oscar Stieglitz	Chicago, Ill.
Henry Paul Talbot	Newton
Richard Chase Tolman	Washington, D. C.
William Hultz Walker	Boston
Willis Rodney Whitney	Schenectady, N. Y.
Robert Seaton Williams	Cambridge
Alpheus Grant Woodman	Watertown

CLASS I., SECTION IV.— *Technology and Engineering*.— 42.

Henry Larcom Abbot	Cambridge
Comfort Avery Adams	Cambridge
Bernard Arthur Behrend	Boston
William Herbert Bixby	Chicago, Ill.
Francis Tiffany Bowles	Boston
Charles Francis Brush	Cleveland, O.
William Hubert Burr	New Canaan, Conn.
John Joseph Carty	New York, N.Y.
Harry Ellsworth Clifford	Newton
Arthur Powell Davis	Washington, D. C.

Theodore Harwood Dillon	Cambridge
Gano Dunn	New York, N. Y.
William Frederick Durand	Palo Alto, Cal.
Frederic Harold Fay	Boston
Desmond FitzGerald	Brookline
John Ripley Freeman	Providence, R. I.
George Washington Goethals	New York, N. Y.
John Hays Hammond	New York, N. Y.
Rudolph Hering	Montclair, N. J.
Ira Nelson Hollis	Worcester
Hector James Hughes	Cambridge
Alexander Crombie Humphreys	New York, N. Y.
Dugald Caleb Jackson	Cambridge
Lewis Jerome Johnson	Cambridge
Arthur Edwin Kennelly	Cambridge
Gaetano Lanza	Philadelphia, Pa.
Charles Thomas Main	Winchester
Lionel Simeon Marks	Cambridge
Edward Furber Miller	Newton
Frederick Law Olmsted	Brookline
Charles Francis Park	Boston
William Barclay Parsons	New York, N.Y.
Harold Pender	Philadelphia, Pa.
Albert Sauveur	Cambridge
Peter Schwamb	Arlington
Henry Lloyd Smyth	Cambridge
Charles Milton Spofford	Brookline
Charles Proteus Steinmetz	Schenectady, N. Y.
George Fillmore Swain	Cambridge
George Chandler Whipple	Cambridge
Robert Simpson Woodward	Washington, D. C.
Joseph Ruggles Worcester	Boston

CLASS II.— *Natural and Physiological Sciences.*— 185.

SECTION I.— *Geology, Mineralogy, and Physics of the Globe.*— 55.

Wallace Walter Atwood	Worcester
George Hunt Barton	Cambridge
Norman Levi Bowen	Washington, D. C.
Isaiah Bowman	Washington, D. C.
John Casper Branner	Palo Alto, Cal.
Thomas Chrowder Chamberlin	Chicago, Ill.
John Mason Clarke	Albany, N. Y.
Henry Helm Clayton	Canton
Herdman Fitzgerald Cleland	Williamstown
William Otis Crosby	Jamaica Plain
Reginald Aldworth Daly	Cambridge
Edward Salisbury Dana	New Haven, Conn.
William Morris Davis	Cambridge
Benjamin Kendall Emerson	Amherst
William Ebenezer Ford	New Haven, Conn.
James Walter Goldthwait	Hanover, N. H.
Louis Caryl Graton	Cambridge
Herbert Ernest Gregory	New Haven, Conn.
William Jackson Humphreys	Washington, D. C.
Ellsworth Huntington	Milton
Oliver Whipple Huntington	Newport, R. I.
Robert Tracy Jackson	Peterborough, N. H.
Thomas Augustus Jaggar	Honolulu, H. I.
Douglas Wilson Johnson	New York, N. Y.
Arthur Keith	Washington, D. C.
James Furman Kemp	New York, N. Y.
Alfred Church Lane	Cambridge
Andrew Cowper Lawson	Berkeley, Cal.
Charles Kenneth Leith	Madison, Wis.
Waldemar Lindgren	Brookline
Frederic Brewster Loomis	Amherst
Alexander George McAdie	Readville
John Campbell Merriam	Washington, D. C.
William John Miller	Northampton

Charles Palache	Cambridge
Raphael Pumpelly	Newport, R. I.
Percy Edward Raymond	Cambridge
William North Rice	Middletown, Conn.
Austin Flint Rogers	Palo Alto, Cal.
Robert Wilcox Sayles	Cambridge
Waldemar Theodore Schaller	Washington, D. C.
Charles Schuchert	New Haven, Conn.
William Berryman Scott	Princeton, N. J.
Hervey Woodburn Shimer	Watertown
Thomas Wayland Vaughan	Washington, D. C.
Charles Doolittle Walcott	Washington, D. C.
Robert DeCourcy Ward	Cambridge
Charles Hyde Warren	New Haven, Conn.
David White	Washington, D. C.
Herbert Percy Whitlock	New York, N. Y.
Bailey Willis	Palo Alto, Cal.
Arthur Winslow	Boston
John Eliot Wolff	Cambridge
Jay Backus Woodworth	Cambridge
Frederick Eugene Wright	Washington, D. C.

CLASS II., SECTION II.—*Botany*.—34.

Oakes Ames	North Easton
Irving Widmer Bailey	Cambridge
Liberty Hyde Bailey	Ithaca, N. Y.
Edward Wilber Berry	Baltimore, Md.
Douglas Houghton Campbell	Palo Alto, Cal.
George Perkins Clinton	New Haven, Conn.
John Merle Coulter	Chicago, Ill.
Bradley Moore Davis	Ann Arbor, Mich.
Edward Murray East	Jamaica Plain
Rollins Adams Emerson	Ithaca, N. Y.
Alexander William Evans	New Haven, Conn.
Merritt Lyndon Fernald	Cambridge
George Lincoln Goodale	Cambridge
Robert Almer Harper	New York, N. Y.

John George Jack	East Walpole
Edward Charles Jeffrey	Cambridge
Fred Dayton Lambert	Tufts College
Jacob Goodale Lipman	New Brunswick, N. J.
Burton Edward Livingston	Baltimore, Md.
George Richard Lyman	Washington, D. C.
Elmer Drew Merrill	Manila, P. I.
Winthrop John Vanleuven Osterhout	Cambridge
Charles Vancouver Piper	Washington, D. C.
Alfred Rehder	Jamaica Plain
Benjamin Lincoln Robinson	Cambridge
Charles Sprague Sargent	Brookline
William Albert Setchell	Berkeley, Cal.
Arthur Bliss Seymour	Cambridge
Erwin Frink Smith	Washington, D. C.
John Donnell Smith	Baltimore, Md.
William Codman Sturgis	New York, N. Y.
Roland Thaxter	Cambridge
William Trelease	Urbana, Ill.
William Henry Weston, Jr.	Cambridge

Class II., Section III.— *Zoölogy and Physiology.*— 57.

Nathan Banks	Cambridge
Thomas Barbour	Cambridge
Francis Gano Benedict	Boston
Henry Bryant Bigelow	Concord
Robert Payne Bigelow	Brookline
William T. Bovie	Milton
John Lewis Bremer	Boston
Charles Thomas Brues	Boston
Hermon Carey Bumpus	Providence, R. I.
Walter Bradford Cannon	Cambridge
Thorne Martin Carpenter	Boston
William Ernest Castle	Belmont
Charles Value Chapin	Providence, R. I.
Benjamin Preston Clark	Boston
Samuel Fessenden Clarke	Williamstown

Edwin Grant Conklin . . . . Princeton, N. J.
Joseph Augustine Cushman . . . . Sharon
William Healey Dall . . . . Washington, D. C.
Charles Benedict Davenport . . . . Cold Spring Harbor, N. Y.
Gilman Arthur Drew . . . . Woods Hole
Cecil Kent Drinker . . . . Boston
Alexander Forbes . . . . Milton
Samuel Henshaw . . . . Cambridge
Leland Ossian Howard . . . . Washington, D. C.
Herbert Spencer Jennings . . . . Baltimore, Md.
Charles Willison Johnson . . . . Brookline
Charles Atwood Kofoid . . . . Berkeley, Cal.
Frederic Thomas Lewis . . . . Waban
Ralph Stayner Lillie . . . . Worcester
Jacques Loeb . . . . New York, N. Y.
Richard Swann Lull . . . . New Haven, Conn.
Edward Laurens Mark . . . . Cambridge
Ernest Gale Martin . . . . Palo Alto, Cal.
Albert Davis Mead . . . . Providence, R. I.
Gerrit Smith Miller . . . . Washington, D. C.
Edward Sylvester Morse . . . . Salem
Herbert Vincent Neal . . . . Tufts College
Henry Fairfield Osborn . . . . New York, N. Y.
George Howard Parker . . . . Cambridge
William Patten . . . . Hanover, N. H.
Raymond Pearl . . . . Baltimore, Md.
John Charles Phillips . . . . Wenham
Henry Augustus Pilsbry . . . . Philadelphia, Pa.
Herbert Wilbur Rand . . . . Cambridge
Arthur Clarence Redfield . . . . Boston
William Emerson Ritter . . . . La Jolla, Cal.
Percy Goldthwait Stiles . . . . Newtonville
John Eliot Thayer . . . . Lancaster
William Lyman Underwood . . . . Belmont
Addison Emory Verrill . . . . Whitneyville, Conn.
John Broadus Watson . . . . Washington, D. C.
Arthur Wisswald Weysse . . . . Boston
William Morton Wheeler . . . . Boston

Harris Hawthorne Wilder . . . . Northampton
Edmund Beecher Wilson . . . . New York, N. Y.
Frederick Adams Woods . . . . Brookline
Robert Mearns Yerkes . . . . Washington, D. C.

CLASS II., SECTION IV.—*Medicine and Surgery.*—39.

Edward Hickling Bradford . . . . Boston
Charles Macfie Campbell . . . . Cambridge
Alexis Carrel . . . . New York, N. Y.
Henry Asbury Christian . . . . Boston
Stanley Cobb . . . . Ponkapoag
Rufus Cole . . . . New York, N. Y.
Harvey Cushing . . . . Boston
David Linn Edsall . . . . Cambridge
Simon Flexner . . . . New York, N. Y.
Joseph Lincoln Goodale . . . . Boston
Robert Battey Greenough . . . . Boston
Ross Granville Harrison . . . . New Haven, Conn.
William Henry Howell . . . . Baltimore, Md.
Reid Hunt . . . . Brookline
Henry Jackson . . . . Boston
Elliott Proctor Joslin . . . . Boston
William Williams Keen . . . . Philadelphia, Pa.
Robert Williamson Lovett . . . . Boston
Frank Burr Mallory . . . . Brookline
William James Mayo . . . . Rochester, Minn.
Samuel Jason Mixter . . . . Boston
Francis Weld Peabody . . . . Brookline
Theophil Mitchell Prudden . . . . New York, N. Y.
William Lambert Richardson . . . . Boston
Milton Joseph Rosenau . . . . Boston
Frederick Cheever Shattuck . . . . Boston
Theobald Smith . . . . Princeton, N. J.
Charles Wardell Stiles . . . . Washington, D. C.
Richard Pearson Strong . . . . Boston
William Sydney Thayer . . . . Baltimore, Md.
Ernest Edward Tyzzer . . . . Wakefield

Frederick Herman Verhoeff . . . . Boston
Henry Pickering Walcott . . . . . . Cambridge
John Warren . . . . Boston
John Collins Warren . . . . Boston
William Henry Welch . . . . . . Baltimore, Md.
Francis Henry Williams . . . . Boston
Simeon Burt Wolbach . . . . Boston
Horatio Curtis Wood . . . . Philadelphia, Pa.

CLASS III.— *Moral and Political Sciences.*— 199.

SECTION I.— *Theology, Philosophy and Jurisprudence.*— 52.

Thomas Willing Balch . . . . Philadelphia, Pa.
Simeon Eben Baldwin . . . . New Haven, Conn.
Willard Bartlett . . . . Brooklyn, N. Y.
Joseph Henry Beale . . . . Cambridge
Charles Henry Brent . . . . Buffalo, N. Y.
Howard Nicholson Brown . . . . Boston
Charles Warren Clifford . . . . New Bedford
Edmund Burke Delabarre . . . . Providence, R. I.
James De Normandie . . . . Roxbury
Frederic Dodge . . . . Belmont
Edward Staples Drown . . . . Cambridge
William Harrison Dunbar . . . . Cambridge
William Herbert Perry Faunce . . . . Providence, R. I.
William Wallace Fenn . . . . Cambridge
Frederick Perry Fish . . . . Brookline
Paul Revere Frothingham . . . . Boston
George Angier Gordon . . . . Boston
Alfred Hemenway . . . . Boston
William Ernest Hocking . . . . Cambridge
Charles Evans Hughes . . . . Washington, D. C.
Frederick John Foakes Jackson . . . . New York, N. Y.
Charles Francis Jenney . . . . Boston
William Lawrence . . . . Boston
Frederick Lawton . . . . Boston
Arthur Lord . . . . Plymouth

William Caleb Loring . . . . Boston
Nathan Matthews . . . . Boston
William McDougall . . . . Cambridge
Samuel Walker McCall . . . . Winchester
Edward Caldwell Moore . . . . Cambridge
John Bassett Moore . . . . New York, N. Y.
James Madison Morton . . . . Fall River
George Herbert Palmer . . . . Cambridge
Charles Edwards Park . . . . Boston
Leighton Parks . . . . New York, N. Y.
Francis Greenwood Peabody . . . . Cambridge
George Wharton Pepper . . . . Philadelphia, Pa.
Roscoe Pound . . . . Belmont
Elihu Root . . . . New York, N. Y.
James Hardy Ropes . . . . Cambridge
Arthur Prentice Rugg . . . . Worcester
Austin Wakeman Scott . . . . Cambridge
Henry Newton Sheldon . . . . Boston
Moorfield Storey . . . . Boston
William Howard Taft . . . . New Haven, Conn.
William Jewett Tucker . . . . Hanover, N. H.
William Cushing Wait . . . . Medford
Eugene Wambaugh . . . . Cambridge
Edward Henry Warren . . . . Brookline
Winslow Warren . . . . Dedham
Samuel Williston . . . . Belmont
Woodrow Wilson . . . . Washington, D. C.

Class III., Section II.— *Philology and Archæology*.— 56.

Francis Greenleaf Allinson . . . . Providence, R. I.
William Rosenzweig Arnold . . . . Cambridge
Maurice Bloomfield . . . . Baltimore, Md.
Franz Boas . . . . New York, N. Y.
Carl Darling Buck . . . . Chicago, Ill.
Eugene Watson Burlingame . . . . Albany, N. Y.
Edward Capps . . . . Princeton, N. J.
George Henry Chase . . . . Cambridge

Walter Eugene Clark . . . . Chicago, Ill.
Roland Burrage Dixon . . . . Cambridge
Franklin Edgerton . . . . Philadelphia, Pa.
William Curtis Farabee . . . . Philadelphia, Pa.
Jesse Walter Fewkes . . . . Washington, D. C.
Jeremiah Denis Mathias Ford . . . . Cambridge
Basil Lanneau Gildersleeve . . . . Baltimore, Md.
Pliny Earle Goddard . . . . New York, N. Y.
Charles Hall Grandgent . . . . Cambridge
Louis Herbert Gray . . . . New York, N. Y.
Charles Burton Gulick . . . . Cambridge
Roy Kenneth Hack . . . . Cambridge
William Arthur Heidel . . . . Middletown, Conn.
George Lincoln Hendrickson . . . . New Haven, Conn.
Bert Hodge Hill . . . . Athens, Greece
Elijah Clarence Hills . . . . Bloomington, Ind.
William Henry Holmes . . . . Washington, D. C.
Edward Washburn Hopkins . . . . New Haven, Conn.
Joseph Clark Hoppin . . . . Pomfret, Conn.
Albert Andrew Howard . . . . Cambridge
William Guild Howard . . . . Cambridge
Aleš Hrdlička . . . . Washington, D. C.
Eugene Xavier Louis Henry Hyvernat . . . . Washington, D. C.
Carl Newell Jackson . . . . Cambridge
Hans Carl Gunther von Jagemann . . . . Cambridge
James Richard Jewett . . . . Cambridge
Alfred Louis Kroeber . . . . Berkeley, Cal.
Kirsopp Lake . . . . Cambridge
Henry Roseman Lang . . . . New Haven, Conn.
Charles Rockwell Lanman . . . . Cambridge
John Livingston Lowes . . . . Cambridge
David Gordon Lyon . . . . Cambridge
Clifford Herschel Moore . . . . Cambridge
George Foot Moore . . . . Cambridge
Hanns Oertel . . . . New Haven, Conn.
Chandler Rathfon Post . . . . Cambridge
Edward Kennard Rand . . . . Cambridge
George Andrew Reisner . . . . Cambridge

Edward Robinson . . . . New York, N. Y.
Fred Norris Robinson . . . . Cambridge
Rudolph Schevill . . . . Berkeley, Cal.
Edward Stevens Sheldon . . . . Cambridge
Herbert Weir Smyth . . . . Cambridge
Franklin Bache Stephenson . . . . Washington, D. C.
Charles Cutler Torrey . . . . New Haven, Conn.
Alfred Marston Tozzer . . . . Cambridge
Clark Wissler . . . . New York, N. Y.
James Haughton Woods . . . . Cambridge

Class III., Section III.—*Political Economy and History.*—45.

Wilbur Cortez Abbott . . . . Cambridge
Brooks Adams . . . . Quincy
George Burton Adams . . . . New Haven, Conn.
Charles McLean Andrews . . . . New Haven, Conn.
John Spencer Bassett . . . . Northampton
Charles Jesse Bullock . . . . Cambridge
Thomas Nixon Carver . . . . Cambridge
Edward Channing . . . . Cambridge
John Bates Clark . . . . New York, N. Y.
Archibald Cary Coolidge . . . . Boston
Richard Henry Dana . . . . Cambridge
Clive Day . . . . New Haven, Conn.
Davis Rich Dewey . . . . Cambridge
Ephraim Emerton . . . . Cambridge
Henry Walcott Farnam . . . . New Haven, Conn.
Max Farrand . . . . New Haven, Conn.
William Scott Ferguson . . . . Cambridge
Irving Fisher . . . . New Haven, Conn.
Worthington Chauncey Ford . . . . Cambridge
Edwin Francis Gay . . . . New York, N. Y.
Frank Johnson Goodnow . . . . Baltimore, Md.
Evarts Boutell Greene . . . . Champaign, Ill.
Arthur Twining Hadley . . . . New Haven, Conn.
Albert Bushnell Hart . . . . Cambridge
Charles Homer Haskins . . . . Cambridge

Isaac Minis Hays . . . . Philadelphia, Pa.
Charles Downer Hazen . . . . New York, N. Y.
George La Piana . . . . Cambridge
Henry Cabot Lodge . . . . Nahant
Abbott Lawrence Lowell . . . . Cambridge
William MacDonald . . . . New York, N. Y.
Charles Howard McIlwain . . . . Cambridge
Roger Bigelow Merriman . . . . Cambridge
Samuel Eliot Morison . . . . Boston
William Bennett Munro . . . . Cambridge
Charles Lemuel Nichols . . . . Worcester
James Ford Rhodes . . . . Boston
William Milligan Sloane . . . . New York, N. Y.
John Osborne Sumner . . . . Boston
Frank William Taussig . . . . Cambridge
William Roscoe Thayer . . . . Cambridge
Frederick Jackson Turner . . . . Cambridge
George Grafton Wilson . . . . Cambridge
George Parker Winship . . . . Cambridge
Allyn Abbott Young . . . . Cambridge

Class III., Section IV.— *Literature and the Fine Arts.*— 46.

Irving Babbitt . . . . Cambridge
George Pierce Baker . . . . Cambridge
William Sturgis Bigelow . . . . Boston
Le Baron Russell Briggs . . . . Cambridge
Charles Allerton Coolidge . . . . Boston
Frederick Shepherd Converse . . . . Boston
Samuel McChord Crothers . . . . Cambridge
Wilberforce Eames . . . . New York, N. Y.
Edward Waldo Emerson . . . . Concord
William Emerson . . . . Cambridge
Arthur Fairbanks . . . . Cambridge
Frank Edgar Farley . . . . Middleton, Conn.
Arthur Foote . . . . Brookline
Edward Waldo Forbes . . . . Cambridge
Kuno Francke . . . . Gilbertsville, N. Y.

Daniel Chester French	New York, N. Y.
Horace Howard Furness	Philadelphia, Pa.
Robert Grant	Boston
Morris Gray	Boston
Chester Noyes Greenough	Cambridge
James Kendall Hosmer	Minneapolis, Minn.
Mark Antony DeWolfe Howe	Boston
Archer Milton Huntington	New York, N. Y.
George Lyman Kittredge	Cambridge
William Coolidge Lane	Cambridge
John Ellerton Lodge	Boston
Charles Martin Tornov Lœffler	Medfield
Charles Donagh Maginnis	Brookline
Allan Marquand	Princeton, N. J.
Albert Matthews	Boston
Harold Murdock	Brookline
William Allan Neilson	Northampton
Thomas Nelson Page	Washington, D. C.
William Lyon Phelps	New Haven, Conn.
Arthur Kingsley Porter	Cambridge
Herbert Putnam	Washington, D. C.
Denman Waldo Ross	Cambridge
Paul Joseph Sachs	Cambridge
John Singer Sargent	London, Eng.
Ellery Sedgwick	Boston
Henry Dwight Sedgwick	Cambridge
Richard Clipston Sturgis	Boston
Charles Howard Walker	Boston
Owen Wister	Philadelphia, Pa.
George Edward Woodberry	Beverly
Charles Henry Conrad Wright	Cambridge

FOREIGN HONORARY MEMBERS.—71.

(Number limited to seventy-five.)

Class I.— *Mathematical and Physical Sciences.*— 24.

Section I.— *Mathematics and Astronomy.*— 8.

Johann Oskar Backlund . . . . Petrograd
Arthur Stanley Eddington . . . . Cambridge
Godfrey Harold Hardy . . . . Oxford
Jacques Salomon Hadamard . . . . Paris
Felix Klein . . . . Göttingen
Tullio Levi-Civita . . . . Rome
Charles Emile Picard . . . . Paris
Charles Jean de la Vallée Poussin . . . . Louvain

Class I., Section II.— *Physics.*— 7.

Svante August Arrhenius . . . . Stockholm
Oliver Heaviside . . . . Torquay
Sir Joseph Larmor . . . . Cambridge
Hendrik Antoon Lorentz . . . . Leyden
Max Planck . . . . Berlin
Sir Ernest Rutherford . . . . Manchester
Sir Joseph John Thomson . . . . Cambridge

Class I., Section III.— *Chemistry.*— 4.

Fritz Haber . . . . Berlin
Henri Louis Le Chatelier . . . . Paris
Wilhelm Ostwald . . . . Leipsic
William Henry Perkin . . . . Oxford

Class I.— Section IV.— *Technology and Engineering.*— 5.

Heinrich Müller Breslau . . . . Berlin
Ferdinand Foch . . . . Paris
Joseph Jacques Césaire Joffre . . . . Paris
Vsevolod Jevgenjevic Timonoff . . . . Petrograd
William Cawthorne Unwin . . . . London

Class II.— *Natural and Physiological Sciences.*— 22.

Section I.— *Geology, Mineralogy, and Physics of the Globe.*— 10.

Frank Dawson Adams . . . . Montreal
Charles Barrois . . . . Lille
Waldemar Christofer Brögger . . . . Christiania
Sir Archibald Geikie . . . . Haslemere, Surrey
Viktor Goldschmidt . . . . Heidelberg
Albert Heim . . . . Zürich
Emanuel de Margerie . . . . Paris
Gustaf Adolf Frederik Molengraaff . . . . Delft
Sir William Napier Shaw . . . . London
Johan Herman Lie Vogt . . . . Trondhjem

Class II, Section II.— *Botany.*— 5.

John Briquet . . . . Geneva
Hugo de Vries . . . . Lunteren
Adolf Engler . . . . Berlin
Ignatz Urban . . . . Berlin
Eugene Warming . . . . Copenhagen

Class II.— Section III.— *Zoölogy and Physiology.*— 3.

Maurice Caullery . . . . Paris
Sir Edwin Ray Lankester . . . . London
George Henry Falkiner Nuttall . . . . Cambridge

Class II., Section IV.— *Medicine and Surgery.*— 4.

Rt. Hon. Sir Thomas Clifford Allbutt . . . . . . Cambridge
Sir Thomas Barlow, Bart. . . . . London
Francis John Shepherd . . . . Montreal
Charles Scott Sherrington . . . . Oxford

Class III.— *Moral and Political Sciences.*— 25.

Section I.— *Theology, Philosophy and Jurisprudence.*— 5.

Rt. Hon Arthur James Balfour . . . . Prestonkirk
Heinrich Brunner . . . . Berlin

Albert Venn Dicey	Oxford
Raymond Poincaré	Paris
Rt. Hon. Sir Frederick Pollock, Bart.	London

SECTION II.— *Philology and Archæology.*— 9.

Friedrich Delitzsch	Berlin
Hermann Diels	Berlin
Wilhelm Dörpfeld	Athens
Henry Jackson	Cambridge
Hermann Georg Jacobi	Bonn
Arthur Anthony Macdonell	Oxford
Alfred Percival Maudslay	Hereford
Ramon Menendez Pidal	Madrid
Eduard Seler	Berlin

SECTION III.— *Political Economy and History.*— 6.

Adolf Harnack	Berlin
Alfred Marshall	Cambridge
Rt. Hon. John Morley, Viscount Morley of Blackburn	London
Henri Pirenne	Ghent
George Walter Prothero	London
Rt. Hon. Sir George Otto Trevelyan, Bart.	London

SECTION IV.— *Literature and the Fine Arts.*— 5.

Georg Brandes	Copenhagen
Thomas Hardy	Dorchester
Jean Adrien Antoine Jules Jusserand	Paris
Rudyard Kipling	Burwash
Sir Sidney Lee	London

STATUTES AND STANDING VOTES

STATUTES

Adopted November 8, 1911: *amended May* 8, 1912, *January* 8, *and May* 14, 1913, *April* 14, 1915, *April* 12, 1916, *April* 10, 1918, *May* 14, 1919, *February* 8, *and April* 12, 1922.

CHAPTER I

THE CORPORATE SEAL

ARTICLE 1. The Corporate Seal of the Academy shall be as here depicted:

ARTICLE 2. The Recording Secretary shall have the custody of the Corporate Seal.

See Chap. v. art. 3; chap. vi. art. 2.

CHAPTER II

Fellows and Foreign Honorary Members and Dues

Article 1. The Academy consists of Fellows, who are either citizens or residents of the United States of America, and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, and each Class is divided into four Sections, namely:

Class I. *The Mathematical and Physical Sciences*
- Section 1. Mathematics and Astronomy
- Section 2. Physics
- Section 3. Chemistry
- Section 4. Technology and Engineering

Class II. *The Natural and Physiological Sciences*
- Section 1. Geology, Mineralogy, and Physics of the Globe
- Section 2. Botany
- Section 3. Zoölogy and Physiology
- Section 4. Medicine and Surgery

Class III. *The Moral and Political Sciences*
- Section 1. Theology, Philosophy, and Jurisprudence
- Section 2. Philology and Archaeology
- Section 3. Political Economy and History
- Section 4. Literature and the Fine Arts

Article 2. The number of Fellows shall not exceed Six hundred, of whom not more than Four hundred shall be residents of Massachusetts, nor shall there be more that Two hundred and ten in any one Class.

Article 3. The number of Foreign Honorary Members shall not exceed Seventy-five. They shall be chosen from among citizens of foreign countries most eminent for their discoveries and attainments in any of the Classes above enumerated. There shall not be more than Twenty-five in any one Class.

Article 4. If any person, after being notified of his election as Fellow or Resident Associate, shall neglect for six months to accept in writing, or, if a Fellow resident within fifty miles of Boston shall neglect to pay his Admission Fee, his election shall be void; and if any Fellow resident within fifty miles of Boston or any Resident Associate shall neglect to pay his Annual Dues for six months after they are due, provided his attention shall have been called to this Article of the Statutes in the meantime, he shall cease to be a Fellow or Resident Associate respectively; but the Council may suspend the provisions of this Article for a reasonable time.

With the previous consent of the Council, the Treasurer may dispense (*sub silentio*) with the payment of the Admission Fee or of the Annual Dues or both whenever he shall deem it advisable. In the case of officers of the Army or Navy who are out of the Commonwealth on duty, payment of the Annual Dues may be waived during such absence if continued during the whole financial year and if notification of such expected absence be sent to the Treasurer. Upon similar notification to the Treasurer, similar exemption may be accorded to Fellows or Resident Associates subject to Annual Dues, who may temporarily remove their residence for at least two years to a place more than fifty miles from Boston.

If any person elected a Foreign Honorary Member shall neglect for six months after being notified of his election to accept in writing, his election shall be void.

See Chap. vii. art. 2.

ARTICLE 5. Every Fellow resident within fifty miles of Boston hereafter elected shall pay an Admission Fee of Ten dollars.

Every Fellow resident within fifty miles of Boston shall, and others may, pay such Annual Dues, not exceeding Fifteen dollars, as shall be voted by the Academy at each Annual Meeting, when they shall become due; but any Fellow or Resident Associate shall be exempt from the annual payment if, at any time after his admission, he shall pay into the treasury Two hundred dollars in addition to his previous payments.

All Commutations of the Annual Dues shall be and remain permanently funded, the interest only to be used for current expenses.

Any Fellow not previously subject to Annual Dues who takes up his residence within fifty miles of Boston, shall pay to the Treasurer within three months thereafter Annual Dues for the current year, failing which his Fellowship shall cease; but the Council may suspend the provisions of this Article for a reasonable time.

Only Fellows who pay Annual Dues or have commuted them may hold office in the Academy or serve on the Standing Committees or vote at meetings.

ARTICLE 6. Fellows who pay or have commuted the Annual Dues and Foreign Honorary Members shall be entitled to receive gratis one copy of all Publications of the Academy issued after their election.

See Chap. x, art. 2.

ARTICLE 7. Diplomas signed by the President and the Vice-President of the Class to which the member belongs, and countersigned by the Secretaries, shall be given to Foreign Honorary Members and to Fellows on request.

ARTICLE 8. If, in the opinion of a majority of the entire Council, any Fellow or Foreign Honorary Member or Resident Associate shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his membership; and if three fourths of the Fellows present, out of a total attendance of not less than fifty at a Stated Meeting, or at a Special Meeting called for the purpose, shall adopt this recommendation, his name shall be stricken from the Roll.

See Chap. iii.; chap. vi. art. 1; chap. ix, art. 1, 7; chap. x. art. 2.

CHAPTER III

ELECTION OF FELLOWS AND FOREIGN HONORARY MEMBERS

ARTICLE 1. Elections of Fellows and Foreign Honorary Members shall be made by the Council in April of each year, and announced at the Annual Meeting in May.

ARTICLE 2. Nominations to Fellowship or Foreign Honorary Membership in any Section must be signed by two Fellows of that Section or by three voting Fellows of any Sections; but in any one year no Fellow may nominate more than four persons. These nominations, with statements of qualifications and brief biographical data, shall be sent to the Corresponding Secretary.

All nominations thus received prior to February 15 shall be forthwith sent in printed form to every Fellow, with the names of the proposers in each case and a brief account of each nominee, and with the request that the list be returned before March 15, marked to indicate preferences of the voter in such manner as the Council may direct.

All the nominations, with any comments thereon and with the results of the preferential indications of the Fellows, received by March 15, shall be referred at once to the appropriate Class Committees, which shall report their decisions to the Council, which shall thereupon have power to elect.

Persons nominated in any year, but not elected, may be placed on the preferential ballot of the next year at the discretion of the Council, but shall not further be continued on the list of nominees unless renominated.

Notice shall be sent to every Fellow, not later than the fifteenth of January, of each year, calling attention to the fact that the limit of time for sending nominations to the Corresponding Secretary will expire on the fifteenth of February.

See Chap. ii.; chap. vi. art. 1; chap. ix. art. 1.

CHAPTER IV

OFFICERS

ARTICLE 1. The Officers of the Academy shall be a President (who shall be Chairman of the Council), three Vice-Presidents (one from each Class), a Corresponding Secretary (who shall be Secretary of the Council), a Recording Secretary, a Treasurer, and a Librarian, all of whom shall be elected by ballot at the Annual Meeting, and shall hold their respective offices for one year, and until others are duly chosen and installed.

There shall be also twelve Councillors, one from each Section of each Class. At each Annual Meeting three Councillors, one from each Class, shall be elected by ballot to serve for the full term of four years and until others are duly chosen and installed. The same Fellow shall not be eligible for two successive terms.

The Councillors, with the other officers previously named, and the Chairman of the House Committee, *ex officio*, shall constitute the Council.

See Chap. x, art. 1.

Article 2. If any officer be unable, through death, absence, or disability, to fulfil the duties of his office, or if he shall resign, his place may be filled by the Council in its discretion for any part or the whole of the unexpired term.

Article 3. At the Stated Meeting in March, the President shall appoint a Nominating Committee of three Fellows having the right to vote, one from each Class. This Committee shall prepare a list of nominees for the several offices to be filled, and for the Standing Committees, and file it with the Recording Secretary not later than four weeks before the Annual Meeting.

See Chap. vi. art. 2.

Article 4. Independent nominations for any office, if signed by at least twenty Fellows having the right to vote, and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call therefor, and shall be mailed to all the Fellows having the right to vote.

See Chap. vi. art. 2.

Article 5. The Recording Secretary shall prepare for use in voting at the Annual Meeting a ballot containing the names of all persons duly nominated for office.

CHAPTER V

The President

Article 1. The President, or in his absence the senior Vice-President present (seniority to be determined by length of continuous fellowship in the Academy), shall preside at all meetings of the Academy. In the absence of all these officers, a Chairman of the meeting shall be chosen by ballot.

ARTICLE 2. Unless otherwise ordered, all Committees which are not elected by ballot shall be appointed by the presiding officer.

ARTICLE 3. Any deed or writing to which the Corporate Seal is to be affixed, except leases of real estate, shall be executed in the name of the Academy by the President or, in the event of his death, absence, or inability, by one of the Vice-Presidents, when thereto duly authorized.

See Chap. ii. art. 7; chap. iv. art. 1, 3; chap. vi. art. 2; chap. vii. art. 1; chap. ix. art. 6; chap. x. art. 1, 2; chap. xi. art. 1.

CHAPTER VI

The Secretaries

ARTICLE 1. The Corresponding Secretary shall conduct the correspondence of the Academy and of the Council, recording or making an entry of all letters written in its name, and preserving for the files all official papers which may be received. At each meeting of the Council he shall present the communications addressed to the Academy which have been received since the previous meeting, and at the next meeting of the Academy he shall present such as the Council may determine.

He shall notify all persons who may be elected Fellows or Foreign Honorary Members, or Resident Associates, send to each a copy of the Statutes, and on their acceptance issue the proper Diploma. He shall also notify all meetings of the Council; and in case of the death, absence, or inability of the Recording Secretary he shall notify all meetings of the Academy.

Under the direction of the Council, he shall keep a List of the Fellows, Foreign Honorary Members, and Resident Associates, arranged in their several Classes and Sections. It shall be printed annually and issued as of the first day of July.

See Chap. ii. art. 7; chap. iii. art. 2, 3; chap. iv. art. 1; chap. ix. art. 6; chap. x. art. 1; chap. xi. art. 1.

ARTICLE 2. The Recording Secretary shall have the custody of the Charter, Corporate Seal, Archives, Statute-Book, Journals, and all literary papers belonging to the Academy.

Fellows or Resident Associates borrowing such papers or documents shall receipt for them to their custodian.

The Recording Secretary shall attend the meetings of the Academy and keep a faithful record of the proceedings with the names of the Fellows and Resident Associates present; and after each meeting is duly opened, he shall read the record of the preceding meeting.

He shall notify the meetings of the Academy to each Fellow and Resident Associate by mail at least seven days beforehand, and in his discretion may also cause the meetings to be advertised; he shall apprise Officers and Committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy.

After all elections, he shall insert in the Records the names of the Fellows by whom the successful nominees were proposed.

He shall send the Report of the Nominating Committee in print to every Fellow having the right to vote at least three weeks before the Annual Meeting.

See Chap. iv. art. 3.

In the absence of the President and of the Vice-Presidents he shall, if present, call the meeting to order, and preside until a Chairman is chosen.

See Chap. i.; chap. ii. art. 7; chap. iv. art. 3, 4, 5; chap. ix. art. 6; chap. x. art. 1, 2; chap. xi. art. 1, 3.

Article 3. The Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them likely to promote its interests.

CHAPTER VII

The Treasurer and the Treasury

Article 1. The Treasurer shall collect all money due or payable to the Academy, and all gifts and bequests made to it. He shall pay all bills due by the Academy, when approved by the proper officers, except those of the Treasurer's office, which may be paid without such approval; in the name of the Academy he shall sign all leases of real estate; and, with the written consent of a member of the Committee on Finance, he shall make all transfers of stocks, bonds, and other securities belonging to the Academy, all of which shall be in his official custody.

He shall keep a faithful account of all receipts and expenditures, submit his accounts annually to the Auditing Committee, and render them at the expiration of his term of office, or whenever required to do so by the Academy or the Council.

He shall keep separate accounts of the income of the Rumford Fund, and of all other special Funds, and of the appropriation thereof, and render them annually.

His accounts shall always be open to the inspection of the Council.

Article 2. He shall report annually to the Council at its March meeting on the expected income of the various Funds and from all other sources during the ensuing financial year. He shall also report the names of all Fellows and Resident Associates who may be then delinquent in the payment of their Annual Dues.

Article 3. He shall give such security for the trust reposed in him as the Academy may require.

Article 4. With the approval of a majority of the Committee on Finance, he may appoint an Assistant Treasurer to perform his duties, for whose acts, as such assistant, he shall be responsible; or, with like approval and responsibility, he may employ any Trust Company doing business in Boston as his agent for the same purpose, the compensation of such Assistant Treasurer or agent to be fixed by the Committee on Finance and paid from the funds of the Academy.

Article 5. At the Annual Meeting he shall report in print all his official doings for the preceding year, stating the amount and condition of all the property of the Academy entrusted to him, and the character of the investments.

Article 6. The Financial Year of the Academy shall begin with the first day of April.

Article 7. No person or committee shall incur any debt or liability in the name of the Academy, unless in accordance with a previous vote and appropriation therefor by the Academy or the Council, or sell or otherwise dispose of any property of the Academy, except cash or invested funds, without the previous consent and approval of the Council.

See Chap. ii. art. 4, 5; chap. vi. art. 2; chap. ix. art. 6; chap. x. art. 1, 2, 3; chap. xi. art. 1.

CHAPTER VIII

The Librarian and the Library.

Article 1. The Librarian shall have charge of the printed books, keep a correct catalogue thereof, and provide for their delivery from the Library.

At the Annual Meeting, as Chairman of the Committee on the Library, he shall make a Report on its condition.

Article 2. In conjunction with the Committee on the Library he shall have authority to expend such sums as may be appropriated by the Academy for the purchase of books, periodicals, etc., and for defraying other necessary expenses connected with the Library.

Article 3. All books procured from the income of the Rumford Fund or of other special Funds shall contain a book-plate expressing the fact.

Article 4. Books taken from the Library shall be receipted for to the Librarian or his assistant.

Article 5. Books shall be returned in good order, regard being had to necessary wear with good usage. If any book shall be lost or injured, the Fellow or Resident Associate to whom it stands charged shall replace it by a new volume or by a new set, if it belongs to a set, or pay the current price thereof to the Librarian, whereupon the remainder of the set, if any, shall be delivered to the Fellow or Resident Associate so paying, unless such remainder be valuable by reason of association.

Article 6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

Article 7. The Librarian shall have the custody of the Publications of the Academy. With the advice and consent of the President, he may effect exchanges with other associations.

See Chap. ii. art. 6: chap. x. art. 1, 2.

CHAPTER IX

The Council

Article 1. The Council shall exercise a discreet supervision over all nominations and elections to membership, and in general supervise all the affairs of the Academy not explicitly reserved to the Academy as a whole or entrusted by it or by the Statutes to standing or special committees.

It shall consider all nominations duly sent to it by any Class Committee, and act upon them in accordance with the provisions of Chapter III.

With the consent of the Fellow interested, it shall have power to make transfers between the several Sections, reporting its action to the Academy.

See Chap. iii. art. 2, 3; chap. x. art. 1.

Article 2. Seven members shall constitute a quorum.

Article 3. It shall establish rules and regulations for the transaction of its business, and provide all printed and engraved blanks and books of record.

Article 4. It shall act upon all resignations of officers, and all resignations and forfeitures of Fellowship or Resident Associateship; and cause the Statutes to be faithfully executed.

It shall appoint all agents and subordinates not otherwise provided for by the Statutes, prescribe their duties, and fix their compensation. They shall hold their respective positions during the pleasure of the Council.

Article 5. It may appoint, for terms not exceeding one year, and prescribe the functions of, such committees of its number, or of the Fellows of the Academy, as it may deem expedient, to facilitate the administration of the affairs of the Academy or to promote its interests.

Article 6. At its March meeting it shall receive reports from the President, the Secretaries, the Treasurer, and the Standing Committees, on the appropriations severally needed for the ensuing financial year. At the same meeting the Treasurer shall report on the expected income of the various Funds and from all other sources during the same year.

A report from the Council shall be submitted to the Academy, for action, at the March meeting, recommending the appropriation which in the opinion of the Council should be made.

On the recommendation of the Council, special appropriations may be made at any Stated Meeting of the Academy, or at a Special Meeting called for the purpose.

See Chap. x. art. 3.

Article 7. After the death of a Fellow or Foreign Honorary Member, it shall appoint a member of the Academy to prepare a biographical notice for publication in the Proceedings.

Article 8. It shall report at every meeting of the Academy such business as it may deem advisable to present.

See Chap. ii art. 4, 5, 8; chap. iv. art. 1, 2; chap. vi. art. 1; chap. vii. art. 1; chap. xi. art. 1, 4.

CHAPTER X

Standing Committees

Article 1. The Class Committee of each Class shall consist of the Vice-President, who shall be chairman, and the four Councillors of the Class, together with such other officer or officers annually elected as may belong to the Class. It shall consider nominations to Fellowship in its own Class, and report in writing to the Council such as may receive at a Class Committee Meeting a majority of the votes cast, provided at least three shall have been in the affirmative.

See Chap. iii. art. 2.

Article 2. At the Annual Meeting the following Standing Committees shall be elected by ballot to serve for the ensuing year:

(i) *The Committee on Finance*, to consist of three Fellows, who, through the Treasurer, shall have full control and management of the funds and trusts of the Academy, with the power of investing the funds and of changing the investments thereof in their discretion.

See Chap. iv. art. 3; chap. vii. art. 1, 4; chap. ix. art. 6.

(ii) *The Rumford Committee*, to consist of seven Fellows, who shall report to the Academy on all applications and claims for the

Rumford Premium. It alone shall authorize the purchase of books, publications and apparatus at the charge of the income from the Rumford Fund, and generally shall see to the proper execution of the trust.

See Chap. iv. art. 3; chap, ix. art. 6.

(iii) *The Cyrus Moors Warren Committee,* to consist of seven Fellows, who shall consider all applications for appropriations from the income of the Cyrus Moors Warren Fund, and generally shall see to the proper execution of the trust.

See Chap. iv. art. 3; chap. ix. art. 6.

(iv) *The Committee of Publication,* to consist of three Fellows, one from each Class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Proceedings and the Memoirs shall be entrusted.

It shall fix the price at which the Publications shall be sold; but Fellows may be supplied at half price with volumes which may be needed to complete their sets, but which they are not entitled to receive gratis.

Two hundred extra copies of each paper accepted for publication in the Proceedings or the Memoirs shall be placed at the disposal of the author without charge.

See Chap. iv. art. 3; chap. vi. art. 1, 3; chap. ix. art. 6.

(v) *The Committee on the Library,* to consist of the Librarian, *ex officio,* as Chairman, and three other Fellows, one from each Class, who shall examine the Library and make an annual report on its condition and management.

See Chap. iv. art. 3; chap. viii. art. 1, 2; chap. ix. art. 6.

(vi) *The House Committee,* to consist of three Fellows, who shall have charge of all expenses connected with the House, including the general expenses of the Academy not specifically assigned to the care of other Committees or Officers.

See Chap. iv. art. 1, 3; chap. ix. art. 6.

(vii) *The Committee on Meetings,* to consist of the President, the Recording Secretary, and three other Fellows, who shall have charge of plans for meetings of the Academy.

See Chap. iv. art. 3; chap. ix. art. 6.

(viii) *The Auditing Committee,* to consist of two Fellows, who shall audit the accounts of the Treasurer, with power to employ an expert and to approve his bill.

See Chap. iv. art. 3; chap. vii. art. 1; chap. ix. art. 6.

ARTICLE 3. The Standing Committees shall report annually to the Council in March on the appropriations severally needed for the ensuing financial year; and all bills incurred on account of these Committees, within the limits of the several appropriations made by the Academy, shall be approved by their respective Chairmen.

In the absence of the Chairman of any Committee, bills may be approved by any member of the Committee whom he shall designate for the purpose.

See Chap. vii. art. 1, 7; chap. ix. art. 6.

CHAPTER XI

MEETINGS, COMMUNICATIONS, AND AMENDMENTS

ARTICLE 1. There shall be annually eight Stated Meetings of the Academy, namely, on the second Wednesday of October, November, December, January, February, March, April and May. Only at these meetings, or at adjournments thereof regularly notified, or at Special Meetings called for the purpose, shall appropriations of money be made or amendments of the Statutes or Standing Votes be effected.

The Stated Meeting in May shall be the Annual Meeting of the Corporation.

Special Meetings shall be called by either of the Secretaries at the request of the President, of a Vice-President, of the Council, or of ten Fellows having the right to vote; and notifications thereof shall state the purpose for which the meeting is called.

A meeting for receiving and discussing literary or scientific communications may be held on the fourth Wednesday of each month, excepting July, August, and September; but no business shall be transacted at said meetings.

ARTICLE 2. Twenty Fellows having the right to vote shall constitute a quorum for the transaction of business at Stated or Special

Meetings. Fifteen Fellows shall be sufficient to constitute a meeting for literary or scientific communications and discussions.

ARTICLE 3. Upon the request of the presiding officer or the Recording Secretary, any motion or resolution offered at any meeting shall be submitted in writing.

ARTICLE 4. No report of any paper presented at a meeting of the Academy shall be published by any Fellow or Resident Associate without the consent of the author; and no report shall in any case be published by any Fellow or Resident Associate in a newspaper as an account of the proceedings of the Academy without the previous consent and approval of the Council. The Council, in its discretion, by a duly recorded vote, may delegate its authority in this regard to one or more of its members.

ARTICLE 5. No Fellow or Resident Associate shall introduce a guest at any meeting of the Academy until after the business has been transacted, and especially until after the result of the balloting upon nominations has been declared.

ARTICLE 6. The Academy shall not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its Publications.

ARTICLE 7. All proposed Amendments of the Statutes shall be referred to a committee, and on its report, at a subsequent Stated Meeting or at a Special Meeting called for the purpose, two thirds of the ballot cast, and not less than twenty, must be affirmative to effect enactment.

ARTICLE 8. Standing Votes may be passed, amended, or rescinded at a Stated Meeting, or at a Special Meeting called for the purpose, by a vote of two thirds of the members present. They may be suspended by a unanimous vote.

See Chap. ii. art. 5, 8; chap. iii. chap. iv. art. 3, 4, 5; chap. v. art. 1; chap. vi. art. 1, 2; chap. ix. art. 8.

STANDING VOTES

1. Communications of which notice has been given to either of the Secretaries shall take precedence of those not so notified.

2. Fellows or Resident Associates may take from the Library six volumes at any one time, and may retain them for three months, and no longer. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

3. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy without the leave of the Librarian.

4. There may be chosen by the Academy, under such rules as the Council may determine, one hundred Resident Associates. Not more than forty Resident Associates shall be chosen in any one Class.

Resident Associates shall be entitled to the same privileges as Fellows, in the use of the Academy building, may attend meetings and present papers, but they shall not have the right to vote. They shall pay no Admission Fee, and their Annual Dues shall be the same as those of Fellows residing within fifty miles of Boston.

The Council and Committees of the Academy may ask one or more Resident Associates to act with them in an advisory or assistant capacity.

5. Communications offered for publication in the Proceedings or Memoirs of the Academy shall not be accepted for publication before the author shall have informed the Committee on Meetings of his readiness, either himself or through some agent, to use such time as the Committee may assign him at such meeting as may be convenient both to him and to the Committee, for the purpose of presenting to the Academy a general statement of the nature and significance of the results contained in his communication.

RUMFORD PREMIUM

In conformity with the terms of the gift of Sir Benjamin Thompson, Count Rumford, of a certain Fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court of Massachusetts for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of the Rumford Fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a Premium to the author of any important discovery or useful improvement in light or heat, which shall have been made and pubished by printing, or in any way made known to the public, in any part of the continent of America, or any of the American Islands; preference always being given to such discoveries as, in the opinion of the Academy, shall tend most to promote the good of mankind; and, if the Academy sees fit, to add to such medals, as a further Premium for such discovery and improvement, a sum of money not exceeding three hundred dollars.

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